IB Mathematics: Design, Content, and Rigor for All IB Students

Fact Sheet for Universities
In 2019, the IB launched two mathematics subjects for students ages 16-19: **Mathematics: applications and interpretation (MAI)** and **Mathematics: analysis and approaches (MAA)**, which replaced the previous mathematics subjects. They support what students need to know, what they can do, and how they think about mathematics to be successful at university, in their chosen career and throughout life.

The IB believes that all students should have access to a mathematics course that best supports their future aspirations and successful transition to higher education. The two IB mathematics subjects are offered at standard level (SL) and higher level (HL), giving students the option of four math courses: MAI SL, MAI HL, MAA SL, MAA HL.

Both subjects are designed to meet the diverse needs, interests and motivations of all Diploma Programme (DP) and Career-related Programme (CP) students. Building on that foundation, the IB has designed each one with the different aspirations of our student community in mind; those who wish to study mathematics as a subject in its own right or to pursue their interests in areas related to mathematics, and those who wish to gain understanding and competence in how mathematics relates to the real world and to other subjects.

Your university recognition policy should include the most appropriate DP mathematics course for each of your programs of study offered.

---

The IB believes that both SL courses prepare students to transition to most arts, social science, life science and medicine university degree programs. The additional engagement in both HL courses prepares students to transition to university degree programs that require substantial mathematics knowledge and skills.
The Courses: Design and Content

The courses are designed to encourage teachers and students to appreciate the international dimensions of mathematics and the multiplicity of its cultural and historical perspectives by integrating theory of knowledge, international-mindedness and other IB values into their mathematical studies. Most importantly, IB mathematics courses focus on critical thinking skills because as technology advances, students must be able to:

- recognize situations in which mathematics can be applied to drive solutions
- understand and synthesise technical documents
- apply relevant mathematical approaches to familiar and unfamiliar situations
- structure logical arguments
- quantify risk
- understand that technology and mathematics can go hand-in-hand, and
- interpret the meaning and relevance of solutions.

IB Course Design

The courses are separated by how they approach mathematics, described generally below:

Mathematics: analysis and approaches

- Emphasis on algebraic methods
- Develops strong skills in mathematical thinking
- Real and abstract mathematical problem solving

Mathematics: applications and interpretation

- Emphasis on modelling and statistics
- Develops strong skills in applying mathematics to the real world
- Real mathematical problem-solving using technology.

Figure 1 shows the number of teaching hours for each of the four courses (MAA SL, HL and MAI SL, HL).
The 30 hours of inquiry, investigation, modelling and problem solving are an integral part of each course; its assessment is compulsory for both SL and HL students. It enables students to apply their skills and knowledge and pursue their personal interests without the time limitations and other constraints that are associated with written examinations. The additional 90 hours of content in the HL courses is meant to deepen student understanding and expose them to more complex concepts.

All four courses cover the same five topics within mathematics, but with varying emphasis in each area:

- number and algebra
- functions
- geometry and trigonometry
- statistics and probability
- calculus.

Figure 2 provides an intuitive view of the content in the four courses. To view the full graphics, use these individual mind maps.

**Figure 2: Mathematics mind map**

For more information, read the subject briefs:

- [Mathematics: Application and interpretation](#)
- [Mathematics: Analysis and approaches](#)

To access the subject guides and the sample exam materials (SEMS) visit our designated [mathematics recognition webpage](#).
An IB mathematics teacher’s perspective on MAI and MAA

MAI emphasises using mathematics in context and interpreting results. Many students do well in applying techniques to solve problems but sometimes cannot understand results they produced. The MAI courses require students to articulate their mathematical reasoning and the meaning behind their results.

This ability to interpret results relates to another significant difference between the MAI and MAA courses: learning through real-world contexts. MAI courses cover a wide range of real-world problems that students might encounter in their other DP subjects and beyond. These might include supply and demand curves in economics, phase shift and voltage in physics and the interpretation of statistical tests in biology or geography. Because the focus is set on interpreting results, students need to develop some knowledge and understanding of multiple and diverse contexts.

In MAI, the largest number of suggested teaching hours is allocated to Statistics and Probability. This is one striking difference from MAA: To succeed, students in MAI courses need to develop a strong ability for statistical thinking and drawing conclusions that are probable rather than certain.

Mechanics in IB DP Mathematics and IB DP Physics

At the IB, some aspects of mechanics are covered in the Mathematics: analysis and approaches course and almost all are covered in the Mathematics: applications and interpretation course. If a student does either mathematics subject at HL along with the IB physics course, they will have a good foundation for the kinematics section of a mechanics course and the necessary background in vectors and calculus to meet the necessary mechanics university entry requirements. Please refer universities to the DP physics subject brief when these questions arise.

“MAI HL, with its technological and statistical approaches, fulfils a need for students for whom mathematics will be a necessary…tool in their further studies. These students will start their university courses more equipped both in terms of reasoning skills and more practical skills.”

– Peter Gray, mathematics teacher at Munich International School
Assessment

Both subjects have similar assessment models, which gives schools and students a common starting point regardless of which mathematics courses are offered.

- The external assessment for both subjects comprises the remaining 80% of a student’s overall score.
- The courses share a common internal assessment, known as “the exploration.” This makes up 20% of the final mark; the same assessment criteria is used for both levels.
- Students are asked to answer a similar number of long and short questions.

In addition, both subjects have the same prior learning requirements, share some common content, and share the same assessment objectives (knowledge and understanding, problem solving, communication and interpretation, technology, reasoning, and inquiry approaches).

Both MAA and MAI subject teams write and standardize exam questions collaboratively to ensure that one subject level does not vary from the corresponding level in the other subject. The document “Examimation questions” includes a variety of questions from past examinations. MAI questions lean towards real-world problem solving. MAA questions are generally more theoretical, although some contextual questions are also included.

Course Rigor

To determine if there is a measurable difference between the outcomes achieved by the candidates and if that difference is statistically significant, the IB applied two data analysis approaches using the May 2021 and May 2022 candidate results: regression analysis and subject pairs analysis.

Both approaches show that, statistically, there is little difference between student achievement in the two mathematics courses at higher level and the two mathematics courses at standard level based on comparative performance of students across subjects. Students who achieve high grades in their other five DP subjects are likely to achieve similar grades in whichever mathematics course they choose to take.
Regression analysis compared each candidate’s performance in mathematics to their average performance in their other five DP subjects. The regression was run for both subjects at the SL and HL levels, and both models were statistically significant; the results showing that there is very little difference between the two subjects at both levels. Therefore, the data suggests that students who averaged a certain grade in their other five subjects would get very similar grades in either of the DP mathematics subjects at the same level. Figures 3 and 4 illustrate this. To learn more about the methods and get more details on results, access Appendix A.

Subject pairs analysis follows each assessment session to give a general idea of how candidates are faring in a particular subject compared to their other subjects. It identifies any anomalies, such as a change in the general pattern of stability. Data from May 2021 and May 2022 shows that in each year there is less than 1/10th of a grade difference between the two DP mathematics SLs and the two DP mathematics HLs.

Both analyses show that, although there are differences in subject content and focus, there is no measurable difference between the outcomes achieved by the candidates. Students are supported at the school level to choose the DP mathematics course that will better help them transition into their chosen university program. Therefore, the IB encourages universities to reflect on what mathematics content and skills are necessary for transition to each degree discipline.

Figure 3: DP Mathematics SL regression analysis

Figure 4: DP mathematics HL regression analysis
Setting Recognition Policies for IB DP Mathematics

IB mathematics courses support students’ varied interests, enthusiasms and future career plans. Not all students need to have deep theoretical math knowledge to be successful in their university studies. For example, a student who wishes to pursue a journalism career will benefit from an increased focus on statistics and storytelling. Thus, the IB created more choice without sacrificing the mathematical problem solving, reasoning and critical thinking skills we value in mathematics. This means that when writing your university’s recognition policy for DP mathematics you can take a holistic approach, confident in the mathematical skills learned in both subjects.

“Part of the excitement of designing these two new DP mathematics subjects was ensuring that within each subject and at each level the content was fit for purpose, and tailored to the modern student, their career aspirations and their university needs.

– Deborah Sutch, IB Mathematics Curriculum Manager

Consider these questions as you write or revise your policy:

• How does the content compare to the skills necessary to complete courses of study at your university?
• What level (SL or HL) of mathematics will contribute to student success in your university’s course offerings?
• What is the appropriate mathematics knowledge students need to know to be successful in your different degree disciplines? For example, does your psychology course require a theoretical mathematics course to be successful or would a more applied mathematics course be better? Or does your engineering course demand theoretical mathematics knowledge and foundation with the extra 90 hours deep dive that MAA HL provides?
• In what way can your university’s recognition policy reflect and support the diversity of your students and their future career paths?
Ultimately, your university recognition policy should include the most appropriate IB mathematics course for each of the different programs of study on offer. In this way, students not only have their choice of focus, but also the appropriate depth and breadth of knowledge to succeed in their chosen program of study. When that happens, mathematics faculty will enjoy teaching students who are confident and ready to engage with the right mathematics content at your university.

Examples of recognition policies:

University of Oxford
“All Oxford courses that state that they require Mathematics will accept both courses at HL, apart from Chemistry, which will require applicants to have taken either of the courses at Higher Level, or the Analysis and Approaches course at SL, depending on what other subjects they are taking.”

University of Toronto
“The following IB Math courses will satisfy the Calculus and Advanced Functions prerequisites for science and business programs: Mathematics: Applications and Interpretations HL; Mathematics: Analysis and Approaches SL; and Mathematics: Analysis and Approaches HL. Mathematics: Applications and Interpretations SL will satisfy the Advanced Functions prerequisite alone.”
Mathematics: applications and interpretations SL

The regression analysis compares each candidate’s performance in mathematics to their average performance in their other five DP subjects. The data for this analysis is taken from the May 2022 examination session.

FOR SL:

Summary statistics generated from this SL data are:

<table>
<thead>
<tr>
<th>AVG_OTHER</th>
<th>Mathematics: analysis and approaches</th>
<th>Mathematics: applications and interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

The coefficients in the first column above give the variables of the regression equation. An illustrative example of how this can be used:

A student scores 5 on average in all their other subjects excluding mathematics, this gives

\[
5 \times 1.040182042 + -0.360431027 = 4.840 \text{ for Mathematics: analysis and approaches}
\]

\[
5 \times 1.040182042 + -0.360431027 + -0.092861955 = 4.748 \text{ for Mathematics: applications and interpretation}
\]

The result from this model is statistically significant and we can be confident that the difference between the two DP mathematics subjects at SL is very small (0.09 of a grade), reflected in the graph on page 7.
FOR HL:

Summary statistics generated from this HL data are:

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>2</td>
<td>25488.59663</td>
<td>12719.5</td>
<td>17422.77</td>
<td>0</td>
</tr>
<tr>
<td>Residual</td>
<td>22607</td>
<td>16504.24282</td>
<td>0.73005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>22609</td>
<td>41992.8395</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The coefficients in the first column above give the variables of the regression equation. An illustrative example of how this is used:

A student scores 5 on average in all their other subjects excluding mathematics, this gives:

\[ 5 \times 1.225249154 + -1.672665952 + 0 = 4.454 \text{ for Mathematics: analysis and approaches} \]

\[ 5 \times 1.225249154 + -1.672665952 + -0.040245452 = 4.413 \text{ for Mathematics: applications and interpretation} \]

The result from this model is statistically significant and we can be confident that the difference between the two DP mathematics subjects at HL is very small (0.04 of a grade), reflected in the graph on page 7.

<table>
<thead>
<tr>
<th>AVG_OTHER</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics: analysis and approaches</td>
<td>6.904</td>
<td>5.679</td>
<td>4.454</td>
<td>3.228</td>
<td>2.003</td>
</tr>
<tr>
<td>Mathematics: applications and interpretation</td>
<td>6.864</td>
<td>5.639</td>
<td>4.413</td>
<td>3.188</td>
<td>1.963</td>
</tr>
</tbody>
</table>
Contact us

To discuss setting fair and equitable recognition policies for your various programs, contact recognition@ibo.org.