



IB MATHEMATICS COMPARABILITY STUDY: CURRICULUM & ASSESSMENT COMPARISON

REPORT FOR IB GLOBAL RECOGNITION, September 2015

Prepared by:

Liz Bergeron, Ph.D.

University of Wisconsin-La Crosse

The summary is based on full reports by Adriana Alcántara and UK NARIC.

Adriana Alcántara, Ph.D.
International Education
Consultant

UK NARIC
The National Recognition Information Centre,
United Kingdom

Introduction

Purpose and Scope of the IB Mathematics Comparability Study

This report summarizes two independently written reports created as a part of the IB Mathematics Comparability Study. The purpose of the IB Mathematics Comparability Study is “to review and compare a selection of well-established mathematics qualifications offered in a range of high-performing education systems around the world whose primary function is to prepare senior secondary students for higher education”.¹ In this study, IB’s four Diploma Program mathematics courses – Mathematical Studies SL, Mathematics SL, Mathematics HL and Further Mathematics HL – are compared with five mathematics qualifications chosen on the basis of their market share, the extent of their university recognition, geographical spread and their performance on PISA tests (where relevant):²

- 1) Alberta Diploma (Canada): *Alberta Math 30-1, Alberta Math 30-2, Alberta 31*
- 2) Advanced Placement (AP) (US and Canada, growing overseas market): *AP Calculus AB, AP Calculus BC*
- 3) GCE A Levels (England and overseas market): *A Level Mathematics and A Level Further Mathematics*
- 4) Singapore-Cambridge GCE A Levels (SIPCAL) (Singapore): *Singapore H1 Math, Singapore H2 Math, Singapore Math H3*
- 5) Gāokǎo (China): *Gāokǎo*

This study is composed of two distinct components: the Curriculum Comparison, which involves a compilation, analysis and comparison of the attributes of these qualifications, and the Assessment Comparison, which involves a comparative analysis of the assessment objectives, assessment methods, and marking guidelines. Dr. Adriana Alcántara performed the curriculum analysis and the National Recognition Information Centre in the United Kingdom (UK NARIC) conducted the assessment comparison. It is important to note the term “curriculum” is used when referring to the content of the qualifications analyzed. Often studies of this nature use the term “standards”. Standards are generally considered the benchmark for judging student progress. Standards outline what students are expected to know at each level. Curriculum is generally understood to be more comprehensive. Curriculum usually refers to the collection of teaching materials used to facilitate student learning so that each student can reach or exceed the standards.

Theoretical framework

This work extends on two previous alignment projects involving the IB DP. In 2014 Dr. Faas explored the extent the IB DP aligns with national standards in selected regions in Germany and Switzerland³. Additionally, Dr. Conley performed an alignment study between the IB DP and the Knowledge and Skills for University Success (KSUS). Faas’ research compares the DP written curriculum to selected German and Swiss regional curriculums with regard to content, cognitive demand and philosophical underpinnings. The findings suggest that while there is a high level of content alignment between the DP and Swiss and the DP and German curriculums in the natural sciences (biology and mathematics), there is less alignment in the social sciences

¹ IB Mathematics Comparability Study Project Plan, 29/01/2015.

² The focus of this study is on the qualifications themselves, not on the national or state systems that sustain them (in the relevant cases).

³ Faas, D, & Friesenhahn, I. (2014a). *Curriculum Alignment between the IB DP and national systems: Germany*. Bethesda, MD, USA. International Baccalaureate Organization; Faas, D, & Friesenhahn, I. (2014b). *Curriculum Alignment between the IB DP and national systems: Switzerland*. Bethesda, MD, USA. International Baccalaureate Organization.

and humanities (history and Spanish). The DP also appears to offer greater flexibility than the Swiss and German curriculums, allowing students to pursue their academic interests.

In 2009 Conley and Ward⁴ analyzed the alignment of the IB DP standards and the Knowledge and Skills for University Success (KSUS) and found IB standards to be highly aligned with the KSUS standards. The study also confirmed complete alignment between the IB Diploma's mathematical studies and the KSUS' algebra, trigonometry and statistics standards.

Methodology

There are numerous methodologies for investigating curriculum comparisons⁵. Researchers often rely on methods like the Webb Model⁶ or the Achieve Model⁷ which convene a panel of judges to score intended curriculum using qualitative and quantitative indicators. This can be difficult to apply to the international comparisons required by this study because consistent information is not available for all the courses examined in this research. It is difficult to gather exactly the same categories of data, in the same degree of detail, from a wide range of international educational qualifications, due to language barriers, different rules about the public accessibility and confidentiality of data, or simply different approaches to laying out curricular requirements. Therefore, it is not feasible to establish the uniform scoring categories required by the Webb Model or Achieve Model. To encourage an objective analysis two independent researchers were commissioned to carry out the curriculum and assessment comparisons. Similar to the Webb Model and the Achieve Model, both researchers utilized the intended curriculum in their analyses. The curriculum comparison was performed by Dr. Adriana Alcántara. Data was gathered mainly from primary sources, consisting of the most recent guidance documents and course specifications or guides provided by the administering authorities of each qualification (available on their official websites). Secondary sources were also used to enrich the data, namely existing international studies that have compared various mathematics qualifications. One particularly useful source for this report is Ofqual's 2012 International Comparisons in Senior Secondary Assessment (hereafter referred to as ICOSSA), which provides a methodological model in some ways, and contains a large amount of detailed data on several of the qualifications included in this study. With regard to concerns over objectivity, the curriculum research highlights similarities and differences between other qualifications and IB qualifications in its analysis and conclusions, but IB qualifications were not used as a benchmark against which to compare the others. In order to establish an external framework for determining certain equivalencies between the qualifications, the textbooks and course outlines used by top global universities in science and engineering were referred to for defining objective categories of mathematics content.

The assessment comparison was carried out by UK NARIC. Again, because each qualification being compared had different details available it was difficult to quantitatively score the qualifications. Therefore, a qualitative comparison approach was utilized. To arrive at a point at which reliable comparisons could be drawn between the international awards; it was important to thoroughly explore each qualification and its mathematics

⁴ Conley, D., Ward, T. (2009). *International Baccalaureate Standards Development and Alignment Project*. Educational Policy Improvement Center, Eugene, Oregon.

⁵ Bloom, B.S., Madaus, G.F., & Hastings, J.T. (1981). *Evaluation to Improve Learning*, New York: McGraw-Hill.; Webb, N.L. (1999). *Alignment of Science and Mathematics Standards and Assessments in Four States*, Washington: Council of Chief State School Officers.; Faas, 2014

⁶ Webb, 1999

⁷ developed by Achieve, Inc. in 2000 (<http://www.achieve.org/>)

programme(s) in isolation, building up comprehensive and objective profiles. The awarding bodies or government websites were reviewed and any publically available documentation was collected. Using this information, or sources provided through the awarding bodies or governments, additional websites providing information pertaining to the international qualifications were reviewed with any relevant information collated. Where documentation was lacking, which was the case for most of the awards, an overall web search was completed for any remaining websites containing information related to the international qualifications. The main documents searched for included:

- Examination papers and mark schemes
- Student or teacher handbooks/guides
- Grade distribution data.

The comparative analysis involved comparing each of the IB DP Mathematics courses with the international counterparts considering factual information gathered, with the aim of highlighting similarities and differences. Some aspects of the comparative analysis were, by their very nature, more subjective. It is important however to highlight that UK NARIC, both in its capacity as a national agency and as a credential evaluation organisation rather than awarding body, undertook a fully independent and objective analysis.

Curriculum: Overview of Mathematics Similarities and Differences

Comprehensive individual profiles for each qualification are available in the full report. Similarities and differences in 1) structure 2) content, 3) cognitive demand, 4) philosophical underpinnings and 5) university recognition for the qualifications are discussed below.

In general, it is clear that the mathematics courses seek to provide a solid preparation for further study in higher education. Specifically, three courses (the IB DP Further Mathematics HL, A Level Further Mathematics and the Singapore Math H3) are intended for those who have a strong aptitude and enthusiasm for mathematics.

Structure

Three of the six qualifications analyzed are baccalaureate-style qualifications (the Alberta Diploma, Singapore-Cambridge A Level Curriculum, and the International Baccalaureate Diploma), whose purpose is to provide students with a holistic education that covers a variety of subjects from different areas. The Chinese Gāokǎo is a comprehensive examination that defines the content of the Senior Secondary curriculum; thus, it also requires students to follow a certain fixed combination of subjects. Advanced Placement and A Levels, in contrast, provide stand-alone qualifications in a variety of subjects that can be taken independently of each other. AP courses are currently offered in 34 subjects; they are stand-alone courses, which can be taken in any number or order, and are not part of any structured Diploma. Nonetheless, it is important to note that a certain number or combination of AP courses are sometimes incorporated by individual schools or school districts into a specific set of requirements for completing an academic track. The College Board itself has also created an AP “Capstone Diploma”, launched at some participating schools in 2014, which is awarded to students who obtain scores of 3 or higher on four AP courses and who also take two new courses, AP Seminar (focusing on analyzing and evaluating information and making sound arguments) and AP Research (producing and defending a scholarly academic thesis).⁸ This Diploma appears to be an attempt to emulate the

⁸ The College Board (2015). *Advances in AP*. Retrieved from <https://advancesinap.collegeboard.org/ap-capstone>.

International Baccalaureate’s Theory of Knowledge and Extended Essay components. Mathematics is a required component of only three of these qualifications: the Alberta Diploma, the Gāokǎo, and the International Baccalaureate Diploma.

Content

With the exception of the Gāokǎo, all other qualifications offer several different mathematics courses that correspond to different higher education pathways. The IB Diploma offers the largest number of pathways (4), providing the most options for students with different needs. Additionally, the IB Diploma offers course options across the spectrum meeting a variety of student needs. For analysis purposes the courses within each qualification were grouped by mathematical background based on the stated purpose and structure of each course. The 15 courses from the 6 qualifications were organized into 5 groups⁹: 1) pre-calculus courses, 2) broad or applied mathematical content, 3) calculus topics, 4) calculus courses equivalent to introductory university course, and 5) advanced courses that prepare students for highly mathematical disciplines (Table 1).

Table 1

Content of courses

Group	Courses	Content coverage and prerequisites	University Preparation
Pre-calculus courses	Alberta Math 30-2 Alberta Math 30-1 IB Math Studies SL	<ul style="list-style-type: none"> • Functions, trigonometry, and geometry • IB Math Studies SL includes an introduction to differential calculus, thus providing a more solid preparation to this group of students 	<ul style="list-style-type: none"> • These are all meant to prepare students for university programs that require less mathematical preparation
Broad or applied mathematical content	Singapore H1 Math IB Math SL	<ul style="list-style-type: none"> • Practical applications • Prerequisites are pre-calculus topics (although Singapore H1 does not require previous knowledge of trigonometry). 	<ul style="list-style-type: none"> • Provide students with a stronger basis for entering university programs that will require mathematics, even if this may be in a more applied form
Calculus topics	A Level Mathematics Singapore H2 Math IB Math HL	<ul style="list-style-type: none"> • Calculus topics • Aim to develop students’ analytical skills in mathematics 	<ul style="list-style-type: none"> • Prepare students for university programs requiring a solid base in mathematics.

⁹ in ascending order of mathematical background required

		<ul style="list-style-type: none"> • Prerequisites¹⁰ are pre-calculus, basic statistics, and probability • Both A Level Math and IB Math HL offer a wide range of topics in applied mathematics • IB Math HL has an individual exploration which encourages thinking about mathematical applications 	<ul style="list-style-type: none"> • Prepare students for entry into mathematics, engineering and science fields
Calculus courses equivalent to introductory university course	Alberta 31 AP Calculus AB AP Calculus BC	<ul style="list-style-type: none"> • Theoretical calculus topics and their applications • Calculus applications are an integral part of the core content of AP Calculus courses • Calculus applications are part of the options Alberta Math 31 offers 	<ul style="list-style-type: none"> • All three courses are equivalent to a first semester or first year introductory university course on the subject
Advanced courses that prepare students for highly mathematical disciplines	Singapore Math H3 A Level Further Math IB Further Math HL Gāokǎo	<ul style="list-style-type: none"> • Calculus, linear algebra, complex numbers and discrete mathematics • Only IB Further Math HL includes technology and inquiry skills as part of the desired cognitive understandings • Singapore Math H3 covers the fewest topics in an equally long time frame, indicating possibly a higher level of depth¹¹ • The Chinese Gāokǎo covers a very large number of topics, but the time required to cover them is not specified 	<ul style="list-style-type: none"> • Prepare students for university programs in highly mathematical disciplines • Prepare students for careers where math will be essential

¹⁰ Singapore H2 has a slightly higher level of pre-requisites than the other two, assuming that students will have some basic knowledge of calculus

¹¹ It could contain a series of sub-topics that are equivalent to the main topics outlined for the other two qualifications, but that was not evident in the review

Cognitive Demand

The cognitive demand required for each course was analyzed based on the course descriptions and available course materials in addition to examining the content. The results below list the courses from lowest to highest level of cognitive demand:

1. Alberta Math 30-2

- **Scope of content areas:** Does not cover trigonometry or calculus; in applied math, only covers basic probability
- **Breadth of study:** Covers the least number of topics in Algebra and Functions; does not require any other areas of Pure Math; no coverage of Applied Math except for one unit on probability

2. Alberta Math 30-1

- **Scope of content areas:** Does not cover calculus but does cover two basic areas of trigonometry
- **Breadth of study:** Like Alberta Math 30-2, covers the least number of topics in Algebra and Functions; does not require any other areas of Pure Math; no coverage of Applied Math except for one unit on probability

3. IB Math Studies SL

- **Scope of content areas:** Requires basic trigonometry topics (2 out of 5) and introduces differentiation in Calculus (2 out of 13 topics)
- **Breadth of study:** Covers a moderate number of topics in Algebra and Functions, some Geometry; requires two other areas of Pure Mathematics, and has a good coverage of Statistics topics (6 out of 8)

4. Singapore H1 Mathematics

- **Scope of content areas:** Does not cover trigonometry (expects some basic knowledge of this area as a prerequisite), but does cover 6 out of 13 calculus topics (including parametric functions)
- **Breadth of study:** Covers a moderate number of topics in Algebra and Functions, does not require any other areas of Pure Mathematics, but has a very complete coverage of Statistics topics (7 out of 8)

5. IB Mathematics SL

- **Scope of content areas:** Covers 4 out of 5 areas of trigonometry, and 8 out of 13 calculus topics (including applications to kinematics)
- **Breadth of study:** Covers all topics in Algebra and Functions; requires two additional areas of Pure Mathematics, and has a good coverage of Statistics topics (5 out of 8)

6. A Level Mathematics

- **Scope of content areas:** Covers all topics in trigonometry and 8 out of 13 calculus topics (including parametric functions and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions; includes study of parametric equations and curves in Geometry; requires 3 other areas of Pure Mathematics, has a good coverage of Statistics topics (6 out of 8); also includes Decision Mathematics and Mechanics

7. IB Mathematics HL

- **Scope of content areas:** Covers all topics in trigonometry; covers 11 out of 13 calculus topics, including infinite sequences and series (two of these topics are optional however; also does not include parametric functions)
- **Breadth of study:** Covers all topics in Algebra and Functions; includes study of parametric equations and curves in Geometry; requires 5 other areas of Pure Mathematics (1 as a prerequisite), and makes one more optional; covers all Statistics topics (8 out of 8, but 4 of these are optional); also provides Discrete Mathematics as an option

8. Singapore H2 Mathematics

- **Scope of content areas:** Does not cover trigonometry, but expects all trigonometry topics to have been covered as a prerequisite; covers 9 out of 13 calculus topics (including parametric functions and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions (some as prerequisites); includes study of parametric equations and curves in Geometry, requires 4 other areas of Pure Mathematics, has a quite complete coverage of Statistics topics (7 out of 8)

9. Singapore H3 Mathematics

- **Scope of content areas:** Does not cover trigonometry, but expects all trigonometry topics to have been covered as a prerequisite; covers 10 out of 13 calculus topics (including parametric functions and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions (some as prerequisites); includes study of parametric equations and curves in Geometry as a prerequisite; requires 5 other areas of Pure Mathematics (4 as prerequisites from H2); does not require any Applied Mathematics topics

10. A Level Further Math

- **Scope of content areas:** Does not cover trigonometry, but expects all trigonometry topics to have been covered as a prerequisite; covers 9 out of 13 calculus topics (including parametric functions, infinite sequences and series, and hyperbolic functions)
- **Breadth of study:** Covers all topics in Algebra and Functions (most as prerequisites); includes study of parametric equations and curves, as well as polar coordinates, in Geometry; requires 6 out of 8 other areas of Pure Mathematics; requires all 8 topics in Statistics, and provides 2 additional areas of Applied Mathematics as options – Decision Mathematics and Mechanics

11. IB Further Math HL

- **Scope of content areas:** Covers one new area in trigonometry, expects all other trigonometry topics to have been covered as a prerequisite; covers 12 out of 13 calculus topics (including applications to kinematics and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions (all as prerequisites); includes study of parametric equations and curves in Geometry; requires 6 out of 8 other areas of Pure Mathematics; requires all 8 topics in Statistics; requires one additional area of Applied Mathematics – Decision Mathematics

Philosophical underpinnings

All of the qualifications include “university preparation” as a part of the description. All of the qualifications except the Gāokǎo describe mathematics learning outcomes as 1) enjoy mathematics, 2) communicate mathematics, 3) apply mathematics across contents and to the real world, 4) incorporate technology with mathematics, and 5) build connections. The Alberta High School Diploma and the IB DP are unique in their inclusion of international or global dimensions. The IB specifically articulates one objective is to enable students to “appreciate the international dimension of mathematics”. The Alberta High School Diploma describes the diploma in general as “global and cultural understanding”, not specifically mathematics courses. The IB, the GCE A Levels and the Singapore-Cambridge GCE A Levels all describe attributes of a learner they hope to develop, specifically, students who take responsibility, are self-directed, concerned citizens, and confident.

Recognition at illustrative universities

At selected top global universities IB, AP, and A Level qualifications have the greatest level of recognition overall; the other qualifications are recognized at a more local level, but do not have explicit policies formulated for them in these top-ranked higher education institutions. Singapore A levels and Gāokǎo (China) are not included as they are tied to specific regions.

Table 2

University Recognition Policies

University	IB Courses	AP	GCE A LEVELS (ENGLAND)
MIT ¹²	A score of a 6 or 7 on the Mathematics HL exam will be given for 12 units of Calculus I. No credit for scores lower than 6.	Score of 4 or 5 on the BC Calculus receive 12 units of Calculus I. No credit for scores lower than 4.	For a grade of A*, A, or B in A-Level Mathematics, credit will be given for 12 units of Calculus I. For grades lower than B or grades in Further Mathematics, no credit is given
Stanford ¹³	Mathematics HL with a score of 5 or higher is eligible for 10 credits	AP Calculus BC with a score of 4 or 5 is eligible for 10. A score of 3 is eligible for 5 credits.	"A" levels may be awarded a maximum of 12 units for some of the subjects which receive College Board AP credit
CALTECH ¹⁴	Does not grant credit for any courses taken prior to enrollment.	Does not grant credit for any courses taken prior to enrollment.	Does not grant credit for any courses taken prior to enrollment.

¹² Massachusetts Institute of Technology (MIT) (2015). *MIT Admissions: About AP*. Retrieved from <http://mitadmissions.org/>.

¹³ Stanford University (2015). *AP Charts*. Retrieved from <http://studentaffairs.stanford.edu/registrar/students/ap-charts>.

¹⁴ California Institute of Technology (2015). *Admissions first year applicants*. Retrieved from <http://www.admissions.caltech.edu/content/how-apply-first-year-applicant>

Princeton ¹⁵	A score of 6 or 7 on an IB HL mathematics exam is considered equivalent to a 4 or 5 on an AP math exam	A score of 4 in AP Calculus BC equals placement in Math 104/Math 175; a score of 5 equals placement in Math 201. A score of 5 together with a score of 750 on the MSAT earns placement in Math 203	A grade of A or B on an A Level mathematics exam is considered equivalent to a 4 or 5 on an AP math exam
Cambridge ¹⁶	All colleges offering Computer Science, Economics, Engineering and Mathematics require a 6 or 7 in IB Mathematics HL	Successful applicants have normally achieved 5s in at least five Advanced Placement Tests in appropriate subjects	All colleges offering Computer Science with Mathematics, Economics, Engineering and Mathematics require an A* or A in A Level Mathematics

Assessment: Overview of Mathematics Similarities and Differences

All six qualifications examined assess students through one or more external written examinations, conducted under exam conditions. The Alberta Diploma and IB DP (with the exception of Further Mathematics HL) also include internal assessment. In the absence of documentation on internal assessment, the analysis focusses exclusively on external assessment. The most notable differences in assessment observed between the mathematics qualifications pertain to item types, marking approaches, and grading systems.

Items are the questions or statements used on exams to gauge student understanding of concepts. Items are considered either constructed responses or selected response. Constructed response requires the test taker to create responses, while selected response requires test takers to choose the best answer(s) from an established list. Common selected response item types are multiple choice, multiple select, matching, and true/false. Common constructed response item types include essay and short answer. Both Alberta Diploma courses (Mathematics 30-1, and 30-2), the AP, and to a lesser extent, the Gāokǎo, employ multiple choice questions which are not used at all within the IB DP mathematics exams. The IB DP and GCE A Level, by contrast, primarily use multi-part, free response questions. The benefits to using multi-part free response questions are numerous. This item type enables students to construct their responses instead of selecting from provided options. By constructing responses students are able to demonstrate their thinking and steps. The Gāokǎo and the AP also use free-response questions, whilst the Alberta Diploma has short numerical response questions. Table 3 displays each qualification with the item types utilized.

¹⁵ Princeton University (2015). *AP admissions*. Retrieved from <http://www.princeton.edu/pub/ap/table/>

¹⁶ Cambridge University (2015). *Entrance requirements*. Retrieved from <http://www.undergraduate.study.cam.ac.uk/applying/entrance-requirements/international-baccalaureate>

Table 3**Comparative review of item types**

Qualification	Courses/programme	Multiple choice	Short answer	Multi-part structured	Extended problem	Combination of multi-part structured and extended problem
IB DP	Mathematical Studies SL		✓	✓		
	Mathematics SL		✓	✓	✓	✓
	Mathematics HL		✓	✓	✓	✓
	Further Mathematics HL		✓	✓	✓	✓
Advanced Placement	Calculus AB	✓		✓		✓
	Calculus BC	✓		✓		
	Statistics	✓		✓		
GCE A level	A Level Mathematics		✓	✓	✓	✓
	A Level Further Mathematics		✓	✓	✓	✓
Gāokǎo	Arts Stream	✓	✓		✓	✓
	Science Stream	✓	✓		✓	✓
Alberta Diploma	Mathematics 30-1	✓	✓			
	Mathematics 30-2	✓	✓			

The second notable difference among qualifications is marking approaches (see Table 4). Two critical aspects of marking are 1) handling partial credit and 2) awarding credit for accurate methods (even with an incorrect final answer). By awarding marks for accurate methods the examination is valuing understanding of the mathematical process and mathematical thinking. Across all mathematics within the IB DP and GCE A Level, the mark schemes provide a significant level of guidance to examiners on how to apply marking principles to incomplete answers or alternative methods. Students are expected to show their work and mark schemes detail the number of marks associated with demonstrating an appropriate method. This approach ensures that students must demonstrate that they can apply the methods learned and the number of accuracy marks (that is, those available for a correct answer) are limited, and in some cases conditional on the method marks. Throughout the IB DP mathematics exams and A Levels students can obtain method marks for a correct or appropriate approach, independent of whether they reach the correct answer. This method is considered beneficial as it provides a more comprehensive record of student understanding¹⁷. By utilizing constructed response item types and the marking scheme allowing for credit for accurate methods the IB is able to provide a detailed account of student mathematical understanding and accuracy. In programmes where only accuracy marks are awarded a minor mathematical error in a student's calculations would have a greater impact on

¹⁷ Popham, W. J. (2003). *Test Better, Teach Better*. Alexandria, VA: ASCD.

their marks than is the case in the IB DP exams. In the case of IB exams the student can receive points for accurately solving the problem and/or demonstrating understanding of the correct approach to solving the problem. This combination of item types and marking schemes also yields more accurate results as students are less likely to guess a correct constructed response answer. Alternatively, in Gāokǎo and AP mathematics, a considerable proportion of the marks (50% in the case of the AP) are assigned to multiple choice questions, where no marks are available for method, thereby placing more weight on providing an accurate answer in order to receive marks. Furthermore, whilst the Gāokǎo includes free response questions (extended and multi-part questions), the mark schemes comprised model answers and a breakdown of marks per sub-question, but no differentiation of marks for method and accuracy, indicating that no part marks would be awarded. The use of multiple choice and numerical response questions in the Alberta Diploma external examinations, enabling machine-marking, also means that there is no facility for method marks and accordingly 50% of students' overall grade will rely on accuracy (with the other 50% coming from internal assessment).

Table 4

Marking approach

Qualification	Machine marked [Answer key]	Marking guidelines used
IB DP	X	✓
Advanced Placement	✓ for multiple choice questions	✓ for free response questions
GCE A Level	X	✓
Gāokǎo	✓ for multiple choice questions	✓ for free response questions
Alberta Diploma	✓	X

The final notable difference identified was grading schemes (Table 5). There was considerable variation in the number and range of distinct grades awarded for each qualification, and not all included the concept of a pass or fail. The IB DP mathematics courses allow for the most granular level of differentiation in student achievement by grade, which can be considered beneficial for facilitating identification of the highest levels of student achievement for competitive higher education admissions purposes. All of the qualifications include external assessment in the format of one or more unseen tests, taken in an exam setting. For the IB DP Further Mathematics HL, the GCE A Level, the AP courses, and the Gāokǎo this external assessment accounts for 100% of the student's final grade. The other IB DP mathematics courses all include internal assessment contributing toward 20% of the final grade and the Alberta Diploma included internal assessment (50%).

Table 5

Grading systems

	IB Diploma Programme Mathematics	GCE A level Mathematics	Advanced Placement Mathematics	Alberta High School Diploma Mathematics 30-1 / 30-2	SIPCAL		Gāokǎo Mathematics (Shanghai municipality)
					H1 and H2	H3	
Grade system used	Numeric	Letter	Numeric	Pass / Fail	Letter	Pass / Fail	Numeric
Total number of grades	7	7	5	3	7	4	Students are graded out of 150 for mathematics, 600 overall
Total number of pass grades	7	6	5	2	5	3	
Highest pass grade	7	A*	5	Standard of Excellence ¹⁸	A	Distinction	150, 600 overall
Lowest pass grade	1	E	1	Acceptable standard	E	Pass	N/A
Fail grades used?	<i>x</i>	✓	<i>x</i>	✓	✓	✓	<i>x</i>
Summary of all grades (in descending order, highest to lowest) Fail grades, where used, are denoted in red.	7 6 5 4 3 2 1	A* A B C D E <i>U</i>	5 4 3 2 1	Standard of Excellence (80-100%) Acceptable standard (50-79%) <i>Fail (below 49%)</i>	A B C D E S (<i>sub-pass</i>) ¹⁹ <i>Ungraded</i>	Distinction Merit Pass <i>Ungraded</i>	

¹⁸ Students are also given an overall percentage. The percentages corresponding to “Standard of Excellence” and “Acceptable Standard” are defined in the summary row.

¹⁹ Students who fail to pass a subject at the lowest pass grade (E) will be awarded an S (sub-pass) or Ungraded. These grades will not appear on the GCE A level certificate, but will however be included on the student’s result slip.

There were some clear differences in the breadth and depth of the external exam content assessed. Overall the AP exams were the most specialized, focusing on one area of mathematics (statistics or calculus); followed by the Alberta Diploma, with each course exam assessing three core areas. The IB DP exams all assess across the curriculum with a broader range of topics covered. The Gāokǎo assessed across a broad range of topics as well. The IB DP and GCE A Level use a skills-based approach to assessment objectives whilst the Alberta Diploma employs content based outcomes in conjunction with underpinning cognitive levels and mathematical processes which are skills-based. The IB DP higher level courses (Mathematics and Further Mathematics) assign a significant proportion of marks to questions considered to be of high demand, more so than many of the other exams reviewed in this study. The IB DP, GCE A Level, and Alberta Diploma had questions at a variety of levels, enabling those at the lower levels to achieve some marks across the low-demand questions whilst allowing for differentiation of the more able students by including some more demanding questions. In contrast, the Gāokǎo questions were all considered to be of at least medium demand-- this coupled with one of the shortest assessments in terms of duration, the use of extended problems, and the importance of accuracy to a student's overall marks presents a particularly demanding assessment.



IB MATHEMATICS COMPARABILITY STUDY: CURRICULUM COMPARISON

REPORT FOR IB GLOBAL RECOGNITION, JULY 2015

Adriana Alcántara, Ph.D.
International Education Consultant

TABLE OF CONTENTS

INTRODUCTION.....	p. 2
Purpose and Scope of the IB Mathematics Comparability Study.....	p. 2
Curriculum Comparison: Scope and Structure of This Report.....	p. 3
Models of Comparative Analysis, Methodology and Sources.....	p. 4
PART A: OVERVIEW OF QUALIFICATIONS: CONTEXT, PURPOSE, STRUCTURE, REFORMS AND RECOGNITION.....	p. 6
International Baccalaureate Diploma.....	p. 6
The Alberta High School Diploma.....	p. 10
Advanced Placement.....	p. 17
GCE A Levels (England).....	p. 23
Singapore-Cambridge GCE A Levels.....	p. 28
Gaokao (China).....	p. 33
Comparative Summary.....	p. 38
PART B: COMPARATIVE ANALYSIS OF CURRICULUM STRUCTURE AND CONTENT.....	p. 40
Comparison of Qualifications According to their Stated Purpose and Basic Structure.....	p. 41
Comparison of Qualifications According to an Objective Framework of Mathematics Content.....	p. 62
Categorization of Qualifications According to a Preliminary Assessment of Level of Demand.....	p. 65
ANNEXES	
Annex I- Comparison of Qualifications: Mathematical Content Areas.....	p. 1
Annex II - Qualifications by Breadth of Mathematical Content.....	p. 7
Annex III: Classifications: Pure Mathematics Topics, Calculus Topics, Applied Mathematics Topics.....	p. 14

INTRODUCTION

Purpose and Scope of the IB Mathematics Comparability Study

The International Baccalaureate Organization (IBO) has commissioned a Mathematics Comparability Study in response to discussions within the organization on the level of demand of IB Diploma Program mathematics courses, both compared to other IB DP courses and to the mathematics qualifications offered by other agencies and countries. The number of IB Diploma Program students who choose to pursue certain IB mathematics courses, the different pathways offered by each of these courses and the ways in which universities recognize them have also been topics of concern. At a broader level, mathematics courses have been in the spotlight internationally due to increasing global emphasis on fields of study in science, technology, engineering and mathematics (STEM). Education providers across the world have increasingly recognized that pre-university and university courses in mathematics must be strengthened in order to provide an adequate basis for successful completion of STEM programs that require more complex skill levels, and ultimately, meet the growing demand for graduates in these fields. This concern is coupled with a focus on global competitiveness: national education systems and independent qualification providers are participating in international comparative tests and studies in order to measure their performance against common global benchmarks and improve their courses accordingly.

The purpose of the IB Mathematics Comparability Study, as set out in the initial Project Plan, is “to review and compare a selection of well-established mathematics qualifications offered in a range of high-performing education systems around the world whose primary function is to prepare senior secondary students for higher education”.¹ In this study, IB’s four Diploma Program mathematics courses – Mathematical Studies SL, Mathematics SL, Mathematics HL and Further Mathematics HL – are compared with five mathematics qualifications chosen on the basis of their market share, the extent of their university recognition, geographical spread and their performance on PISA tests (where relevant):²

- 1) Alberta Diploma (Canada)
- 2) Advanced Placement (US and Canada, growing overseas market)
- 3) GCE A Levels (England and overseas market)
- 4) Singapore-Cambridge GCE A Levels (Singapore)
- 5) Gaokao (China)

¹ IB Mathematics Comparability Study Project Plan, 29/01/2015.

² The focus of this study is on the qualifications themselves, not on the national or state systems that sustain them (in the relevant cases).

This study focuses on the general academic mathematics qualifications offered by each of these providers that are oriented towards entry into 4-year universities; it does not include qualifications oriented towards vocational tracks or practical specializations.

This study is composed of two distinct components: the Curriculum Comparison, which involves a compilation, analysis and comparison of the attributes of these qualifications, and the Assessment Comparison, which involves a comparative analysis of question papers and mark schemes.

Curriculum Comparison: Scope and Structure of this Report

In accordance with the specifications of the Project Plan, the author has attempted to conduct an objective review of the attributes of the selected mathematics qualifications according to broad categories and present the findings concurrently rather than consecutively in order to facilitate comparative analysis.

The main aims of this Curriculum Comparison, in accordance with the aims set out in the IB Mathematics Comparability Study Project Plan, are to:

- 1) Provide consistent information on the broad context, purpose and structure of each of the qualifications, in order to gain a detailed understanding of other recognized senior secondary qualifications in mathematics
- 2) Provide an overview of relevant reforms and recognition of each of the qualifications, in order to understand the common issues associated with mathematics qualifications, including the broader challenge of maintaining strong, relevant and innovative curriculum
- 3) Provide information on the specific curriculum structure and mathematical content of each qualification and conduct an initial comparative analysis across all qualifications, in order to understand the extent of alignment between IB qualifications and other qualifications both at the macro and more detailed level.

This analysis, together with the analysis conducted in the Assessment Comparison, can contribute to an overall comparison of the level of demand of these qualifications (according to the notion of “demand” defined in the section below), to be ultimately reviewed and determined by mathematics subject experts. Thus, it is expected that this Curriculum Comparison component will contribute to overall reflections in the final IB Mathematics Comparability Study on the relative strengths of IB’s mathematics qualifications and ways in which IB’s provision might develop over time.

This report is structured according to the main aims outlined above: Part A will provide an overview of the context, reforms, purpose, structure and recognition of each of the broad senior secondary qualifications and their specific mathematical components; Part B will present a detailed comparative analysis of the curriculum content of each. The annexes contain charts that allow some of the main information contained in this report to be viewed concurrently;

they also contain detailed descriptions of the syllabus content of each qualification that has been included under each general category of mathematics.

Models of Comparative Analysis, Methodology and Sources

Researchers and analysts have approached the task of conducting comparative analyses of educational qualifications and curriculum from various angles. While some comparative studies have taken a broad, macro-level view and focused on the cultural, political and economic context of educational structures and how this influences concrete programs and practices,³ others have taken a micro-level view that focuses only on the technical aspects of educational programs and on comparing specific curricular content. Likewise, some studies are more academic in nature, while others are conducted by government agencies, international bodies or educational NGOs for specific policy-making purposes.⁴

In all of these types of comparative study, researchers often face challenges related to data gathering techniques, objectivity and the analytical challenge of establishing equivalencies.⁵ With regard to data, it may be difficult to gather exactly the same categories of data, in the same degree of detail, from a wide range of educational qualifications, due to language barriers, different rules about the public accessibility and confidentiality of data, or simply different approaches to laying out curricular requirements. Maintaining objectivity is a key issue, particularly when comparative studies are commissioned for policy purposes. Analysts may consciously or inadvertently utilize a biased framework that is based primarily on the ideological focus, philosophical orientation or policy preferences of the specific national system or educational qualification that is at the center of the policy review process. Finally, an inherent difficulty of international comparisons is the issue of establishing equivalencies among qualifications that may have very varied sets of requirements and structures. Relying on an external framework or benchmarks according to which all qualifications or curricula can be aligned or categorized is one way of engaging in a more objective comparison. Categories or levels of qualifications, such as those established by UNESCO in its ISCED classification, are an example of this kind of external framework. International assessment bodies such as the OECD or international professional associations for given topics can also provide an “external” set of benchmarks (although these are naturally not free of their own biases in the ways in which they categorize educational content).

Ultimately, completely objective or precise equivalencies cannot be established between international qualifications, but this does not reduce the importance of attempting to compare them by gathering relevant information and finding significant areas of similarity and

³ One key example is Robin Alexander, *Culture and Pedagogy: International Comparisons in Primary Education* (2000).

⁴ UNESCO, the OECD and the World Bank are the leaders in this area, and have produced numerous technical studies comparing specific areas of national education systems and curricula.

⁵ Michael Crossley and Keith Watson, *Comparative and International Research in Education* (2003), among others, engage in a discussion on these issues in comparative research.

difference. In order for comparative studies of qualifications to be of practical use, some assessment must eventually be made of the level of demand of their content. “Demand” can be defined as “the level of knowledge, skills and competence required by the typical learner” in a given subject course or examination.⁶ According to an extensive Ofqual report comparing Senior Secondary assessments, demand is reflected in each qualification’s syllabus content (level of abstract thinking required, complexity of topics), structure (breadth and depth of topics covered) and assessment (cognitive skills students must use to generate responses, levels of expected performance, weighting of each component).⁷ Ultimately, making informed and balanced judgements on the level of demand of various qualifications can only be done through experienced external experts on the topic of comparison. As Pollit et al (2007) state, “there is no statistical indicator of demands, and no prospect of our developing objective scales for assessing them. Instead we rely on the judgement of experienced professionals”.⁸

In the case of the qualifications analyzed for this study, data was gathered mainly from primary sources, consisting of the most recent guidance documents and course specifications or guides provided by the administering authorities of each qualification (available on their official websites). Secondary sources were also used to enrich the data, namely existing international studies that have compared various mathematics qualifications. One particularly useful source for this report is Ofqual’s 2012 International Comparisons in Senior Secondary Assessment (hereafter referred to as ICOSSA), which provides a methodological model in some ways, and which contains a large amount of detailed data on several of the qualifications included in this study.

In this study, the amount and type of data gathered for each qualification varies somewhat based on various factors. Firstly, the amount of publicly available information was much greater for Advanced Placement, A Levels, and the Alberta Diploma than for Singapore A Levels and the Chinese Gaokao. Indeed, publicly available data in English was extremely limited for Gaokao; information on this qualification has thus been drawn from secondary sources, mainly ICOSSA. Secondly, the structure of Advanced Placement qualifications is much less prescribed than that of the other qualifications; thus, AP guidelines provide relatively less specific information on prerequisites and time allocation for each qualification, for example. Likewise, the content of A Level qualifications is broadly defined by the U.K. Department for Education, but these qualifications are administered by several different providers who establish their own structural and content variations. For the purpose of this study, Pearson's Edexcel A Level qualifications were chosen, following the model of the ICOSSA study. Finally, information on internal reviews of these qualifications was not publicly available, as it is generally considered confidential or restricted to members of the administering organizations. Hence, this report draws only upon

⁶ Elliot, 2011, p.11, cited in Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 20.

⁷ Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 20.

⁸ Pollitt et al., 2007, p.168, cited in Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 26.

the summaries of reform and review processes that have been made available to the public, as well as on external media reports and analyses of the reform processes.

With regard to concerns over objectivity, this report naturally highlights similarities and differences between other qualifications and IB qualifications in its analysis and conclusions, but IB qualifications were not used as a benchmark against which to compare the others. In order to establish an external framework for determining certain equivalencies between the qualifications, the textbooks and course outlines used by top global universities in science and engineering were referred to for defining objective categories of mathematics content. The methodology used for the comparative analysis of specific curriculum structure and content will be outlined in more detail at the beginning of that section.

PART A

OVERVIEW OF QUALIFICATIONS: CONTEXT, PURPOSE, STRUCTURE, REFORMS AND RECOGNITION

INTERNATIONAL BACCALAUREATE DIPLOMA

Context

The International Baccalaureate (IB) is a non-profit organization founded in 1968 in Geneva, Switzerland. Today, IB has offices worldwide and works with over 4,000 schools in numerous countries to develop and offer three challenging programmes to students aged 3 to 19 years. IB does not own or manage any schools; rather, it works with both publicly and privately funded schools that share IB's commitment to quality international education and authorizes them to offer one or more of its' programmes. The Primary Years Programme (PYP), the Middle Years Programme (MYP) and the Diploma Programme all encourage students to be active learners, well-rounded individuals and engaged world citizens. One of IB's most important recent innovations has been the creation of a career-related certificate that applies IB's curricular strengths to a more accessible programme.

A recognized leader in the field of international education, IB's work encompasses curriculum development, assessments, professional development and other services. There are no "typical" IB World Schools: the IB community is composed of a diverse group of schools that are geographically widespread, publicly and privately funded, and that serve students from a wide variety of social and ethnic backgrounds. IB is committed to expanding access to high-quality educational programmes and services, working with governments, foundations and other organizations in order to do so. The IB curriculum is compatible with many local and national

standards, and its assessments often meet or exceed the requirements of government-mandated exams. Indeed, IB World Schools receive different forms of support from various local or national governments.

IB is motivated by a strong mission to promote education for a better world, an idealistic vision that defines its core values and has very concrete practical outcomes. The IB Learner Profile translates the IB mission into a set of key attributes that are widely recognized as essential traits for successful engagement in the 21st century. According to the IB Learner Profile, IB learners strive to be: inquirers, knowledgeable, thinkers, communicators, principled, open-minded, caring, risk-takers, balanced and reflective.

The IB curriculum, across all programmes, includes a key set of features that reflect its basic educational principles: a strong international dimension, a broad and balanced range of subjects, interdisciplinary areas, the development of skills for inquiry-based, lifelong learning, and community engagement.

Purpose and structure of IB Diploma qualifications

The Diploma Programme was established in 1968 to provide students with a balanced education, to facilitate geographic and cultural mobility, and to promote international understanding. The Diploma Programme has also always had a very pragmatic goal of helping students to gain entry into universities all over the world. The Diploma incorporates IB's basic curricular features in the following manner:

An international dimension. Each Diploma course includes content that illustrates examples and forms of knowledge from different parts of the world. Students are required to study two languages: one for which they have a native ability, and a second language, learned up to near-native level. Around 100 languages are available for study at these two levels. The overall Diploma programme can be taught in English, Spanish and French.

A broad and balanced range of individual subjects. IB Diploma students study a total of six courses; five of these must be chosen from five subject groups, thus ensuring breadth of experience in a first language, a second language, individuals and societies, the experimental sciences and mathematics and computer science. The sixth subject may be an arts subject chosen from a sixth group, or the student may choose another subject from the first five groups. At least three subjects are taken at "higher" level (recommended 240 teaching hours), and the others at "standard" level (150 teaching hours). In addition to the choice of languages, 29 subjects are available in groups 3 to 6. IB provides detailed course outlines for each subject.

A framework that encourages interdisciplinary learning. In the Diploma, interdisciplinary thinking is encouraged in all areas, and particularly through the Theory of Knowledge course, which is a core component of the programme and encourages students to reflect on different kinds of knowledge and make connections between them.

A focus on developing students' inquiry skills and their capacity to "learn how to learn". IB Diploma students are required to take a Theory of Knowledge course in which students reflect on the nature of knowledge by critically examining different ways of knowing (perception, emotion, language and reason) and different kinds of knowledge (scientific, artistic, mathematical and historical). Students examine truth claims in the different disciplines, looking into the paradigm shifts that occur in all areas of human understanding.

Opportunities to learn through experience and serve the community. Creativity, action and service (CAS) is at the heart of the Diploma. It consists of arts and other experiences that involve creative thinking (Creativity), physical activity contributing to a healthy lifestyle (Action), and an unpaid and voluntary service to the community that also provides learning benefits (Service).

A culminating experience. The Extended Essay, the third core component of the Diploma, is an in-depth, individual inquiry into a focused topic, normally from one of the student's six chosen subject areas.

The Diploma Programme includes summative, externally-defined performance assessments, in the form of final examinations with specific scores that are used for the purpose of receiving globally-recognized qualifications. Throughout the two years of the programme, teachers must also engage in internal assessments that identify students' learning needs. The IB Diploma's assessment is considered highly reliable and consistent, since it consists of a combination of final examinations that are marked by IB-appointed, experienced external examiners, and internal assessments that are also moderated by external examiners. This reduces the possibility of partiality or inaccuracy. Assessments are also criterion-referenced, meaning that students around the world are measured against the same pre-specified criteria for each subject, rather than against other students' performance. This eliminates the possibility of "grade inflation", and provides internationally-benchmarked results that are trusted and welcomed by universities worldwide.

Purpose and structure of IB Diploma mathematics qualifications

The aims of all mathematics courses in group 5 are to enable students to:

- "enjoy mathematics, and develop an appreciation of the elegance and power of mathematics
- develop an understanding of the principles and nature of mathematics

- communicate clearly and confidently in a variety of contexts
- develop logical, critical and creative thinking, and patience and persistence in problem-solving employ and refine their powers of abstraction and generalization
- apply and transfer skills to alternative situations, to other areas of knowledge and to future developments
- appreciate how developments in technology and mathematics have influenced each other
- appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
- appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
- appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course.⁹

Students are required to take one Group 5 (mathematics and computer science) course within the IB Diploma. The options available are mathematical studies SL, mathematics SL, mathematics HL, and further mathematics HL; each of these is oriented to different needs and abilities of students, and fulfills different university and career aspirations.

Recognition of IB Diploma mathematics qualifications

Universities and colleges recognize the value of the IB Diploma by actively recruiting IB Diploma students, providing advantages for admission, acknowledging the rigour of IB exams by granting advanced placement and/or course credit, and providing scholarships for IB Diploma students. Over 1,500 universities around the world list their IB recognition policies on the IB website, which may include one or more of the above advantages.

The top 5 global universities in engineering and technology (according to the 2014-15 Times Global Ranking) recognize IB mathematics qualifications as follows:

- **MIT:** “For a score of a 6 or 7 on the Mathematics HL exam, credit will be given for 12 units of subject 18.01, Calculus I. For scores lower than 6, no credit is awarded.”¹⁰
- **Stanford:** Mathematics HL with a score of 5 or higher is eligible for 10 credits and advanced placement in the Math 51 course.¹¹

⁹ IB DP Subject Brief: Mathematics SL

¹⁰ <http://web.mit.edu/firstyear/2018/subjects/incomingcredit/intl/ib.html>

¹¹ <http://studentaffairs.stanford.edu/registrar/students/baccalaureate-credit>

- **CALTECH:** Does NOT “grant credit for AP, IB, A Level, Pre-U, or college courses taken prior to enrollment. Each student accepted to Caltech will take a math and physics placement exam prior to enrolling.”¹²
- **Princeton:** A score of 6 or 7 on an IB HL mathematics exam is considered equivalent to a 4 or 5 on an AP math exam (whether Calculus AB or BC is not specified) and eligible for the equivalent credit (there is no specific chart for IB courses).¹³
- **Cambridge:** All colleges offering Computer Science, Economics, Engineering and Mathematics require a 6 or 7 in IB Mathematics HL; some of these colleges require a 6 or 7 in IB Further Mathematics HL.¹⁴

THE ALBERTA HIGH SCHOOL DIPLOMA

Provincial context

The Alberta public education system is based on the premise that “schools have the responsibility to provide instructional programs that ensure students will meet the provincial high school completion requirements and are prepared for entry into the workplace or post-secondary studies”. Public schools also fulfill the central task of ensuring that “students understand the rights and responsibilities of citizenship and have the skills and attitudes to pursue learning throughout their lives”.¹⁵ Provincial education is governed by the *School Act* and guided by Alberta Education’s periodically renewed *Three-year Business Plan*.

According to the Student Learning Ministerial Order (#001/2013), based on Section 39(1) of the School Act, K-12 education should enable all students to achieve the following learning outcomes: “1) be Engaged Thinkers and Ethical Citizens with an Entrepreneurial Spirit; 2) strive for engagement and personal excellence in their learning journey; 3) employ literacy and numeracy to construct and communicate meaning; and 4) discover, develop and apply competencies across subject and discipline areas for learning, work and life”. These competencies include knowing how to learn, thinking critically, identifying and solving complex problems, managing information, being innovative, creating opportunities with an

¹² <http://www.admissions.caltech.edu/content/how-apply-first-year-applicant>

¹³ <http://www.princeton.edu/pub/ap/table/>

¹⁴ Taken from various Cambridge websites by subject area, and <http://www.undergraduate.study.cam.ac.uk/applying/entrance-requirements/international-baccalaureate>

¹⁵ Alberta Guide to Education 2014, p. 1.

entrepreneurial spirit, applying multiple literacies, demonstrating good communication and cooperation skills, demonstrating global and cultural understanding, and applying career and life skills through personal growth.¹⁶

The structure of the Alberta education system consists of Early Childhood Services (including Kindergarten), Grades 1 through 9 (which includes the Elementary Program – grades 1 through 6 – and the Junior High School Program – grades 7 through 9), and Senior High School (grades 10 through 12). Francophone programming is available for all grade levels, parallel to English language programming.

The Junior High School Program includes the following required subject areas: English Language Arts, French Language Arts, Mathematics, Science, Social Studies, Physical Education, and Health and Life Skills. Students must also take optional courses which include second languages, arts, career and technology studies and other subjects, in order to complete a total of 950 hours of instruction by the end of Junior High School.¹⁷

The Senior High School Program requires a minimum of 1,000 hours of instruction over three years (10th through 12th grades). Local school authorities can develop their own senior high school programming. However, these programs must be “articulated with a variety of programs in colleges and technical institutes”; they must also meet the requirements of the Alberta High School Diploma (described in the next section).¹⁸ Several pathways are available to students at the Senior High School level: Career and Technology Studies (CTS), oriented towards careers in areas such as administration, health, media and manufacturing; Knowledge and Employability Studies, for students who are up to three grade levels below their age-appropriate grade; and Advanced Placement or International Baccalaureate pathways which do not replace Alberta Diploma requirements but rather provide students with the opportunity to study courses beyond the Alberta Diploma level.¹⁹

The mainstream Senior High School Program consists of a sequence of courses from 10th to 12th grades that link together to form distinct pathways at different levels. “10” level courses are taken in 10th grade, and are available at several different levels (10-1, 10-2, 10-3 or 10-4, ranging from courses for higher academic pathways to more basic career training); these link with the equivalent “20” level courses taken in 11th grade, and “30” level courses taken in 12th grade.

¹⁶ Alberta Guide to Education 2014, p. 4.

¹⁷ Alberta Guide to Education 2014, p. 37.

¹⁸ Alberta Guide to Education 2014, p. 43.

¹⁹ Alberta Guide to Education 2014, p. 50.

Recent reforms

Province-wide public consultations which took place in 2010 led to a “Curriculum Redesign” process that has been underway since 2011. The goals of the Curriculum Redesign process are “to develop revised standards and guidelines for future curriculum (programs of study, assessment, and learning and teaching resources); [and] to develop a cohesive and collaborative process for curriculum development that will ensure that curriculum is responsive to students in a rapidly changing world”.²⁰

The new curriculum that is being developed will “be more student-centered and inclusive of economic, social and cultural perspectives; focus on students’ development of competencies; emphasize a foundation of literacy and numeracy; and encourage more flexibility to address local needs”. It is hoped that these changes will allow students to: “1) learn at their own pace using a variety of formats; 2) demonstrate learning in different ways; 3) apply what they learn to real-life situations; 4) explore topics of interest in greater detail; and 5) use technology to support the creation and sharing of knowledge.”²¹

Purpose and structure of the Alberta High School Diploma

The Alberta High School Diploma is the academic pathway for high school completion, and requires a total of 100 credits across various subject areas. As a Diploma-style qualification, it consists of a set of six minimum required courses, in addition to optional courses, and requires students to complete 100 credits in total. Given that each high school course credit consists of 25 hours,²² this comes to a total of 2,500 hours of instruction for the entire Diploma. The minimum requirements for the Alberta High School Diploma are as follows:

²⁰ Alberta Education, *From Knowledge to Action*, 2012, Executive Summary, p. 1.

²¹ Alberta Education, *Curriculum Redesign Overview*, March 12, 2014.

²² “Instructional services [...] must ensure that students have access to at least 25 hours of instruction per high school credit.” *Alberta Guide to Education 2014*, p. 42.

ALBERTA HIGH SCHOOL DIPLOMA: GRADUATION REQUIREMENTS (ENGLISH)
The requirements indicated in this chart are the <u>minimum</u> requirements for a student to attain an Alberta High School Diploma. The requirements for entry into post-secondary institutions and workplaces may require additional and/or specific courses.
100 CREDITS including the following:
ENGLISH LANGUAGE ARTS – 30 LEVEL (English Language Arts 30-1 or 30-2)
SOCIAL STUDIES – 30 LEVEL (Social Studies 30-1 or 30-2)
MATHEMATICS – 20 LEVEL (Mathematics 20-1, Mathematics 20-2 or Mathematics 20-3)
SCIENCE – 20 LEVEL ¹ (Science 20, ² Science 24, Biology 20, Chemistry 20 or Physics 20)
PHYSICAL EDUCATION 10 (3 CREDITS) ³
CAREER AND LIFE MANAGEMENT (3 CREDITS) ⁴
10 CREDITS IN ANY COMBINATION FROM:
<ul style="list-style-type: none"> • Career and Technology Studies (CTS) courses • Fine Arts courses • Second Languages ⁵ courses • Physical Education 20 and/or 30 • Knowledge and Employability courses • Registered Apprenticeship Program courses • Locally developed/acquired and authorized courses in CTS, fine arts, second languages or Knowledge and Employability occupational courses ⁶
10 CREDITS IN ANY 30-LEVEL COURSE (IN ADDITION TO A 30-LEVEL ENGLISH LANGUAGE ARTS AND A 30-LEVEL SOCIAL STUDIES COURSE AS SPECIFIED ABOVE) ⁷
<p>These courses may include:</p> <ul style="list-style-type: none"> • 30-level locally developed/acquired and authorized courses • Advanced level (3000 series) in Career and Technology Studies courses • 30-level Work Experience courses ⁸ • 30-level Knowledge and Employability courses • 30-level Registered Apprenticeship Program courses • 30-level Green Certificate Specialization courses • Special Projects 30

Source: *Alberta Guide to Education 2014*, p. 85.

There is no central body for university applications in Alberta; higher education institutions determine their own admissions requirements independently. Thus, the Alberta High School Diploma does not guarantee admission to higher education; each university may require additional or specific courses at the more demanding “30” level.

Not all of the components of the Alberta Diploma are examined externally – only “30” level courses in Biology, Chemistry, English, Mathematics, French, Physics, Science and Social Studies culminate in Diploma examinations. The Grade 12 Diploma Examinations Program was established in 1984 in order to “certify the level of individual student achievement in selected grade 12 courses, ensure that province-wide standards of achievement are maintained, [and] report individual and group results”.²³

²³ Q & A Related to Diploma Exam Weighting March 2015, http://education.alberta.ca/media/14442388/q-a-dip-weighting-english_20150318.pdf

Currently, the final mark for “30” level courses that culminate in examinations is made up of a school-awarded mark (50%), determined by classroom work over the entire year, and the Diploma examination mark (50%). To pass the course, students must obtain 50% or more of total marks. “Acceptable Standard” is reached by students with 50-79% score, and a “Standard of Excellence” is reached by those with over 80% score.

Several changes to the Alberta High School Diploma have been made as part of the “Curriculum Redesign” process (described above): Diploma examinations are available online since May 2013, and measures have been taken to strengthen the curriculum of various courses and improve high school graduation rates. The weighting of marks has recently been modified, and as of September 1, 2015, the school-awarded mark will make up 70% of the final mark, reducing the weight of the examination mark to only 30%. According to the Alberta Education Ministry, this change will increase the fairness of Diploma assessments, since it allows for “a more accurate reflection of student performance by enabling those who work most closely with students on a daily basis (teachers) to assess student performance over a longer period of time, rather than relying so heavily on exams lasting only a few hours”.²⁴ It is hoped that graduation rates may increase and there may be less reliance on test-preparation companies, at the same time as the overall rigor of the “30” level Diploma courses is maintained.

Reforms, purpose and structure of Alberta Diploma mathematics qualifications

In 2007, a new mathematics curriculum from K-12 was approved by the Minister of Education. The new curriculum was rolled out across the province in various stages between 2008 and 2013.²⁵ There are now seven basic “Mathematical Processes” that permeate the entire K-12 Mathematics curriculum. According to these processes, students are expected to “1) use *communication* in order to learn and express their understanding; 2) make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines; 3) demonstrate fluency with *mental mathematics and estimation*; 4) develop and apply new mathematical knowledge through *problem solving*; 5) develop mathematical *reasoning*; 6) select and use *technology* as a tool for learning and for solving problems; 7) develop *visualization* skills to assist in processing information, making connections and solving problems”.²⁶

²⁴ Q & A Related to Diploma Exam Weighting March 2015, http://education.alberta.ca/media/14442388/q-a-dip-weighting-english_20150318.pdf

²⁵ <http://education.alberta.ca/teachers/program/math/educator/progstudy.aspx>

²⁶ <http://education.alberta.ca/teachers/program/math/educator/>

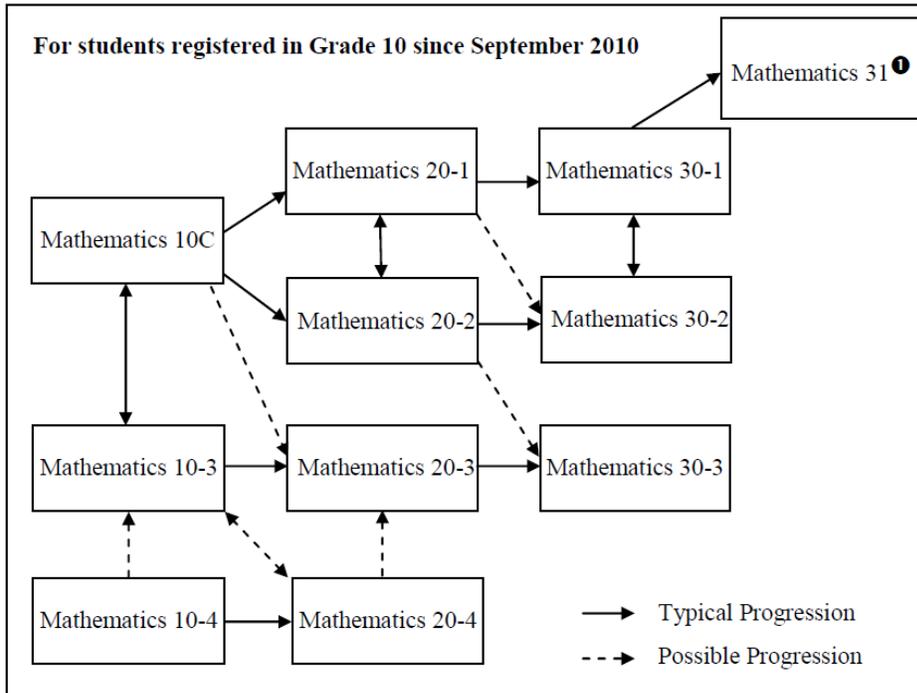
There are four different course sequences that students can take in mathematics, beginning in 10th grade. Each course sequence is in accordance with the pathway students hope to take throughout high school and beyond, into a practical career or into higher education. In 10th grade, Mathematics 10-4 is related to the “Knowledge and Employability” pathway, which is not a part of the Alberta High School Diploma.

Pathways that can culminate in earning the Alberta High School Diploma include:

- 1) The course sequence 10-3, 20-3, 30-3 (corresponding to 10th, 11th and 12th grades), which is meant for students who want to enter a trade or the workforce after high school, and covers topics such as finance, geometry, measurement and trigonometry
- 2) the course sequence 10-2, 20-2, 30-2, which is meant for students who want to attend a university, college or technical institute but will not need calculus skills in their chosen field of study. This series includes topics such as functions and equations, probability and statistics, and trigonometry. Students can switch from this sequence to a higher or lower sequence in 11th grade
- 3) the course sequence 10-1, 20-1, 30-1, which is meant for students who plan to enter post-secondary programs that require more math skills, such as engineering, mathematics or science subjects.

At least one 20-level (11th grade) mathematics course is a required component of the Alberta High School Diploma, and one additional 30-level course is highly recommended for entry into university. The 30-2 and 30-1 mathematics courses culminate in a Diploma examination, and both are more likely to be accepted by higher education institutions for admissions purposes (particularly Math 30-1). The course Math 31, for which Math 30-1 is a prerequisite, is the only course which includes calculus topics, and is recommended for students hoping to enter university programs that require a strong base in mathematics.

Mathematics Course Sequences



Source: Alberta Guide to Education 2014, p. 62.

The appropriate use of technology is one of the seven Mathematical Processes that permeates the entire K-12 mathematics curriculum. It is considered that technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems”. At the same time, “the use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding”.²⁷ Graphing calculators are not only allowed but required for mathematics Diploma examinations. These calculators must be devices “designed primarily to perform mathematical computations, including logarithmic, trigonometric, and graphing functions”.²⁸

Recognition of Alberta Diploma mathematics qualifications

Alberta Math 30-1 and Math 31 courses are recognized for admission and credit purposes by some of the top Canadian universities in engineering and technology and/or physical sciences

²⁷ Alberta Education, Mathematics Grades 10-12, 2008, p.7.

²⁸ Writing Diploma Examinations Using Calculators, http://education.alberta.ca/media/6902701/06-dip-gib-2014-15_using-calculators-computers.pdf

(according to the 2014-15 Times Global Ranking), as well as by the province’s premier higher education institution, the University of Alberta:

- **University of Alberta:** For admission, the BSc in Engineering requires Math 31; the general Bsc in any area (math, physics, statistics, etc.) and even the BSc with Honors only requires Math 30-1 (and a choice of other subjects that may include Math 31).²⁹
- **University of Toronto:** For the fields of physical/mathematical/chemical sciences, computer science, and applied science/engineering “Mathematics 31 or Calculus AP (AB or BC) is required [for students from Alberta] for programs with the Calculus prerequisite [...]” No credit is given to Alberta Diploma maths courses.³⁰
- **University of British Columbia:** For students from Alberta, either Math 30-1 or 31 are acceptable for admission to science or engineering related programs; no transfer credit is mentioned for any of these courses.³¹

ADVANCED PLACEMENT

Context

Advanced Placement qualifications are administered by the College Board, a non-for-profit educational services provider based in the United States. Founded in 1900, the College Board has striven to fulfill its mission to expand access to higher education through various programs that have become an integral part of the U.S. education system. Indeed, the College Board serves 7 million students as they transition to college each year – nationally and internationally – through one of its various programs.³² The SAT has become nearly synonymous with college admissions, and Advanced Placement courses are well known in every school district as a recommended preparation for higher education. The extensive reach of the College Board has sometimes caused controversy, particularly the emphasis placed on standardized testing for college admissions and the consequent reliance on test-preparation services, as well as the implications of this for educational equity. The SAT has been reformed multiple times in order to give greater emphasis to critical thinking, and U.S. universities have also revised their admissions policies in order to diversify the criteria for admissions and reduce reliance on SAT scores.

²⁹ <http://apps.admissions.ualberta.ca/programs/>

³⁰ <http://www.adm.utoronto.ca/admissions/> and http://www.adm.utoronto.ca/admissions/?domain=adm&page=TC_MAIN&p1=true

³¹ <http://you.ubc.ca/admissions/canadian-highschools/ab-nwt-nunavut/>

³² www.collegeboard.org

At the same time, SAT and Advanced Placement results have become a useful barometer of trends in entry to higher education, and a way to identify issues related to access and equity at the district, state and national level. The College Board engages in research and advocacy on a variety of educational issues, particularly those related to giving students access to “an affordable and successful college experience”. To this end, the organization advocates for strengthening federal Pell Grants for students, and for simplifying and rethinking the federal financial aid for higher education overall.³³ Some of the College Board’s programs have received local, state or federal government support. Notably, school districts across the country have received considerable support from the federal government for implementing Advanced Placement courses over the past decades, and various recent federal funding grants and competitions such as “Race to the Top” have also rewarded schools that incorporate AP courses into their reform plans.

Reforms, purpose and structure of Advanced Placement qualifications

The purpose of the Advanced Placement Program is to “enable willing and academically prepared students to pursue college-level studies – with the opportunity to earn college credit, advanced placement or both – while still in high school”.³⁴ According to the College Board, benefits of AP qualifications for students include: improving chances of admission to higher education by showing that they have “sought the most rigorous curriculum available to them”, acquiring skills that will be useful in college (such as learning to think critically and analyze issues from many angles), and thus, generally having greater academic success in college.

AP courses are currently offered in 34 subjects; they are stand-alone courses, which can be taken in any number or order, and are not part of any structured Diploma. Nonetheless, it is important to note that a certain number or combination of AP courses are sometimes incorporated by individual schools or school districts into a specific set of requirements for completing an academic track. The College Board itself has also created an AP “Capstone Diploma”, launched at some participating schools in 2014, which is awarded to students who obtain scores of 3 or higher on four AP courses and who also take two new courses, AP Seminar (focusing on analyzing and evaluating information and making sound arguments) and AP Research (producing and defending a scholarly academic thesis).³⁵ This Diploma appears to be an attempt to emulate the International Baccalaureate’s Theory of Knowledge and Extended Essay components.

³³ <https://www.collegeboard.org/advocacy>

³⁴ <https://www.collegeboard.org/releases/2014/class-2013-advanced-placement-results-announced>

³⁵ <https://advancesinap.collegeboard.org/ap-capstone>

Every College Board course culminates in an examination that is externally developed by committees of experts. Each examination consists of a free-response section as well as a multiple-choice section. The latter is graded by machine, while the former is scored by college faculty and experienced AP teachers. Weighted raw scores on exams are converted into scores on a scale of 1 to 5, where 1 = no recommendation; 2 = possibly qualified; 3 = qualified; 4 = well qualified; and 5 = extremely well qualified. In addition, 5 is considered equivalent to a college grade of A, 4 to A-/B+, 3 to B-/C+/C/.

The College Board provides guidance on the content and structure of AP courses in each subject, but each school is ultimately responsible for the way in which these courses are implemented. Participation in AP courses has nearly doubled over the past decade, however, and this rapid expansion has led to concerns about maintaining the same standards of quality.³⁶ In response, the College Board has strengthened its requirement for all participating schools to submit their AP course syllabi for auditing. The AP Course Audit was created in order to “provide AP teachers and administrators with clear guidelines on curricular and resource requirements that must be in place for AP courses [and] give colleges and universities confidence that AP courses are designed to meet the same clearly articulated college-level criteria across high schools”.³⁷ Every year teachers submit their syllabi for approval or renewal; each syllabus is evaluated and approved by college and university faculty with expertise in the particular subject of the course.

Despite these concerns over maintaining quality standards, the expansion of AP courses is generally considered a positive development by the College Board because it indicates greater access for a larger and more diverse pool of students. It is considered that taking an AP course is in itself an enriching experience for students, regardless of whether they are able to obtain high marks on it. Pre-requisite learning required for each individual AP course is specified in each AP course guide, but eligibility for AP courses and access to them depends entirely on the policies of each school or district, however, and these vary from extremely restrictive to whole-school accessibility.

³⁶ According to the AP Report to the Nation 2014, in 2013 there were just over one million AP examinees. <http://media.collegeboard.com/digitalServices/pdf/ap/rtn/10th-annual/10th-annual-ap-report-to-the-nation-two-page-spread.pdf> The reach of AP courses across the U.S. is so extensive that their content can easily become the topic of national debate. Revisions to the AP U.S. History course, for example, have recently caused controversy and criticism from conservatives for being too “left-leaning”, “revisionist” and “liberal”.

³⁷ <http://www.collegeboard.com/html/apcourseaudit/index.html>

Since 2011, the College Board has embarked on a comprehensive redesign of many AP courses, in order to improve alignment with college-level learning. This redesign places greater emphasis on inquiry, reasoning and communication skills; it also emphasizes rigorous curricular content that strikes a balance between breadth and depth, and improving standards based on recommendations from national disciplinary organizations and studies conducted at 4-year higher education institutions. New curriculum is based on the “Understanding by Design” methodology, which begins with “clearly defined learning outcomes and then articulate[s] the evidence needed to confirm that the learning outcomes have been met”.³⁸

Reforms, purpose and structure of Advanced Placement mathematics qualifications

Advanced Placement offers three mathematics qualifications: AP Calculus AB, AP Calculus BC, and AP Statistics. Both AP Calculus AB and Calculus BC courses focus on a “multi-representational approach to calculus with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally.” They also use the unifying themes of calculus to produce a cohesive whole. AP Calculus AB is equivalent to first semester college calculus course; AP Calculus BC is equivalent to first semester college calculus course *and* subsequent single-variable calculus course; AP Statistics is equivalent to one semester of college introductory statistics. In 2014, 4,176,200 students took AP exams, of which 7% took Calculus AB, 2.7% took Calculus BC, and 4.4% took Statistics.³⁹

The overarching conceptual framework for both AP Calculus courses is the set of Mathematical Practices for AP Calculus (MPACs), an interrelated set of skills drawn from the National Council of Teachers of Mathematics (NCTM) Process Standards and the Association of American Colleges and Universities (AAC&U) Quantitative Literacy VALUE Rubric. The MPAC learning objectives are: 1) Reasoning with definitions and theorems; 2) Connecting concepts; 3) Implementing computational/algebraic processes; 4) Connecting multiple representations; 5) Building notational fluency; and 6) Communicating.⁴⁰

From 2016-17 onwards, both AP Calculus courses are structured according to the “Understanding by Design” model, where content is aligned with learning objectives and essential maths practices are made explicit (rather than laying out content in a traditional topic list), all in accordance with the Mathematical Practices for AP Calculus (MPACs). Subject matter is now organized around 4 “Big Ideas”, each of which sets out “Enduring Understandings”,

³⁸ <https://aphighered.collegeboard.org/exams/course-exam-revision>

³⁹ College Board, AP Report to the Nation 2014, State and Subject Supplements

⁴⁰ <https://advancesinap.collegeboard.org/stem/calculus/mathematical-practices>

“Learning Objectives” and “Essential Knowledge Statements”.⁴¹ The changes to actual content areas are few, however: L’Hopital Rule was added to Calculus AB, and the limit comparison test, absolute and conditional convergence, and alternating series error-bound was added to Calculus BC.⁴²

In order to validate the new AP Calculus courses, in Spring 2014 the AP Program collaborated with Harris Interactive to conduct a study of 27 representative AP score-receiving colleges and universities. Study participants were department chairs and/or faculty members who have taught introductory-level calculus during the past three years or who have an influence on the department’s AP credit and placement policy. The majority of participants agreed that “the learning objectives of the updated courses were very or somewhat similar to those in their introductory calculus courses [and] indicated that, in light of the course and exam updates, they would recommend that their departments grant credit to students who succeed on the AP Calculus Exams”.⁴³

The AP Calculus Development Committee has discussed the need for a “post Calculus BC” course that would deepen and broaden the mathematical knowledge of those students who have already covered Calculus BC even before their senior year of high school, and who are especially “mathematically talented”.⁴⁴

With regard to technology, use of a graphing calculator is considered an integral part of both Calculus courses, since “professional mathematics organizations have strongly endorsed the use of calculators in mathematics instruction and testing”. Thus, “teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results”.⁴⁵ Graphing calculators are only allowed for some sections of the final examinations, however. In those sections where graphing calculators are required, they are expected to have the capacity to “plot the graph of a function within an arbitrary viewing window; find the zeros of functions (solve equations numerically); numerically calculate the derivative of a function; and numerically calculate the value of a definite integral”.⁴⁶

⁴¹ <https://advancesinap.collegeboard.org/stem/calculus>

⁴² Curriculum Framework Calculus AB and Calculus BC 2016-17, <https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf>

⁴³ <https://advancesinap.collegeboard.org/stem/calculus/higher-ed-validation-study>

⁴⁴ AP Calculus Teachers Guide 2007, p.2:

http://apcentral.collegeboard.com/apc/members/repository/ap07_calculus_teachersguide_2.pdf

⁴⁵ AP Calculus BC overview

⁴⁶ AP Calculus Course Description 2012, p.14, <http://media.collegeboard.com/digitalServices/pdf/ap/ap-calculus-course-description.pdf>

Recognition of Advanced Placement mathematics qualifications

According to the College Board, more than 3,300 institutions worldwide annually receive AP scores. A large number of four-year colleges and universities in the U.S. recognize AP courses for admissions purposes or for granting credit.⁴⁷

The top 5 global universities in engineering and technology (according to the 2014-15 Times Global Ranking) recognize AP mathematics qualifications as follows:

- **MIT:** “For a score of 4 or 5 on the BC Calculus examination, credit is given for 12 units of [18.01, Calculus I](#). For scores lower than 4, no credit is given.”⁴⁸
- **Stanford:** AP Calculus BC with a score of 4 or 5 is eligible for 10 credits and advanced placement in the Math 51 course. AP Calculus BC with a score of 3 is eligible for 5 credits and placement in the Math 20 or Math 42 course. ⁴⁹
- **CALTECH:** Does NOT “grant credit for AP, IB, A Level, Pre-U, or college courses taken prior to enrollment. Each student accepted to Caltech will take a math and physics placement exam prior to enrolling.”⁵⁰
- **Princeton:** A score of 4 in AP Calculus BC is worth placement in Math 104/Math 175; a score of 5 in AP Calculus BC is worth placement in Math 201. A score of 5 in AP Calculus BC together with a score of 750 on the MSAT is worth placement in Math 203.⁵¹
- **Cambridge:** “Prospective applicants from Canada and the USA taking SATs and Advanced Placement Tests should note that offers are usually made on an individual basis. In addition to high passes in the High School Diploma and the SAT, successful applicants have normally achieved 5s in at least five Advanced Placement Tests in appropriate subjects.”⁵²

⁴⁷ www.collegeboard.com

⁴⁸ <http://web.mit.edu/firstyear/2018/subjects/incomingcredit/ap.html>

⁴⁹ <http://studentaffairs.stanford.edu/registrar/students/ap-charts>

⁵⁰ <http://www.admissions.caltech.edu/content/how-apply-first-year-applicant>

⁵¹ <http://www.princeton.edu/pub/ap/table/>

⁵² <http://www.undergraduate.study.cam.ac.uk/applying/entrance-requirements/other-qualifications>

GCE A LEVELS (ENGLAND)

National context

The structure of the education system in England consists of eleven years of compulsory schooling (up to grade 11), divided into Early Years Education, Primary Education, and Secondary Education. Primary Education goes up to age 11 or 12 (6th grade), while Secondary Education culminates at age 16 (11th grade). At the end of this grade, students take General Certificate of Secondary Education (GCSE) examinations that qualify them for entry into Further Education (senior secondary school).

After the age of 16, education is not compulsory in England, and students apply for different educational pathways in senior secondary school (grades 12 and 13) according to their interests and capacities. These pathways include the National Vocational Qualifications, the BTEC Diploma, academic Diploma-style qualifications such as the Cambridge Pre-U and the International Baccalaureate, or the Advanced Level General Certificates of Education (known as GCE A Levels), stand-alone academic qualifications.

The majority (92%) of UK students complete some form of senior secondary education, and over 55% of students go on to study at universities, primarily in the UK.⁵³ The Education and Skills Act of 2008 makes some form of either education or training compulsory for young people up to the age of 18, and will come fully into effect in 2015.

Reforms, purpose and structure of A Level qualifications

General Certificate of Education (GCE) A Levels are taken by approximately 40% of the 16-18 age group that is enrolled in some form of education or training,⁵⁴ and they are considered the primary pre-university qualification in England. A Level qualifications are regulated by the Office of Qualifications and Examinations Regulation (Ofqual), which determines the basic criteria regarding structure and desired learning outcomes, and monitors quality. The U.K. Department for Education also provides basic content guidelines, in accordance with the broader aims of the entire education system, thus ensuring articulation with higher levels of education and with workforce development needs. A Levels are not administered directly by the Department for Education, however. Rather, five government-approved providers are charged with providing and awarding A Level qualifications: the Assessment and Qualifications Alliance (AQA); the Council for Curriculum, Examinations and Assessment (CCEA); Oxford, Cambridge and RSA

⁵³ OECD, 2011a and UNESCO, 2008, cited in Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 49.

⁵⁴ Nuffield Foundation, International Comparison of Upper Secondary Mathematics Education, p. 20.

Examinations (OCR); the Welsh Joint Education Committee (WJEC); and Pearson’s Edexcel. All of these providers are members of the U.K. Joint Council for Qualifications (JCQ).

Senior secondary schools that offer A Levels set their own admissions criteria, but generally students must have passed at least five General Certificate of Secondary Education (GCSE) examinations as a pre-requisite for admission to A Level courses. A Level courses and examinations are offered in more than 45 subjects, and they can be taken in any combination.⁵⁵ Typically, senior secondary students will take three or four A Levels that will be relevant for entry into their chosen higher education program; there is no required structure or set of core courses for A Levels, however, as there would be for a Diploma-style qualification.

A Levels are commonly accepted for university entrance, but are not the only qualification available for this purpose. They also do not guarantee admission, since universities set their own requirements based on tariff points which include exam results and other criteria. These tariff points are allocated by the Universities and Colleges Admissions Service (UCAS) a centralized organization which handles all university admission procedures across the UK.

A Levels are studied over a two-year period, culminating in an AS (Advanced Subsidiary) exam at the end of the first year, and in the full A Level exam (A2) at the end of the second year. Until the implementation of recent reforms (described below), students could choose to end their course at the AS Level, considered to be worth half of a full A Level and consisting of 180 hours of coursework, or build upon the AS exam by continuing with a second year of coursework to obtain a full A Level consisting of 360 hours of coursework.

A Level examinations are externally set and marked by each qualification provider, with quality assurance measures taken by JCQ and Ofqual. Passing grades for A Levels are A*, A, B, C, D and E. The A* grade was introduced in 2010 to distinguish students who achieved an A with a score of at least 90% on a full A Level (A2) exam.⁵⁶ JCQ’s Uniform Marks Scale (UMS) converts candidates’ component raw marks into uniform marks. UMS grade boundaries for GCE are A=80%, B=70%, C=60%, D=50%, E=40%.⁵⁷

Various reforms to both GCSE and A Level qualifications have taken place over the past few years, in response to concerns over the U.K.’s ability to set educational standards that can match up to those of the highest-performing countries. Reforms to A Levels in particular, announced in 2012, strive to ensure the appropriate alignment of skills between senior secondary school and higher education. Thus, according to the Department for Education, “the new A levels will be linear qualifications that encourage development of the skills and

⁵⁵ Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 80.

⁵⁶ Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 81.

⁵⁷ Joint Council for Qualifications (JCQ) Examination System

knowledge students need for progression to undergraduate study”.⁵⁸ Revised AS and A Level courses will be taught beginning in 2015, and phased in for many subjects through 2017. These reforms include 5 main features:

- 1) Assessment will take place mainly through external examinations
- 2) Assessments will take place at the end of each year of courses (rather than at the end of each module, as is currently the case)
- 3) AS and A Level will be separated into two distinct qualifications, with AS Level exams decoupled from A Levels and no longer counting towards them
- 4) Since AS Level courses will be considered as qualifications in their own right, they can be developed independently by providers and taught *alongside* the first year of A Level courses
- 5) A Levels will include updated subject content, defined with greater input from universities.⁵⁹

Reforms, purpose and structure of A Level mathematics qualifications

The overall purpose of A Level mathematics courses, as defined by the Department for Education, is to encourage students to: “*understand* mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study; *extend their range of mathematical skills* and techniques; *understand coherence and progression* in mathematics and how different areas of mathematics are connected; *apply mathematics in other fields* of study and be aware of the relevance of mathematics to the world of work and to situations in society in general; use their mathematical knowledge to *make logical and reasoned decisions* in solving problems both within pure mathematics and in a variety of contexts, and *communicate* the mathematical rationale for these decisions clearly; reason logically and recognize incorrect reasoning; generalise mathematically; *construct mathematical proofs*; use their mathematical skills and techniques to *solve challenging problems* which require them to decide on the solution strategy; recognise when mathematics can be used to analyse and *solve a problem in context*; *represent situations mathematically* and understand the relationship between problems in context and mathematical models that may be applied to solve them; *draw diagrams and sketch graphs* to help explore mathematical situations and interpret solutions; *make deductions and inferences and draw conclusions* by using mathematical reasoning; *interpret solutions* and

⁵⁸ Department for Education, Reformed GCSE and A Level Subject Content Consultation: Government Response, January 2015, p. 5.

⁵⁹ <https://www.gov.uk/government/publications/get-the-facts-gcse-and-a-level-reform/get-the-facts-as-and-a-level-reform>

communicate their interpretation effectively in the context of the problem; *read and comprehend mathematical arguments*, including justifications of methods and formulae, and communicate their understanding; read and *comprehend articles concerning applications of mathematics* and communicate their understanding; *use technology such as calculators and computers effectively* and recognise when such use may be inappropriate; *take increasing responsibility for their own learning* and the evaluation of their own mathematical development”.⁶⁰

The two main mathematics qualifications offered are A Level Mathematics and A Level Further Mathematics; each one of these courses should cover 360 hours of course instruction, and each culminates in a final examination at the end of two years. The purpose of both of these qualifications is to support students’ entry into higher education programs; A Level Mathematics provides students with a broad but solid basis for continued study in a wide range of university subjects, while A Level Further Mathematics builds upon A Level Mathematics to provide deeper coverage of some areas of mathematics and support students’ employment or further study in more highly mathematical disciplines. Unlike some of the other qualifications analyzed in this study, these two qualifications do not represent radically different pathways for students, nor are they based on clear distinctions between pure and applied content: they both provide students with a pure mathematics core as well as a variety of applied mathematics topics.

The use of technology is encouraged in these two A Level qualifications. The effective use of technology (and recognition of cases when its use is inappropriate) is one of the overarching aims of mathematics learning, as stated above. The Department for Education also states that “the use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics. Calculators used must include the following features: an iterative function [and] the ability to compute summary statistics and access probabilities from standard statistical distributions”.⁶¹ For A Level Further Mathematics, calculators used must also include “the ability to perform calculations with matrices up to at least order 3×3 ”.⁶²

In accordance with the broader A Level reforms that are currently taking place (described above), mathematics curriculum has also been facing changes, and revised A Level Mathematics

⁶⁰ Department for Education, Mathematics AS and A Level Content, December 2014 (for teaching from 2017), p. 3-4.

⁶¹ Department for Education, Mathematics AS and A Level Content, December 2014 (for teaching from 2017), p. 6.

⁶² Department for Education, Further Mathematics AS and A Level Content, December 2014 – for teaching from 2017, p. 4

and A Level Further Mathematics courses will be introduced in classrooms from 2017 onward. In response to growing concerns in industry and higher education institutions over the need for more STEM graduates, the Department for Education has made it a priority to “encourage more 16-18 year olds to take up mathematics and science subjects.” Subject content for Math and Further Math was reviewed by the A Level Content Advisory Board (ALCAB, constituted by members of the Russell Group of Universities), which emitted its recommendations in December 2014. The Department for Education has accordingly emitted new content guidelines for these two courses, but Pearson Edexcel has not yet produced a modified syllabus.

In terms of content, the most striking change proposed by ALCAB is that A Level Mathematics will no longer offer students a choice between different areas of applied mathematics (statistics, mechanics and decision mathematics). This qualification will now contain 100% required content, which will include statistics and mechanics (and omit decision mathematics). A public consultation among teachers, administrators and other stakeholders in education reveals that 52% of respondents considered the revised A Level Mathematics content to be inappropriate; concerns were raised especially regarding the change to completely prescribed content and the ways in which this might discourage student uptake and make the qualification inflexible by eliminating students’ ability to tailor the course to their preferred area of specialization.⁶³

ALCAB has responded to these concerns by stating that the changes will strengthen the A Level Mathematics qualification, since optional modules “were often selected based on a teacher’s familiarity with subject content and not on students’ plans for further study [...] university tutors had reported that students who had studied the current mathematics A level aware often inadequately prepared for their chosen areas of study”.⁶⁴ The experts also considered it appropriate to eliminate decision mathematics from this qualification, but strengthen it in A Level Further Mathematics (where optional content is still allowed).

Recognition of A Level Mathematics qualifications

The top 5 global universities in engineering and technology (according to the 2014-15 Times Global Ranking) recognize A Level Mathematics qualifications as follows:

⁶³ Department for Education, Reformed GCSE and A Level Subject Content Consultation: Government Response, January 2015, p. 25.

⁶⁴ Department for Education, Reformed GCSE and A Level Subject Content Consultation: Government Response, January 2015, p. 26.

- **MIT:** “For a grade of A*, A, or B in A-Level Mathematics, credit will be given for 12 units of subject 18.01, Calculus I. For grades lower than B or grades in Further Mathematics, no credit is given.”⁶⁵
- **Stanford:** “International advanced placement examinations such as British General Certificate of Education "A" levels [...] may be awarded a maximum of 12 units per discipline for some of the subjects which receive College Board AP credit.”⁶⁶
- **CALTECH:** Does NOT “grant credit for AP, IB, A Level, Pre-U, or college courses taken prior to enrollment. Each student accepted to Caltech will take a math and physics placement exam prior to enrolling.”⁶⁷
- **Princeton:** A grade of A or B on an A Level mathematics exam (Maths or Further Maths are not specified) is considered equivalent to a 4 or 5 on an AP math exam (whether Calculus AB or BC is not specified) and eligible for the equivalent credit (there is no specific chart for A Level courses).⁶⁸
- **Cambridge:** *All* colleges offering Computer Science with Mathematics, Economics, Engineering and Mathematics require an A* or A in A Level Mathematics; **some of these colleges require an A* or A in AS or A Level Further Mathematics.**⁶⁹

SINGAPORE-CAMBRIDGE GCE A LEVELS

National context

The Singapore education system has been highlighted as one of the highest-performing systems globally, particularly in the area of mathematics and science. Singapore has consistently scored particularly well on international assessments such as the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). The guiding principles of the education system are: 1) to provide a broad-based and holistic

⁶⁵ http://web.mit.edu/firstyear/2018/subjects/incomingcredit/intl/a_levels.html

⁶⁶ <http://studentaffairs.stanford.edu/registrar/students/ap-international>

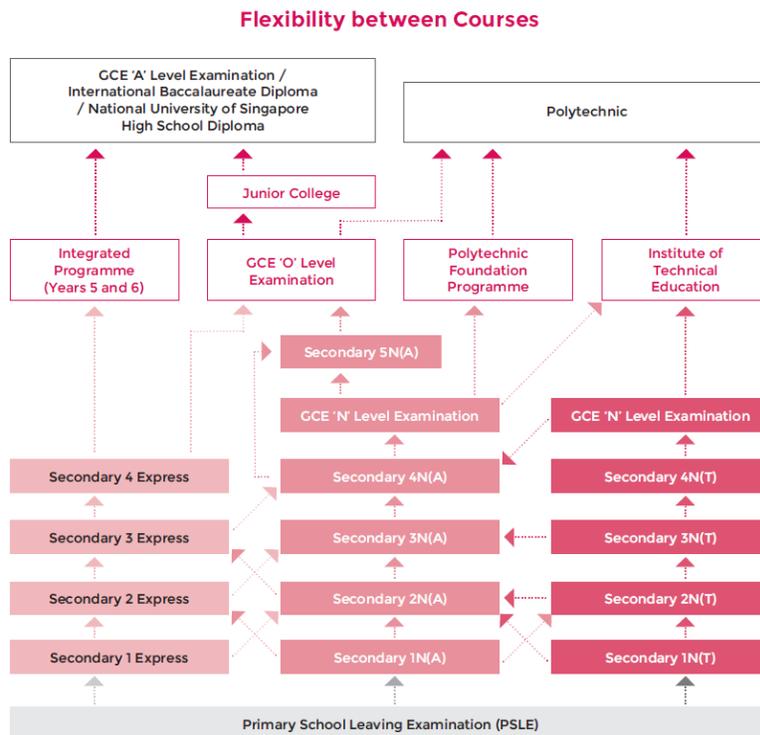
⁶⁷ <http://www.admissions.caltech.edu/content/how-apply-first-year-applicant>

⁶⁸ <http://www.princeton.edu/pub/ap/table/>

⁶⁹ Taken from various Cambridge websites by subject area, and <http://www.undergraduate.study.cam.ac.uk/applying/entrance-requirements/alevels>

education; 2) to encourage bilingualism; 3) to ensure that teachers are well-trained and supported; 4) to integrate information and communication technologies into learning; and 5) to build strong partnerships with parents.⁷⁰ The Desired Outcomes of Education, first formulated in 1997, expect students to be: 1) confident people; 2) self-directed learners; 3) active contributors; and 4) concerned citizens.⁷¹

The structure of Singapore’s education system consists of Primary School (grades 1-6, from age 6 to 11), Secondary School (secondary grades 1-4/5, ages 12/13 to 15/16), and Pre-University Education (2 grades, ages 16/17-18/19). At the Secondary level, students have a variety of schooling options, ranging from 4-year technical schools to 6-year specialized independent schools. The main options for students hoping to follow an academic pathway to universities are “normal” academic 5-year schools or “Integrated Programme” 6-year schools (which cater to students with strong academic aptitudes and provide broader learning experiences). The first of these pathways culminates in the Singapore-GCE O Level examinations (typically, at the age of 16), after which students may enter Pre-University education, consisting of 2 years of Junior College that lead to the Singapore-Cambridge GCE A Level examinations. The Integrated Program pathway leads directly to the A Level examinations, bypassing O Levels.



Source: Singapore Ministry of Education, *Bringing Out the Best in Every Child: Education in Singapore, 2015*, p. 24.

⁷⁰ Singapore Ministry of Education, *Bringing Out the Best in Every Child: Education in Singapore, 2015*, p. 5-6.

⁷¹ <http://www.moe.gov.sg/education/21cc/>

Since the latest 2010 curriculum review, the Singaporean education system has been striving to achieve a better balance between content and skills, to strengthen “21st century competencies”, to improve the use of ICTs for collaborative learning, and to develop assessments that support learning. The focus of reforms is not on content, which is well defined and has changed very little, but rather on the process of learning: the ability to foster skills and competencies among students that will prepare them better for the 21st century.⁷² The 21st Century competencies that have been fostered across the curriculum over the past few years (and which support the Desired Outcomes of Education mentioned above) are: 1) civic literacy, global awareness and cross-cultural skills; 2) critical and inventive thinking; and 3) communication, collaboration and information skills.⁷³ In terms of structure, the education system has moved towards greater flexibility for students, as well as towards a broad-based education that ensures more holistic development.

Reforms, purpose and structure of Singapore-Cambridge GCE A Level qualifications

At the Pre-University level, Junior Colleges (JC) and Centralized Institutes (CI) have traditionally provided academic preparation that culminates in A Level examinations. Since 2006, three levels of qualifications have replaced the AS and A Levels that were previously offered in various subjects, although they are still based on the same model. H1 level qualifications “offer students breadth and sufficient depth for them to acquire foundational knowledge and skills in a subject area”;⁷⁴ they require a similar amount of time as the previous AS level courses. H2 level qualifications are equivalent to the previous A Level subjects, providing broader coverage of the subject matter in double the amount of time as H1 qualifications. Finally, H3 level qualifications provide diverse opportunities for in-depth study of each subject area, building upon H2 level qualifications. H1 and H2 level courses are graded on a range from A-E as well as S for sub-pass and U for Ungraded. H3 courses receive a grade of Distinction, Merit or Pass.

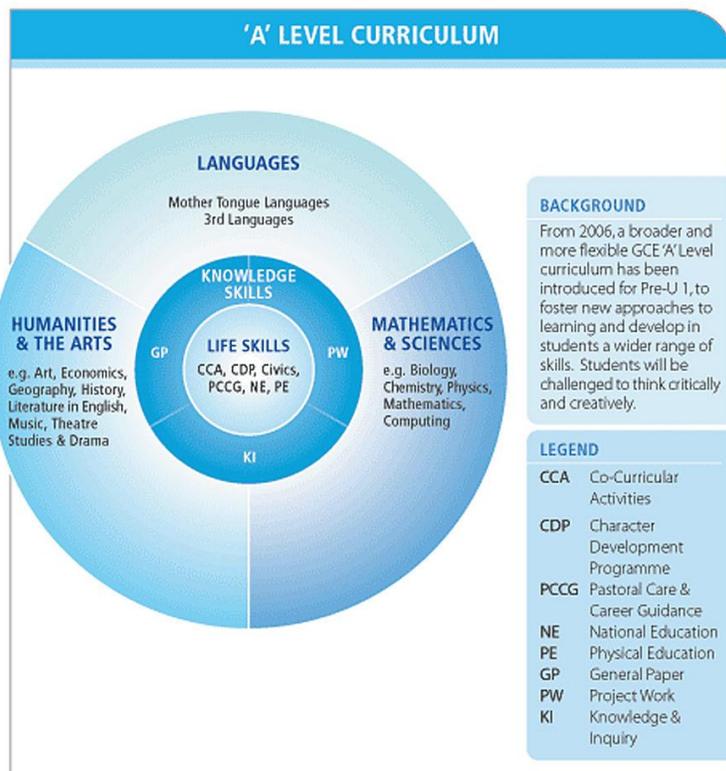
Since 2006, these three levels of courses are offered at JC and CI as part of a comprehensive and integrated “A Level Curriculum”, which is a baccalaureate-style qualification. This required curriculum aims to provide a broad and balanced overall preparation for higher education, in accordance with reforms that emphasize 21st century skills and holistic education (described above). It thus consists of Life Skills (co-curricular activities, civics, pastoral care, etc.), three compulsory Knowledge Skills components (General Paper, Knowledge & Inquiry and Project Work), and a combination of five Content Based Subjects – one must be a Mother-Tongue

⁷² Singapore Ministry of Education, Additional Mathematics (O and N-A Level) Teaching and Learning Syllabus, 2012, p. 8.

⁷³ <http://www.moe.gov.sg/education/21cc/>

⁷⁴ Singapore Ministry of Education, Bringing Out the Best in Every Child: Education in Singapore, 2015, p. 30.

Language, and the other four A Level courses (3 at H2 level and 1 at H1 level) are chosen from three areas: Languages; Humanities and Arts; and Science and Math. One of these four optional A Level courses must be a “contrasting subject” to the other three (coming from a different area), in order to ensure a balance between subjects.



Source: Singapore Ministry of Education: <http://www.moe.gov.sg/education/pre-u/curriculum/>

The University of Cambridge International Examinations (CIE), the Ministry of Education, Singapore (MoE) and the Singapore Examinations and Assessment Board (SEAB) are the joint examining authorities for the Singapore-Cambridge GCE A-Level examination. Completing this pre-university curriculum leads to a School Graduation Certificate. In addition to being a qualification certifying upper secondary completion, the A Level Curriculum also supports university entrance, particularly since higher education institutions adopted a new admissions framework in 2008 which aligns with the objectives of the new curriculum.⁷⁵

⁷⁵ <http://www.moe.gov.sg/cpdd/alevel2006/university/uni.htm>

Reforms, purpose and structure of Singapore-Cambridge GCE A Level mathematics qualifications

The aims of mathematics education in Singapore are to enable students to: acquire and apply mathematical concepts and skills; develop cognitive and metacognitive skills through a mathematical approach to problem solving; and develop positive attitudes towards mathematics. These basic aims are elaborated upon in different ways as the curriculum progresses from primary to pre-university. At the pre-university level, the purpose of H1 Mathematics is to enable students to acquire mathematical concepts and skills to support their tertiary studies in business and the social sciences; H2 Mathematics should enable students to acquire mathematical concepts and skills to support their tertiary studies in mathematics, sciences and engineering; and H3 Mathematics is meant to enable students with a high aptitude in mathematics to acquire advanced mathematical concepts and skills, in order to deepen their understanding of mathematics, and to widen the scope of applications of mathematics.⁷⁶

The mathematics curriculum consists of a set of connected syllabi from primary to pre-university; at each level, the skills acquired in the previous level are built upon and elaborated. At the secondary school level, students who take S1-4 Mathematics (O Level) progress either directly to H1 Mathematics (AS Level equivalent), or to S3-4 Additional Mathematics (also O Level). Those completing S3-4 Additional Mathematics are then qualified to enter the more rigorous H2 mathematics (A Level).⁷⁷

None of the mathematics courses available at the pre-university level (H1, H2 or H3) are a compulsory part of the Singapore A Level Curriculum. Thus, those students who choose one of these qualifications has a particular interest in preparing themselves for certain areas of higher education, or in taking the lower-level option (H1) as a contrasting course to their other areas of main interest. Data for 2009 indicates that the majority (89.7%) of those students who chose a mathematics A Level course took H2 Mathematics, while only 9.5% took H1 Mathematics and 2.6% took H3 Mathematics.⁷⁸

The use of technology is encouraged in mathematics, and students are assumed to have access to graphing calculators during the examinations for all three levels (H1, H2 and H3). In these

⁷⁶ Singapore Ministry of Education, Additional Mathematics (O and N-A Level) Teaching and Learning Syllabus, 2012, p. 10.

⁷⁷ Singapore Ministry of Education, Additional Mathematics (O and N-A Level) Teaching and Learning Syllabus, 2012, p. 11.

⁷⁸ Nuffield Foundation, An International Comparison of Upper Secondary Mathematics Education: 24 Country Profiles, 2010, p. 80.

examinations, “unsupported answers obtained from a GC are allowed unless the question specifically states otherwise”. For H3 Mathematics, it is specified that the use of a GC “without computer algebra system” is expected in the examination.⁷⁹

Recognition of Singapore-Cambridge GCE A Level mathematics qualifications

Universities in Singapore require the fulfillment of the main elements of the A Level Curriculum in order to be considered for admission. The National University of Singapore (NUS), Nanyang Technological University (NTU), Singapore Management University (SMU), Singapore University of Technology and Design (SUTD), Singapore Institute of Technology (SIT) and SIM University (UniSIM), require, at a minimum, two H2 passes and attempted General Paper (GP) or Knowledge & Inquiry (KI) in the same sitting, and fulfillment of the Mother Tongue Language (MTL) requirement.⁸⁰

With regard to mathematics courses in particular, the National University of Singapore (ranked #13 globally in engineering & tech) considers H1 and H2 courses as minimum academic requirements for admission (although these do not guarantee entry). The prerequisite H1 and H2 courses are specified for various degrees. Surprisingly, H3 courses are not mentioned at all (with regard to admissions or advanced placement).⁸¹

GAOKAO (CHINA)

National Context

The Chinese education system consists of 9 years of compulsory schooling, split into two levels: 6 years for Primary School and 3 years for Junior Secondary School. In some districts, such as Shanghai, students remain in Primary School for 5 years, and in Junior Secondary School for 4 years. At the end of Junior Secondary School, students take the Zhongkao examination, a summative assessment of subject knowledge covered through 9 years of compulsory schooling which also determines entry into Senior Secondary School. Nearly 90% of Junior Secondary School students go on to this level; they attend key senior high school, ordinary senior high

⁷⁹ Mathematics H3 Syllabus 2016, p.3

https://www.seab.gov.sg/content/syllabus/alevel/2016Syllabus/9824_2016.pdf

⁸⁰ <http://www.moe.gov.sg/cpdd/alevel2006/university/appl.htm>

⁸¹ <http://www.nus.edu.sg/oam/apply/local/admissionreq/BYA-admission.html> and <http://www.nus.edu.sg/oam/apply/local/prerequisites/BYA-prerequisites.html>

school or vocational school, depending on the scores they obtained on the Zhongkao.⁸² The Senior Secondary level of schooling consists of three years of coursework that lead to a Senior Secondary Graduation Exam (Huikao). The ultimate aim of Senior Secondary School, however, is preparation for the Gaokao, the National Higher Education Entrance Exam. Approximately 80% of Senior Secondary students enter some form of higher education.⁸³

18-	University, college, high vocational schools, employment	
Gaokao		
15-17	Senior high school, secondary vocational school	
Zhongkao		
6-15	13-15 (12-15)	Junior high school
	6-12 (6-11)	Elementary school
Nine-year compulsory education		

Figure 1. School education system in China

Source: Yingkang Wu, “The Examination System in China: The Case of Zhongkao Mathematics”, 12th International Congress on Mathematical Education, July 2012, p. 3.

The Chinese education system is extremely exam-driven, and both secondary and senior secondary level schooling is fundamentally oriented towards preparing students for the two major examinations, Zhongkao and Gaokao. The content of these exams exerts the greatest influence on the topics that students study throughout these two levels of schooling. China’s curriculum is highly centralized, with all schools following national guidelines which focus on building strong foundational knowledge and mastering core concepts. Content knowledge is generally emphasized over competencies or skills.⁸⁴

Reforms, purpose and structure of Gaokao qualifications

The Gaokao (National Higher Education Entrance Examination) is administered by provincial level authorities, under the control of the National Education Examinations Authority (NEEA). It is an annual examination, generally taken in the last year of Senior Secondary School, which

⁸² Ministry of education of China, 2010, cited in Yingkang Wu, “The Examination System in China: The Case of Zhongkao Mathematics”, 12th International Congress on Mathematical Education, July 2012

⁸³ Ministry of education of China, 2010, cited in Yingkang Wu, “The Examination System in China: The Case of Zhongkao Mathematics”, 12th International Congress on Mathematical Education, July 2012

⁸⁴ Aiping Xu, Investigation of International Mathematical Cultures, 2012, p. 23.

selects students for university admission. The Gaokao has been the principal determinant for entrance to higher education since its inception in 1950.

The Gaokao consists of three compulsory subjects: Chinese, Mathematics and Foreign Language; in addition, students take a module that consists of liberal arts subjects (history, geography and politics), one that consists of science subjects (chemistry, biology and physics), or one that consists of a mix of both these areas.⁸⁵ Each of these four subject areas is scored out of 150, and the overall mark is generally a weighted sum of all subject marks. Each provincial authority sets the cutoff marks for each category of achievement, which in turn determines the higher education courses that students are eligible for.⁸⁶ The Gaokao is thus an extremely high-stakes examination, since results determine the kind of college or university that students can attend.

This examination has been criticized for promoting “teaching to the test” across all levels of schooling, for stifling students’ creativity and critical thinking skills, for creating excessive stress among students, and for impeding universities from being able to select students who may have desirable specialized skills but were not able to do well on all areas of the comprehensive examination. On the other hand, the examination has been upheld by many as an example of meritocratic selection and as a way of truly leveling the playing field for all students in a country where connections and social networks (*guanxi*) as well as corruption could otherwise be the main determinants for entrance to higher education.⁸⁷

Most importantly, the Gaokao examination is not merely an educational assessment tool; it is deeply rooted in Chinese culture and society, with key implications for social mobility. Thus, its pros and cons are the subject of heated debates, but it has ultimately not been possible for the Chinese government to propose replacing the exam with an entirely different system. According to various authors who have analyzed attempts at Gaokao reform, the key dilemma is between the current need to introduce a more flexible, differentiated admissions system that might better serve the needs of higher education and of the economy, and the need to maintain a uniform, objective admissions system that has long been considered by the Chinese as the fairest mechanism for social mobility.⁸⁸

⁸⁵ Zhuran Yu and Yingzi Hu, “Walking a Policy Tightrope: The Dilemma of Balancing Diversification and Equality in Chinese College Entrance Examination Reform”, Higher Education Policy, 2013, no. 26, p.26.

⁸⁶ Ofqual, International Comparisons in Senior Secondary Assessment, 2012

⁸⁷ Zhuran Yu and Yingzi Hu, “Walking a Policy Tightrope: The Dilemma of Balancing Diversification and Equality in Chinese College Entrance Examination Reform”, Higher Education Policy, 2013, no. 26, p. 314-315.

⁸⁸ See Haifeng Liu and Qiong Wu, “Consequences of College Entrance Exams in China and the Reform Challenges”, KEDI Journal of Education Policy, 2006, no. 3:1, and Zhuran Yu and Yingzi Hu, “Walking a Policy Tightrope: The Dilemma of Balancing Diversification and Equality in Chinese College Entrance Examination Reform”, Higher Education Policy, 2013, no. 26.

Attempts to reform the Gaokao, which are integrally related to attempts at broader educational reform, began in 1999, when the Chinese government introduced the idea of “Quality Education” to refer to a more well-rounded education that could foster greater innovation and critical thinking – necessary components for a growing, externally competitive economy. A New Curriculum was launched in 2001, but it was not well implemented in Chinese schools and also showed a decline in Gaokao results among those schools that did adopt it. After this, the Chinese government has attempted less ambitious reforms, and has slowly changed only certain aspects of the Gaokao.⁸⁹

The Gaokao has been gradually decentralized, with provinces developing their own versions; by 2013, 16 provinces and municipalities had been authorized to hold their own Gaokao exams. This measure was meant to increase flexibility and adapt the exam to local conditions; it has been criticized, however, for increasing the existing inequalities among regions since the cutoff points set by universities may now differ according to the province the student comes from. In 1999, the Gaokao was also reduced from 6 separate subject tests to the “3 + 1” model that exists today, in order to ease the burden on students. Finally, in 2002 the government allowed several provinces to begin offering a second round of Gaokao examinations during the year, which offer students a second chance to pass. This may have reduced the stress on students, but the “second Gaokao” is not considered as rigorous as the first and is thus not accepted for admissions at 4-year public colleges.⁹⁰

Reforms, purpose and structure of Gaokao mathematics qualifications

Mathematics education is highly emphasized in China, throughout K-12 education. At the primary school level, it is not uncommon for students to learn mathematics from a specialized teacher, rather than the classroom teacher. At this level, “making connections between content areas is regarded as crucial [and] the use of multiple methods to solve a mathematical problem is widely applied.”⁹¹

At the Junior Secondary School level, compulsory mathematics content includes algebra (identities - laws of indices, laws of square root, logarithms); Equations and Inequalities (first degree, quadratic, systems of equations -linear and quadratic, irrational, logarithmic); Sequences and Series (arithmetic, geometric); Geometry (congruency and similarity, notable

⁸⁹ Zhuran Yu and Yingzi Hu, “Walking a Policy Tightrope: The Dilemma of Balancing Diversification and Equality in Chinese College Entrance Examination Reform”, Higher Education Policy, 2013, no. 26, p. 313.

⁹⁰ Zhuran Yu and Yingzi Hu, “Walking a Policy Tightrope: The Dilemma of Balancing Diversification and Equality in Chinese College Entrance Examination Reform”, Higher Education Policy, 2013, no. 26., p. 316-317.

⁹¹ Aiping Xu, Investigation of International Mathematical Cultures, 2012, p. 23.

lines and points of triangle, angles of triangle, relation between the angles and sides, Pythagorean theorem, circle – Thales theorem, circumferential and central angles, chord quadrangle, tangent quadrangle); and Probability and Statistics.⁹² In 2009, Shanghai 15-year old students ranked top in the mathematics portion of the PISA international assessment, indicating the high level of quality of Junior Secondary School mathematics in that province.

The mathematics portion of the Gaokao examination emphasizes problem-solving and pure mathematics; according to the ICOSSA study, “the questions cover a very narrow and formal range of mathematics” with little calculus or trigonometry, and a significant amount of geometric reasoning, three-dimensional geometry, formal proof and sequences and series. Calculators are not allowed in the examination.⁹³

The Senior Secondary School curriculum has been reformed over the past years to allow for some choice of topics and to add new content in the areas of calculus, statistics and probability and algorithms, among others. At the same time, the complexity of formulas and terminology has been decreased, although there continues to be a very analytical focus and an emphasis on formal proof.⁹⁴ Recently, it has been reported that AP-style courses are being planned for Chinese high schools: “more than a hundred mathematicians at high schools across China, including the [High School attached to Tsinghua University](#), are drafting plans to pilot college credit courses by the 2015 graduation year; Potential courses include linear algebra, integral calculus, and inferential statistics”.⁹⁵

Recognition of Gaokao mathematics qualifications

Since 2001, the Ministry of Education has authorized top-ranking universities to establish their own entry exams as supplements to the Gaokao exam; by 2011, 80 higher education institutions had autonomous entry exams. These exams have been criticized for being exclusionary of students who may not have the resources to come to the university from other provinces; they are also costly for universities to implement on a wider scale and are used only for some subject areas; thus, the Gaokao remains the mainstream form of admission. Since 2010, three separate consortia of universities have emerged which join the resources of several institutions in order to implement entrance exams that can be used across all participating institutions. In addition, Southern University of Science and Technology of China has decided to admit students through its own independent admission policies, even if this means that its

⁹² Wang, Changing Framework for Teaching of Mathematics in China, p. 2.

⁹³ Ofqual, International Comparisons in Senior Secondary Assessment, 2012

⁹⁴ Aiping Xu, Investigation of International Mathematical Cultures, 2012, p. 25.

⁹⁵ <http://thediplomat.com/2014/03/china-education-reforms-to-emulate-advanced-placement-system/>

graduates will not have official recognition from the Ministry of Education.⁹⁶ All of these developments indicate that the higher education sector is attempting to open up greater spaces of autonomy, and may ultimately lead the government to make broader changes to the Gaokao system. Indeed, in 2010 the Ministry of Education published a ten-year plan for Chinese education, which includes the “exploration of a subject-based multi-test system and ‘societally-adapted’ testing.”⁹⁷

Some universities abroad explicitly recognize the Gaokao for admission purposes, particularly in Australia. At the same time, a growing number of Chinese students at high-performing secondary schools are opting to prepare themselves for SAT in order to go abroad for higher education, thus bypassing the Gaokao entirely.⁹⁸

COMPARATIVE SUMMARY

- **Context:** Four of the qualifications analyzed in this report are provincial or national – the Alberta Diploma, A Levels for the U.K., the Gaokao for China, and Singapore-Cambridge A Levels – while two are wholly independent from state or provincial control and are implemented at individual schools locally and internationally – Advanced Placement and the International Baccalaureate. The first four qualifications are thus fundamentally influenced by changes in government policy, and respond to overall education reform programs. Due to its large presence in schools across the US, Advanced Placement is also considerably influenced by US education policy (and is a key actor in influencing it as well). Both AP and the IB work with governments across the world through various partnerships.
- **Structure:** Three of the six qualifications analyzed are baccalaureate-style qualifications (the Alberta Diploma, Singapore-Cambridge A Level Curriculum, and the International Baccalaureate Diploma), whose purpose is to provide students with a holistic education that covers a variety of subjects from different areas. The Chinese Gaokao is a comprehensive examination that defines the content of the Senior Secondary curriculum; thus, it also requires students to follow a certain fixed combination of subjects. Advanced Placement and A Levels, in contrast, provide stand-alone qualifications in a variety of subjects that can be taken independently of each other. Nonetheless, a given set of AP or A Level subjects may be required for high school graduation by certain districts or schools that offer them. It is also notable that AP has been moving towards a Diploma-style qualification with its new Capstone courses.

⁹⁶ Zhuran Yu and Yingzi Hu, “Walking a Policy Tightrope: The Dilemma of Balancing Diversification and Equality in Chinese College Entrance Examination Reform”, *Higher Education Policy*, 2013, no. 26, p. 319.

⁹⁷ <http://ednewschina.com/?p=408>

⁹⁸ Aiping Xu, *Investigation of International Mathematical Cultures*, 2012, p. 26.

- **Purpose:** All of these qualifications are essentially high-school leaving certificates, with the exception of the Chinese Gaokao, which is an examination for university admission. Nonetheless, all of the qualifications are meant to go beyond senior secondary level studies by providing students with a solid preparation for university, and they are all used by higher education institutions for the purpose of admission and allocating advanced placement.
- **Reforms:** The overall curriculum review processes taking place in Alberta, in the UK (with regard to A Levels), in Advanced Placement courses, in the Chinese Gaokao and in Singapore all respond to the broader need to better align Senior Secondary education with the needs of higher education. This requires, on the one hand, a higher level of preparation for STEM careers (more academic rigor among incoming students), and on the other hand, students who are more well-rounded and possess the “21st century skills” (including the capacity for creativity and innovation) that are needed for success in a globalized world. Thus, all of these reforms emphasize the development of competencies more than the mere addition of more content, the inter-relation of themes, the applicability of knowledge to real-life issues, and the effective integration of technology into learning. They do so to varying degrees, however, with the Chinese Gaokao having the most conservative approach and advancing slowly, and the Alberta reforms making the broadest moves in this direction. AP and A Level reforms are perhaps the most concerned with having more input from higher education institutions and linking courses more closely with college-level courses. It is notable that A Level reforms are focused on creating more rigorous, focused qualifications, even at the cost of eliminating flexibility and student choice, while Alberta (and to some extent, Singapore and China) has moved in the opposite direction, emphasizing greater flexibility in order to diversify pathways.
- **Mathematics component:** Mathematics is a required component of only three of these qualifications: the Alberta Diploma, the Gaokao, and the International Baccalaureate Diploma.
- **Mathematics pathways:** With the exception of the Gaokao, all other qualifications offer several different mathematics qualifications that respond to different higher education pathways. The IB Diploma offers the largest number of pathways (4); the other qualifications offer two or three. AP is the only provider that offers specialized mathematics courses (in Calculus and Statistics), rather than broader courses oriented towards different levels of preparation.

- **Broader aims of mathematics education:** With the exception of the Gaokao, all of these qualifications hope to encourage a broader appreciation of mathematics as a notable human endeavor; the use of mathematics for strengthening logical reasoning skills; the ability to apply mathematics to daily life problems; and the ability to see the ways in which mathematics is connected to other subject areas, among other overarching aims. The International Baccalaureate and the Alberta Diploma are the only qualifications that include an appreciation of the international dimensions and multicultural history of mathematics, however.
- **Technology policies:** With the exception of the Gaokao, all of these qualifications encourage the use of technology in mathematics, and all of them allow (and even require) the use of graphing calculators in at least some portions of their final examinations.
- **Recognition:** At top global universities, AP, IB and A Level qualifications have the greatest level of recognition overall; the other qualifications are recognized at a more local level, but do not have explicit policies formulated for them in these top-ranked higher education institutions. AP's Calculus BC is the most widely accepted for advanced credit, and AP courses overall seem to have the most detailed credit policies at all of the US-based universities. It is notable that top global universities (or national ones, in the case of Canada and Singapore) do not seem to explicitly recognize the merits of some of the highest-level qualifications (particularly, IB's Further Mathematics HL, Singapore's H3 and Alberta's Math 31; in some cases, A Level Further Mathematics as well).

PART B: COMPARATIVE ANALYSIS OF CURRICULUM STRUCTURE AND CONTENT

As mentioned in the Introduction, the purpose of this section is to provide detailed information on the specific curriculum structure and mathematical content of each qualification and conduct an initial comparative analysis across all qualifications, in order to understand the extent of alignment between IB qualifications and other qualifications. This can also contribute to an overall comparison of the level of demand of these qualifications.

Notes on the qualifications

Each of the broad qualifications analyzed in this study contains several different specific mathematics qualifications, as described in previous sections. Thus, a total of 16 qualifications were analyzed for this report, including one specialized qualification, AP Statistics. The majority of the comparisons will refer only to the 15 qualifications that provide general mathematics tracks. Other aspects of these qualifications to be noted are as follows:

- With regard to UK A Levels, Edexcel’s Pure Mathematics and Further Mathematics (Additional) were excluded from this study because they represent specific variations of the two mainstream GCE Advanced Level qualifications: A Level Mathematics and A Level Further Mathematics (as defined by the UK’s Department of Education). The Department for Education’s new syllabus guidelines, to be implemented from 2017 (see discussion in Part A of this report) were also not included, since Pearson Edexcel’s syllabi remain unchanged up to now. ***It is important to note that these new guidelines would considerably alter the way in which A Level Mathematics is categorized and analyzed in the following charts, since they make statistics and mechanics topics compulsory, and eliminate decision mathematics.***
- Alberta Mathematics 31 is a course that does not culminate in a final Diploma examination; it is considered an additional course that supersedes the requirements of the Alberta Diploma. It has nonetheless been included because it shares the same objective as the other qualifications included in this study – to provide students with a sound mathematical base for entry into higher education.
- The data included in this comparative analysis for the Chinese Gaokao is taken from Ofqual’s 2012 ICOSSA, which indicates the overall content strands covered by this qualification. More detailed information on the precise content strands covered by the Chinese Gaokao was not available in English. Thus, it is not included in many aspects of the content analysis carried out below; when “all qualifications” are referred to, it is to be assumed that the Gaokao is not included, unless otherwise specified.
- The pre-requisites for AP Calculus AB and BC are not specified, since the College Board does not define the specific areas of pre-calculus that must be covered; it assumes that each school that decides to implement AP courses will be responsible for preparing students accordingly. Required learning hours are also not specified for AP courses.

COMPARISON OF QUALIFICATIONS ACCORDING TO THEIR STATED PURPOSE AND BASIC STRUCTURE

Methodology

The purpose of this first comparison is to ascertain which qualifications appear to be more closely aligned with others and can be considered relatively equivalent. In order to do so, this analysis compares all of the qualifications based on their “face comparability”⁹⁹ – the similarity across a set of qualifications with regard to their stated purpose and the group of students they are oriented towards. Based on this similarity, the qualifications are categorized into different

⁹⁹ Elliott, 2011, p. 10, cited in Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 18.

groups. The term “group” is used rather than the term “level”, in order to avoid making premature judgements that would establish a hierarchy among them; rather, they are simply assumed to be serving different purposes.

The following features of each qualification are described and compared, in order to determine key similarities and differences and establish a certain equivalency within each group:

- **Intended purpose:** the scope of mathematical content and the preparation for university that the qualification hopes to provide
- **Type of student:** the level of preparation (pre-requisites), motivation and aspirations of the students the qualification caters to
- **Content structure:** the proportion of course content and examination topics that are required or optional; this indicates whether the student will have the opportunity to tailor the qualification towards a certain desired specialization
- **Assessment structure:** the proportion of content that is internally or externally assessed, and whether any independent research project is also required. On the one hand, more varied forms of assessment can indicate that the qualification encourages multi-dimensional and holistic mathematical development among students¹⁰⁰; on the other hand, internal assessment alone may reduce the level of rigor required to meet externally defined and fixed objectives
- **Cognitive skills:** the broad cognitive understandings and mathematical thinking skills that are required by each qualification provide an indication of its overall orientation
- **Type of mathematical content:** the proportion of content that is dedicated to pure mathematics and to applied mathematics; this is closely linked to the overall purpose of the qualification and the type of student it is oriented towards, since it can broaden or restrict the fields of study that students may go into
- **Level of mathematical content:** whether the mathematical content is mostly at a pre-calculus level, whether it covers calculus, or whether it goes beyond calculus into other advanced areas of mathematics. This provides a clearer indication of the level of demand of each qualification (to be discussed in the final section of this report)

¹⁰⁰ The ICOSSA study emphasizes this point

- **Breadth and depth of content:** the number of main units or areas covered, together with the number of hours required or recommended for each course, provides an idea of the balance between breadth and depth

GROUP I: QUALIFICATIONS THAT PREPARE STUDENTS FOR PRACTICAL APPLICATIONS AND UNIVERSITY PROGRAMS WITH LESS MATHEMATICAL CONTENT

	Alberta Math 30-2	Alberta Math 30-1	IB Math Studies SL
Type of student/purpose	<p>This course “emphasizes the mathematical understandings and critical thinking skills for daily life, direct entry into the workforce, and post-secondary studies in programs that do not require the study of calculus”.</p> <p><i>(Information Bulletin Math 30-2, 2014-15)</i></p>	<p>Purpose is “to provide students with the knowledge base, mathematical understanding and critical thinking skills identified for entry into post-secondary programs that require the study of calculus”.</p> <p><i>(Information Bulletin Math 30-1, 2014-15)</i></p>	<p>The course emphasizes “applications of mathematics and statistical techniques. It is designed to offer students with varied mathematical backgrounds and abilities the opportunity to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics, preparing them to solve problems in a variety of settings, develop more sophisticated mathematical reasoning and enhance their critical thinking”. <i>(IB Diploma Subject Brief: Mathematical Studies SL, 2014-2020)</i></p> <p>“The students most likely to select this course are those whose main interests lie outside the field of mathematics, and for many students this course will be their final experience of being taught formal mathematics. Students likely to need mathematics for the achievement of further qualifications should be advised to consider an alternative mathematics course.”</p> <p>http://www.ibo.org/</p>
Total recom. course teaching time	125 hours <i>(Icossa study)</i>	125 hours <i>(Icossa study)</i>	Total recommended teaching hours, including individual project work = 150 hours
Pre-requisite content areas	Mathematics 20-2: 1) Measurement 2) Geometry 3) Number & Logic	Mathematics 20-1: 1) Algebra & Number 2) Trigonometry 3) Relations & Functions	1) Basic arithmetic, algebraic expressions, linear equations, inequalities

	4) Statistics 5) Relations & Functions + Math Research Project		2) Collection of data and its representation 3) Basic geometric concepts
Content areas (required & optional)	Required topics: 1) Logical Reasoning 2) Probability 3) Relations & Functions Total areas: 3 + Research Project	Required topics: 1) Trigonometry 2) Relations & Functions 3) Permutations, Combinations & Binomial Theorem Total areas: 3	Required topics and rec. teaching hours: 1) Number and Algebra (20) 2) Descriptive Statistics (12) 3) Logic, Sets & Probability (20) 4) Statistical Applications (17) 5) Geometry and Trigonometry (18) 6) Mathematical Models (20) 7) Introduction to Differential Calculus (18) 58 hrs for pure math topics; 49 hrs. for statistics & probability; 18 hrs for calculus Total areas: 7 + Individual Project (25 hours)
Broad assessment objectives: Expected cognitive skills/ math processes, understanding	Cognitive levels: 1) Conceptual 2) Procedural 3) Problem-solving Math processes: Communication, Connection, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization	Cognitive levels: 1) Conceptual 2) Procedural 3) Problem-solving Math processes: Communication, Connection, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization	1) Knowledge and understanding 2) Problem-solving 3) Communication and interpretation 4) Technology 5) Reasoning 6) Inquiry approaches <i>(IB Diploma Subject Brief: Mathematical Studies SL, 2014-2020)</i>
Final assessment/ exam: % emphasis on each content area	Content area: 1) Logical Reasoning 17% 2) Probability 33% 3) Relations & Functions 50% Total time: 3 hrs.	1) Trigonometry – 29% 2) Relations & Functions - 55% 3) Permutations, Combinations & Binomial Theorem - 16% Total exam time: 3 hrs.	Each part of the final examination covers material from all sections of the syllabus. Total examination time: 3 hours
Internal/ external assessment	50% internal (school-based); 50% external (final Diploma exam)	50% internal (school-based); 50% external (final Diploma exam)	20% Internal assessment (component internally assessed by the teacher and externally moderated by the IB at the end of the course); 80% external assessment (final Diploma exam)

- **Intended purpose:** These three qualifications are all meant to prepare students for university programs that require less mathematical preparation, although Alberta Math 30-1 states that it does intend to prepare students for courses with basic calculus content.
- **Type of student:** All three courses are oriented towards students who have a basic level of mathematical background knowledge – its prerequisites consist of basic algebra and functions, geometry and statistics. Only Alberta Math 30-1 requires some knowledge of trigonometry as well.
- **Content structure:** These three qualifications all contain required content, without any options available to students, indicating an emphasis on covering the basics rather than on offering students routes towards specializations.
- **Assessment structure:** All three qualifications have an internally assessed component, with an especially high proportion of internal assessment for the two Alberta math courses. The total exam time is the same for all three.
- **Cognitive skills:** All three qualifications expect the same types of cognitive skills to be developed; however, the Alberta math courses also include Mental Mathematics and Estimation, while IB Math Studies SL includes Inquiry Approaches.
- **Type of mathematical content:** All three of these qualifications emphasize Pure Mathematics more than Applied Math topics (as shown by number of teaching hours and emphasis given on final exam). However, IB Math Studies SL emphasizes statistics much more than the other two qualifications. Alberta Math 30-2 and IB Math Studies SL both include an independent research component, which can be seen as emphasizing practical applications.
- **Level of mathematical content:** These are all “pre-calculus” courses that cover functions, trigonometry, and geometry. Only IB Math Studies SL includes an introduction to differential calculus, thus perhaps providing a more solid preparation to this group of students.
- **Breadth and depth of content:** IB Math Studies SL covers a broader range of topics than the two Alberta courses, in approximately the same amount of time; this indicates that it may not be able to cover topics with the same depth.

GROUP II: QUALIFICATIONS THAT COVER BASIC CALCULUS TOPICS AND PREPARE STUDENTS FOR UNIVERSITY PROGRAMS REQUIRING BROAD OR APPLIED MATHEMATICAL CONTENT

	Singapore H1 Math	IB Math SL
Type of student/ purpose	<p>“The syllabus provides a foundation in mathematics for students who intend to enroll in university courses such as business, economics and social sciences [...] The main focus of the syllabus will be the understanding and application of basic concepts and techniques of statistics. This will equip students with the skills to analyze and interpret data, and to make informed decisions.”</p> <p>The general aims of the syllabus include enabling students to “acquire the necessary mathematical concepts and skills for everyday life, and for continuous learning in mathematics and related disciplines”. <i>(H1 Mathematics Syllabus, 2016)</i></p>	<p>The intention of the course is “to introduce students to [important mathematical concepts] in a comprehensible and coherent way, rather than insisting on the mathematical rigor required for mathematics HL. Students should, wherever possible, apply the mathematical knowledge they have acquired to solve realistic problems set in an appropriate context”. <i>(IB Diploma Subject Brief: Mathematics SL, 2014-2020)</i></p> <p>“This course caters for students who already possess knowledge of basic mathematical concepts [...] The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.” <i>(IB Mathematics SL Guide, 2014)</i></p>
Total recommended course teaching time/ guided learning hours	H1 is taken in half the time of H2; thus, it is considered equivalent to the GCE AS Level = 180 hours (1 year) <i>(Singapore MOE)</i>	Total recommended teaching hours, including individual exploration work = 150 hours
Pre-requisite content areas	GCSE Level Mathematics: 1) Number & Algebra 2) Geometry & Measurement 3) Statistics & Probability	1) Number (arithmetic) 2) Sets and Numbers 3) Algebra 4) Trigonometry 5) Geometry 6) Coordinate Geometry 7) Statistics and Probability
Content areas (required & optional)	Required topics: A. Pure Mathematics: 1) Functions & Graphs 2) Calculus B. Statistics 1) Probability 2) Binomial & Normal Distributions 3) Sampling & Hypothesis Testing	Required topics and rec. teaching hours: 1) Algebra (9) 2) Functions & Equations (24) 3) Circular Functions & Trigonometry (16) 4) Vectors (16) 5) Statistics & Probability (35) 6) Calculus (40) 65 hrs for pure mathematics topics;

	4) Correlation & Regression Total broad areas: 2 Total sub-areas: 6	35 hrs. for statistics & probability; 40 hrs for calculus Total areas: 6 + Individual Exploration (10 hours)
Broad assessment objectives: Expected cognitive skills/ mathematical processes, understanding	“1) understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphic calculators 2) reason and communicate mathematically through writing mathematical explanation, arguments and proofs, and inferences 3) solve unfamiliar problems; translate common realistic contexts into mathematics; interpret and evaluate mathematical results, and use the results to make predictions or comment on the context” <i>(H1 Mathematics Syllabus, 2016)</i>	1) Knowledge and understanding 2) Problem-solving 3) Communication and interpretation 4) Technology 5) Reasoning 6) Inquiry approaches <i>(IB Diploma Subject Brief: Mathematics SL, 2014-2020)</i>
Final assessment/ exam: % emphasis on each content area	Final exam consists of one 3-hour paper consisting of 95 marks as follows: 1) Pure Mathematics – 35 marks 2) Statistics – 60 marks Total time: 3 hours	Final exam consists of two 1.5 hour papers; each one contains both short-response and extended-response questions based on the whole syllabus. Total exam time: 3 hrs.
Internal/external assessment	External assessment through final examination	20% Internal assessment is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. 80% external assessment through final Diploma exam.

- **Intended purpose:** The purpose of both of these qualifications is to provide students with a stronger basis for entering university programs that will require mathematics, even if this may be in a more applied form. They also emphasize using math for practical applications.
- **Type of student:** These qualifications are oriented towards students who have some background in math – they require knowledge of standard pre-calculus topics as a prerequisite (although Singapore H1 does not require previous knowledge of trigonometry).
- **Content structure:** Neither of these qualifications provide optional content areas.

- **Assessment structure:** Both qualifications are mostly assessed externally.
- **Cognitive skills:** Expected cognitive skills cover essentially the same areas.
- **Type of mathematical content:** Both of these qualifications cover pure math and applied math; however, they place different degrees of emphasis on both. Singapore H1 places the greatest emphasis on applied math (Statistics and Probability - 60% of all final exam marks). IB Math SL only dedicates 23% of the time to an applied topic (Statistics & Probability). However, IB Math SL includes an independent math project, which provides an additional opportunity to apply mathematical concepts.
- **Level of mathematical content:** Both of these qualifications provide an introduction to calculus topics; Singapore H1 includes parametric equations, IB Math SL does not. Both include applications of calculus, but IB Math SL is the only one to include kinematics (refer to mathematical content charts in next section for these details).
- **Breadth and depth of content:** both of these qualifications cover the same number of topics (6) in a similar amount of time. IB Math SL requires a little less time, however, and also must allow students 10 hours of this time for their individual projects, indicating that it may not cover the 6 topics in as much detail as Singapore H1.

GROUP III: QUALIFICATIONS THAT COVER CALCULUS TOPICS AND PREPARE STUDENTS FOR UNIVERSITY PROGRAMS REQUIRING A SOLID BASE IN MATHEMATICS

	A Level Mathematics	Singapore H2 Math	IB Math HL
Type of student/ purpose	“A level mathematics builds from GCSE level mathematics and introduces calculus and its applications. It emphasizes how mathematical ideas are interconnected and how mathematics can be applied to model situations mathematically using algebra and other representations, to help make sense of data, to understand the physical world and to solve problems in a variety of contexts, including social sciences and business. It prepares students for further study and	“The syllabus prepares students adequately for university courses including mathematics, physics and engineering, where more mathematics content is required. The syllabus aims to develop mathematical thinking and problem solving skills in students. [Students] will also learn to work with data and perform statistical analyses.” <i>(SEAB, Mathematics Higher 2 Syllabus, 2015)</i>	“This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and

	employment in a wide range of disciplines involving the use of mathematics.” (DfE, GCE A Level Mathematics, December 2014 – for teaching from 2017, p. 3)		enjoy meeting its challenges and engaging with its problems.” (IB Mathematics HL Guide, 2014)
Total recom. course teaching time	Guided learning for A Levels generally requires 360 hours (2 years) (Icossa study, DfE)	H2 Level courses are covered in 360 hours, “equivalent to A Level subjects prior to 2006” (Singapore MOE)	240 hours
Pre-requisite content areas	GCSE Level Mathematics (Pearson Edexcel): 1) Number 2) Algebra 3) Ratio, proportion and rates of change 4) Geometry and measures 5) Probability 6) Statistics	GCE ‘O’ Level Additional Mathematics, covering: 1) Algebra 2) Geometry & Trigonometry 3) Calculus (basic differentiation and integration)	Students should be familiar with: 1) sets & numbers 2) algebra 3) trigonometry 4) geometry 5) coordinate geometry 6) statistics & probability (IB Mathematics HL Guide, 2014, p.15-16)
Content areas (required & optional)	Pearson’s Edexcel A Level Mathematics course sequence includes the following compulsory units: C1-C4 (which include Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration; trigonometry; exponentials and logarithms; numerical methods and vectors). The curriculum includes a choice from among the following combinations: 1) Mechanics 1 (M1) and Statistics 1 (S1) 2) M1 and Decision Math 1 (D1) 3) S1 and D1 4) S1 and Statistics 2 (S2) 5) M1 and Mechanics 2 (M2) 6) D1 and Decision Math 2 (D2) (Pearson Edexcel Specification, 2014)	Required topics: A) Pure Mathematics: 1) Functions & Graphs 2) Sequences & series 3) Vectors 4) Complex numbers 5) Calculus B) Statistics 6) Permutations, combinations & probability 7) Binomial, Poisson and normal distributions 8) Sampling and hypothesis testing 9) Correlation and Regression (SEAB, Mathematics Higher 2 Syllabus, 2015) Total broad areas: 2 Total required sub-areas: 9	Required topics: 1) Algebra 2) Functions & Equations 3) Circular Functions & Trigonometry 4) Vectors 5) Statistics and Probability 6) Calculus Optional topics: 1) Statistics & probability 2) Sets, relations & groups 3) Calculus 4) Discrete mathematics (IB Mathematics HL Guide 2014) Total topics: 7 (6 required and 1 optional) + Individual Exploration

	<p>Total areas: 6 Includes 4 required units of pure mathematics (C1-C4) and 2 optional units of applied mathematics</p>		
<p>Broad assessment objectives: Expected cognitive skills/ math processes, understanding</p>	<p>Students must demonstrate the following overarching skills:</p> <ol style="list-style-type: none"> 1) Mathematical argument, language and proof 2) Mathematical problem solving 3) Mathematical modelling <p><i>(DfE, GCE A Level Mathematics, December 2014 – for teaching from 2017)</i></p>	<p>“1) understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphic calculators 2) reason and communicate mathematically through writing mathematical explanation, arguments and proofs, and inferences 3) solve unfamiliar problems; translate common realistic contexts into mathematics; interpret and evaluate mathematical results, and use the results to make predictions or comment on the context”</p> <p><i>(SEAB, Mathematics Higher 2 Syllabus, 2015)</i></p>	<ol style="list-style-type: none"> 1) Knowledge and understanding 2) Problem-solving 3) Communication and interpretation 4) Technology 5) Reasoning 6) Inquiry approaches <p><i>(IB Mathematics HL Guide 2014, p.9)</i></p>
<p>Final assessment/ exam: % emphasis on each content area</p>	<p>Each of the six units is assessed by a one-and-a-half-hour closed examination (thus, each is given equal weight).</p> <p><i>(Pearson Edexcel Specification, 2014)</i></p> <p>Total exam time: 9 hours</p>	<p>Final exam consists of two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100:</p> <p>PAPER 1 (3 hours): based on the Pure Mathematics section of the syllabus</p> <p>PAPER 2 (3 hours), consisting of 2 sections: Section A (Pure Mathematics – 40 marks) Section B (Statistics – 60 marks)</p> <p>Total exam time: 6 hours</p>	<p>Final exam consists of:</p> <p>Paper 1 (2 hours) worth 120 marks, covering core syllabus; Paper 2 (2 hours) worth 120 marks, covering core syllabus; Paper 3 (1 hour) worth 60 marks, based mainly on the syllabus options</p> <p>Total exam time: 5 hours</p>
<p>Internal/ external assessment</p>	<p>A Level Mathematics is 100% externally assessed through examinations for each unit.</p>	<p>External assessment through final examination</p>	<p>20% internal assessment is an individual exploration: written work that involves investigating an area of mathematics. It is internally assessed by the teacher and externally moderated by the IB at the end of the course.</p> <p>80% external assessment through final Diploma exam</p>

- **Intended purpose:** All three of these qualifications prepare students for entry into mathematics, engineering and science fields, and aim to build up students' analytical skills in mathematics. A Level Math aims to provide students with a broad preparation in mathematics that can be used for a wide range of disciplines, including disciplines that include knowledge of applied mathematics, such as social sciences, business and economics. It is the only qualification in this category to do so; in this respect, its aims and purpose could place it in the previous category of qualifications. (However, it differs considerably in all other respects from Singapore Math H1 and IB Math SL; it was thus not considered part of the same group).
- **Type of student:** All three qualifications assume that students will come into the course with an already fairly solid mathematical background consisting of pre-calculus, as well as basic statistics and probability. Singapore H2 has a slightly higher level of pre-requisites than the other two, assuming that students will have some basic knowledge of calculus.
- **Content structure:** A Level Math and IB Math HL both offer students the possibility of tailoring their course of study to a chosen specialization, since they offer various topics to choose from (3 and 4, respectively). Ultimately, A Level Math provides students with the broadest range of choice, since it requires students to complete *two* optional units (while IB Math HL requires only one).
- **Assessment structure:** External assessment predominates among these three qualifications.
- **Cognitive skills:** Expected cognitive processes and mathematical understandings are essentially similar for all three qualifications; the use of technology is not specified for A Level Math however.
- **Type of mathematical content:** Both A Level Math and IB Math HL offer a wide range of topics in applied mathematics to choose from. IB Math HL offers 4 topics while A Level Math offers only 3; IB Math HL also has an individual exploration, which encourages thinking about mathematical applications. Two of these applied math topics are similar: Decision/Discrete Math, and Statistics. A Level Math is quite unique in offering Mechanics as an option, as highlighted in the ICOSSA study. Singapore H2, in contrast, does not offer any options; it focuses only on Pure Mathematics and Statistics, and gives greater overall weight to Pure Mathematics. All three of these qualifications ultimately place greater emphasis on Pure Mathematics, as

shown by the distribution of required/optional areas, and by the structure of the final exam.

- Level of mathematical content:** All three of these qualifications cover trigonometry, functions, calculus, vectors and statistics; each one also includes a couple of advanced topics that are not covered by the other two – numerical methods and mechanics for A Levels, complex numbers for Singapore H2, Discrete Math for IB Math HL. Students in IB Math HL could potentially cover more calculus than those in the other two qualifications, since IB Math HL offers Calculus as both the core and as an option; however, the learning time is less overall for this qualification.
- Breadth and depth of content:** A Level Math and Singapore H2 recommend a substantially greater number of teaching hours than IB Math HL. Singapore H2 covers a greater number of topics during this longer period of time (9 versus 7 for IB Math HL). In contrast, A Level Math covers a lesser number of topics (6) than the other two qualifications over a longer period of time, which suggests that this qualification has more possibilities of going into greater depth into each area. The number of exam hours is also much greater for A Level Math than for the other two qualifications.

GROUP IV: QUALIFICATIONS THAT FOCUS IN DEPTH ON CALCULUS AND ITS APPLICATIONS

	Alberta 31	AP Calculus AB	AP Calculus BC
Type of student/purpose	<p>“The course is designed to introduce students to the mathematical methods of calculus [and] acts as a link between the outcomes of the Mathematics 10–20–30 program and the requirements of the mathematics encountered in post-secondary programs.[...] The methods of calculus are applied to problems encountered in the areas of science, engineering, business and other fields of endeavor.” <i>(Mathematics 31 course description, p. 5)</i></p>	<p>“AP Calculus AB and AP Calculus BC focus on students’ understanding of calculus concepts and provide experience with methods and applications [...] Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the courses become cohesive rather than a collection of unrelated topics.”</p> <p>“AP Calculus AB is roughly equivalent to a first semester college calculus course devoted to topics in differential and integral calculus [...] The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to</p>	<p>“AP Calculus AB and AP Calculus BC focus on students’ understanding of calculus concepts and provide experience with methods and applications [...] Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the courses become cohesive rather than a collection of unrelated topics.”</p> <p>“AP Calculus BC is roughly equivalent to both first and second semester college calculus courses and extends the content learned in AB to different types of equations and introduces the topic of sequences and series.” <i>(AP Calculus BC overview)</i></p>

		make connections amongst these representations.” <i>(AP Calculus AB overview)</i>	
Total recommended course teaching time	<p>This course may go beyond the required number of hours (125 hours for 5 credits) depending on how many electives are chosen</p> <p>The required component [4 sections] is intended to take the larger proportion of the instructional time. [...] the number of elective units covered will vary, depending on local needs. In general, most students will do one or two elective units; however, some students must do as many as four in order to integrate the requirements of external agencies into the Mathematics 31 course.” <i>(Mathematics 31 course description, p. 5)</i></p>	<p>The College Board does not provide specific guidelines on the number of guided teaching hours; this is left up to each individual school; Sample syllabi in the Teacher’s Guide indicate around 150 hours for 1 school year. <i>(AP Calculus Teacher’s Guide, 2007)</i></p>	<p>The College Board does not provide specific guidelines on the number of guided teaching hours; this is left up to each individual school; Sample syllabi in the Teacher’s Guide indicate around 150 hours for 1 school year.</p>
Pre-requisite content areas	<p>Mathematics 30-1:</p> <ol style="list-style-type: none"> 1) Trigonometry 2) Relations & Functions 3) Permutations, Combinations & Binomial Theorem 	<p>“Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions.</p> <p>These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. [...] students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions [as well as] the language of functions [...] and know the values of the trigonometric functions at the numbers 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and their multiples.” <i>(AP Calculus AB overview)</i></p>	<p>“Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions.</p> <p>These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. [...] students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions [as well as] the language of functions [...] and know the values of the trigonometric functions at the numbers 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and their multiples.” <i>(AP Calculus BC overview)</i></p>

<p>Content areas (required & optional)</p>	<p>Required: 1) precalculus and limits 2) derivatives and derivative theorems 3) applications of derivatives 4) integrals, integral theorems and integral applications.</p> <p>Options: 1) calculus of exponential and logarithmic functions 2) numerical methods 3) volumes of revolution 4) applications of calculus to physical sciences and engineering 5) applications of calculus to biological sciences 6) applications of calculus to business and economics 7) calculus theorems 8) further methods of integration</p> <p>Total available areas: 12 Total required areas: 4 Total optional areas: 8 Total expected areas to be covered: 5-6</p>	<p>From 2016-17 onwards, both AP Calculus courses are structured around 4 main areas (with the fourth exclusively for Calculus BC). For Calculus AB, these areas are essentially the same as the ones currently taught:</p> <p>1) Limits 2) Derivatives 3) Integrals & the Fundamental Theorem of Calculus</p> <p><i>(Curriculum Framework Calculus AB and Calculus BC 2016-17)</i></p> <p>Total areas: 3</p>	<p>From 2016-17 onwards, both AP Calculus courses are structured around 4 main areas (with the fourth exclusively for Calculus BC). These areas are essentially the same as the ones currently taught (with a few additions for Calculus BC):</p> <p>1) Limits (Calculus BC includes parametric, vector and polar functions) 2) Derivatives (Calculus BC includes numerical solution of differential equations using Euler’s method; use of L’Hopital’s Rule in determining limits and convergence of improper integrals and series; derivatives of parametric, vector and polar functions) 3) Integrals & the Fundamental Theorem of Calculus (Calculus BC includes application of integrals; antiderivatives by substituting values and improper integrals; solving logistic differential equations, using them in modeling) 4) Series (Calculus BC only)</p> <p>From 2016, the limit comparison test, absolute and conditional convergence, and alternating series error-bound have been added to Calculus BC. <i>(Curriculum Framework Calculus AB and Calculus BC 2016-17)</i></p> <p>Total areas: 4</p>
<p>Broad assessment objectives: Expected cognitive skills/ mathematical processes, understanding</p>	<p>Mathematical processes:</p> <ol style="list-style-type: none"> attitudes towards mathematics – appreciation for beauty and power of mathematics problem solving skills communication of solutions and solution procedures 	<p>Mathematical Practices for AP Calculus (MPACS) are:</p> <ol style="list-style-type: none"> Reasoning with definitions & theorems Connecting concepts Implementing algebraic/computational processes 	<p>Mathematical Practices for AP Calculus (MPACS) are:</p> <ol style="list-style-type: none"> Reasoning with definitions & theorems Connecting concepts Implementing algebraic/computational processes

	<p>4) reasoning – analysis, making and testing conjectures 5) Connections among math concepts, between math and other subjects, and between math and everyday life 6) Awareness of technology, using it appropriately</p> <p><i>(Mathematics 31 course description, p. 2 & 3)</i></p>	<p>4) Connecting multiple representations 5) Building notational fluency 6) Communicating <i>(Curriculum Framework Calculus AB and Calculus BC 2016-17)</i></p>	<p>4) Connecting multiple representations 5) Building notational fluency 6) Communicating <i>(Curriculum Framework Calculus AB and Calculus BC 2016-17)</i></p>
<p>Final assessment/ exam: % emphasis on each content area</p>	<p>Not applicable</p>	<p>“The AP Calculus BC Exam questions measure students’ understanding of the concepts of calculus, their ability to apply these concepts, and their ability to make connections among graphical, numerical, analytical, and verbal representations of mathematics.” <i>(AP Calculus AB overview)</i></p> <p>The Calculus BC exam consists of 2 sections, each worth 50% of the total score: Section I: 45 multiple-choice questions, to be completed in 105 minutes. Section II: 6 free-response questions, to be completed in 90 minutes.</p> <p>Total exam time: 3 hours 15 minutes</p>	<p>“The AP Calculus BC Exam questions measure students’ understanding of the concepts of calculus, their ability to apply these concepts, and their ability to make connections among graphical, numerical, analytical, and verbal representations of mathematics.” <i>(AP Calculus BC overview)</i></p> <p>The Calculus BC exam consists of 2 sections, each worth 50% of the total score: Section I: 45 multiple-choice questions, to be completed in 105 minutes. Section II: 6 free-response questions, to be completed in 90 minutes.</p> <p>Total exam time: 3 hours 15 minutes</p>
<p>Internal/ external assessment</p>	<p>100% internal (school-based); this course does not have a final Diploma examination (goes beyond Diploma)</p>	<p>Assessment of this course is entirely external, through a final exam consisting of 2 sections, each weighing 50% of the final course grade.</p>	<p>Assessment of this course is entirely external, through a final exam consisting of 2 sections, each weighing 50% of the final course grade.</p>

- Intended purpose:** The purpose of all three of these courses is to provide students with an in-depth calculus course that is equivalent to a first semester or first year introductory university course on the subject.

- **Type of students:** Alberta Math 31 may be oriented towards a broader group of students, since it provides students with an area of pre-calculus to strengthen their mathematical base, in addition to calculus. In contrast, AP Calculus courses appear to have more rigorous pre-requisites, and launch directly into calculus topics.
- **Content structure:** Alberta Math 31 offers students a wide range of options (8), most of which are related to applications of calculus. This qualification certainly offers students the possibility of steering their course towards a specialization that may be useful to them in university, while AP Calculus courses do not provide any options.
- **Assessment structure:** In this respect, there is again a contrast between the qualifications in this group – Alberta Math 31 is entirely internally assessed, while AP Calculus courses are entirely externally assessed.
- **Cognitive skills:** All three qualifications include reasoning, making connections and communicating as essential cognitive skills; Alberta Math 31 also includes having an appreciation of mathematics and using technology appropriately, making its range of expected cognitive abilities a little broader than for AP Calculus courses.
- **Types of mathematical content:** All three courses cover both theoretical calculus topics and their applications. Calculus applications are an integral part of the core content of AP Calculus courses; in Alberta Math 31, these are also part of the core content but mostly part of the options it offers.
- **Level of mathematical content:** All three qualifications cover very similar Calculus content. Only AP Calculus BC includes a whole unit dedicated exclusively to series, a topic not covered in such depth by any of the other qualifications. Alberta Math 31, in contrast, covers some pre-calculus topics in addition to calculus ones.
- **Breadth and depth of content:** All three qualifications require a similar amount of time, but Alberta Math 31 requires the greatest number of topics (5-6) among this group, indicating that it may not cover them in the same depth as AP Calculus courses, which include only 3 and 4 topics. Some of the topics included in Alberta Math 31 may in fact be subtopics in AP Calculus courses however, which would make them more comparable in terms of both time and number of topics.

GROUP V: QUALIFICATIONS THAT COVER CALCULUS AND A RANGE OF ADVANCED TOPICS, TO PREPARE STUDENTS FOR UNIVERSITY PROGRAMS IN HIGHLY MATHEMATICAL DISCIPLINES

	Singapore Math H3	A Level Further Math	IB Further Math HL	Gaokao
Type of student/purpose	<p>“The H3 Mathematics syllabus provides an opportunity for students with an aptitude and passion in the subject to pursue it at a higher level and in greater depth. It challenges students to think critically and creatively and be independent and self-directed learners.”</p> <p><i>(Mathematics H3 Syllabus 2016)</i></p>	<p>“Further mathematics is designed for students with an enthusiasm for mathematics, many of whom will go on to degrees in mathematics, engineering, the sciences and economics [...] A level further mathematics prepares students for further study and employment in highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.”</p> <p><i>(DfE, Further Mathematics, December 2014 – for teaching from 2017, p. 3)</i></p>	<p>The course “caters for students with a very strong background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students will expect to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications.”</p> <p><i>(IB Diploma Subject Brief: Further Mathematics HL, 2014-2020)</i></p>	<p>Mathematics is a required part of the national entrance examination for higher education, for students across China. Its purpose is to ensure a highly rigorous and equitable admission procedure to all Chinese universities.</p>
Total recom. course teaching time	<p>Between 112 and 210 hours (in addition to completing H2 course)</p> <p><i>(Singapore MOE)</i></p>	<p>360 hours</p> <p><i>(Icossa study)</i></p>	<p>240 hours (after completing/ concurrently with HL course)</p> <p>Six topics are covered, 48 recommended teaching hours for each. It is expected that one of these topics will have been covered in the Math HL course, so the total number of teaching hours is 240 (not 288).</p> <p><i>(IB DP Subject Brief, Further Mathematics HL)</i></p>	<p>Not available</p>

<p>Pre-requisite content areas</p>	<p>Must be taken after or alongside Mathematics H2, which covers:</p> <p>A) Pure Mathematics:</p> <ol style="list-style-type: none"> 1) Functions & Graphs 2) Sequences & series 3) Vectors 4) Complex numbers 5) Calculus <p>B) Statistics</p> <ol style="list-style-type: none"> 6) Permutations, combinations & probability 7) Binomial, Poisson and normal distributions 8) Sampling and hypothesis testing 9) Correlation and Regression 	<p>A Level Further Mathematics builds on the subject content in A Level Mathematics.</p> <p>In Pearson Edexcel courses, 4 Core Mathematics units (C1-C4) must have been previously covered in an A Level Mathematics course sequence. These units include: algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; trigonometry; exponentials and logarithms; differentiation; integration; and vectors.</p> <p><i>(Pearson Edexcel Specification GCE Mathematics, 2014)</i></p>	<p>“It is expected that students taking this course will also be taking mathematics HL [...] Students should be equipped at this stage in their mathematical progress to begin to form an overview of the characteristics that are common to all mathematical thinking, independent of topic or branch.”</p> <p><i>(IB DP Subject Brief, Further Mathematics HL)</i></p> <p>“Students registering for further mathematics HL will be presumed to know the topics in the core syllabus of mathematics HL and to have studied one of the options, irrespective of whether they have also registered for mathematics HL.”</p> <p><i>(IB Further Mathematics HL Guide, 2014)</i></p> <p>IB Math HL Required topics:</p> <ol style="list-style-type: none"> 1) Algebra; 2) Functions & Equations; 3) Circular Functions & Trigonometry; 4) Vectors 5) Statistics and Probability; 6) Calculus <p>Optional topics: 1) Statistics & probability; 2) Sets, relations & groups; 3) Calculus; 4) Discrete mathematics</p>	<p>Junior School covers:</p> <ol style="list-style-type: none"> 1) Algebra 2) Equations & Inequalities 3) Sequences & Series 4) Geometry 5) Probability & Statistics <p><i>(Wang, Changing Framework for Teaching of Mathematics in China)</i></p> <p>The entrance examination to Senior High School (Zhongkao) covers: Algebra (39%), Geometry (36%), Statistics (15%), synthesis of other areas (10%).</p> <p><i>(Wu, Examination System in China)</i></p>
<p>Content areas (required & optional)</p>	<p>Required topics:</p> <p>1) The first of these includes extensions of 3 topics covered in Math H2: Functions & Graphs; Sequences & Series; Calculus</p>	<p>Pearson’s Edexcel course must include Further Pure Mathematics unit FP1 (Series; complex numbers; numerical solution of equations; coordinate systems, matrix algebra, proof), and one additional</p>	<p>Required topics:</p> <ol style="list-style-type: none"> 1) Linear Algebra 2) Geometry 3) Statistics & Probability 4) Sets, Relations & Groups 5) Calculus 	<p>Required content:</p> <ol style="list-style-type: none"> 1) Sets 2) Functions 3) Preliminary 3-dimensional geometry 4) Preliminary plane analytical geometry

	<p>2) Combinatorics 3) Differential Equations as Mathematical Models</p> <p><i>(Mathematics H3 Syllabus 2016)</i></p> <p>Total required topics: 3</p>	<p>FP unit – either FP2 (Inequalities; series, first order differential equations; second order differential equations; further complex numbers, Maclaurin and Taylor series) or FP3 (Further matrix algebra; vectors, hyperbolic functions; differentiation; integration, further coordinate systems). Students can also take both FP2 and FP3 (and one less optional area below).</p> <p>The rest of the course can consist of any 3 or 4 optional units from Mechanics (5 units available), Statistics (4 units available), or Decision Mathematics (2 units available). <i>(Pearson Edexcel Specification GCE Mathematics, 2014)</i></p> <p>Total core areas offered: 3 Total optional areas offered: 11</p> <p>Total required areas for the course (including core and optional ones): 6</p>	<p>6) Discrete Mathematics <i>(IB Further Mathematics HL Guide 2014)</i></p> <p>Total required topics: 6</p>	<p>5) Preliminary algorithms 6) Statistics 7) Probability 8) Trigonometric functions 9) Plane vectors 10) Trigonometric transformation 11) Solution of triangles 12) Progression 13) Inequality 14) Common logic expressions 15) Conic curve 16) Space vectors & solid geometry 17) Differential coefficient 18) Inference and proof 19) Series and introduction of complex numbers 20) Principle of counting constants (?)</p> <p>Optional content: 1) Geometric proof 2) Coordinate system and parametric equations 3) Inequalities</p> <p><i>(Icosia study annex, pp. 21-22)</i></p>
<p>Broad assessment objectives: Expected cognitive skills/ math processes, understanding</p>	<p>Candidates should be able to: “1) understand and apply mathematical concepts and skills in a variety of contexts [...] 2) understand and translate common realistic contexts into mathematics; interpret and evaluate</p>	<p>Students must demonstrate the following overarching skills: 1) Mathematical argument, language and proof 2) Mathematical problem solving 3) Mathematical modelling</p> <p><i>(DfE, GCE A Level Further Mathematics, December 2014 – for teaching from 2017)</i></p>	<p>1) Knowledge and understanding 2) Problem-solving 3) Communication and interpretation 4) Technology 5) Reasoning 6) Inquiry approaches</p> <p><i>(IB DP Subject Brief, Further Mathematics HL)</i></p>	<p>“The emphasis of the course is to achieve three levels of attainment: knowledge, comprehension and mastery. There is an emphasis on problem-solving in the qualification.” <i>(Icosia Study, p. 130)</i></p>

	<p>mathematical results [...] 3) reason and communicate mathematically through making inferences and writing mathematical arguments and proofs”</p> <p><i>(Mathematics H3 Syllabus 2016)</i></p>			
<p>Final assessment / exam: % emphasis on each content area</p>	<p>Examination consists of one 3-hour paper marked out of 100. The marks are distributed as follows:</p> <p>Topic 1 Functions & Graphs, Sequences & Series, and Calculus 40% Topic 2 Combinatorics 25% Topic 3 Differential Equations as Mathematical Models 35%</p> <p><i>(Mathematics H3 Syllabus 2016, p.3)</i></p> <p>Total exam time: 3 hours</p>	<p>Each of the six modules which comprise further mathematics is assessed by a one-and-a-half-hour closed examination.</p> <p>Total exam time: 9 hours</p>	<p>Assessment is composed of 2 final exam papers: each is 2.5 hours long and includes questions based on the whole syllabus.</p> <p>Total exam time: 5 hours</p>	<p>External assessment is a 2-hour paper consisting of:</p> <p>Part 1: multiple-choice: 12 questions (5 marks each) Part 2: 4 short-answer questions (5 marks each) and 5 longer questions (12 marks each) Optional section: complete 1 from a choice of 3 questions (10 marks each) (Total: 150 marks)</p> <p><i>(Icosa study annex)</i></p> <p>Total exam time: 2 hours</p>
<p>Internal/ external assessment</p>	<p>External assessment through final examination</p>	<p>A Level Further Mathematics is 100% externally assessed through examinations for each unit.</p>	<p>Assessment of this course is entirely external, through 2 final exam papers, each weighing 50% of the final course grade.</p> <p><i>(IB DP Subject Brief, Further Mathematics HL)</i></p>	<p>External assessment based purely on exam results</p>

- **Intended purpose:** All of these qualifications are aimed at preparing students for careers where math will be quite essential.
- **Type of student:** All of these qualifications are oriented towards students who have a “passion” or “enthusiasm” for math, and who are prepared to take on very sophisticated mathematical ideas and see the relations between them. Indeed, the prerequisites for all of these qualifications are the higher level courses in the previous group – they are thus well versed in calculus topics and other mathematical areas.
- **Content structure:** Only A-Level Further Math and the Chinese Gaokao offer options to students; A Level Further Math offers a considerable number, from a wide variety of applied math topics, allowing students scope for specialization.
- **Assessment structure:** This is the only group in which the assessment for all qualifications is entirely external, indicating perhaps that greater importance is given to meeting external benchmarks for courses with more complex mathematical content.
- **Cognitive skills:** In this group, only IB Further Math HL includes technology and inquiry skills as part of the desired cognitive understandings. Otherwise, the expected cognitive skills are similar.
- **Type of mathematical content:** All of these qualifications emphasize pure mathematics except for A Level Further Math, which provides a wide range of applied mathematics topics and requires students to choose 2 or 3 of these units.
- **Level of mathematical content:** All of these qualifications cover calculus thoroughly, and also include other advanced topics such as linear algebra, complex numbers and discrete mathematics.
- **Breadth and depth of content:** A Level Further Math has a much longer duration than IB Further Math HL, but it covers the same amount of topics (6), indicating that it may cover these topics in greater depth. Singapore Math H3 covers fewer topics in an equally long time frame as A Level Further Math, indicating an even higher level of depth (unless it contains a series of sub-topics that are equivalent to the main topics outlined for the other two qualifications). The Chinese Gaokao covers a very large number of topics, but the time required to cover them is not specified. Its final exam is shorter than that of the other qualifications.

COMPARISON OF QUALIFICATIONS ACCORDING TO AN OBJECTIVE FRAMEWORK OF MATHEMATICS CONTENT

Methodology

After analyzing and comparing the qualifications according to their own structure and self-defined content areas, this report attempts to provide an additional comparison of these qualifications based on an objective, external framework. This framework, an independent categorization of mathematics topics, is based on textbooks that are either recommended as pre-requisites for, or used in, introductory first-year mathematics courses at some of the top five globally-ranked universities in science and engineering (according to the Times Global Ranking 2015). It is considered that these materials provide a solid and relevant framework for categorizing mathematics content, since the overarching purpose of all of the upper secondary qualifications analyzed in this study is to provide students with an appropriate level of preparation for entry into university and, in some cases, for entry into top-ranked programs such as these.

The textbooks and course materials used to define the framework of mathematics categories are the following:

Pre-calculus and Pure Mathematics

- Sheldon Axler, *Pre-calculus: A Prelude to Calculus*, John Wiley & Sons, 2009 (used in Princeton course MAT100: Precalculus/Prestatistics):
<http://precalculus.axler.net/Contents2.pdf>
- ALEKS *Pre-calculus Course Syllabus*, 2013-2014 (Stanford's Pre-calculus study tool):
https://precalculus.stanford.edu/sites/default/files/CourseSyllabus_1.pdf

Calculus

- James Stewart, *Single Variable Calculus: Concepts and Contexts*, Brooks/Cole, 2010 (used in Stanford course Math 19: Introductory Calculus)
- Gilbert Strang, *Calculus*, Wellesley-Cambridge Press, 1991 (used in MIT open courseware courses) <http://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/>
- Tom M. Apostol, *Calculus Volume I: One Variable Calculus with an Introduction to Linear Algebra*, John Wiley & Sons, 1967 (used in MIT course 18.014 Calculus with Theory (2010); Caltech course Math 1A: Calculus of One and Several Variables)

Probability and Statistics

- Jeremy Orloff and Jonathan Bloom, *MIT Course 18.05: Introduction to Probability and Statistics*, Spring 2014: <http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/index.htm>
- John A Rice, *Mathematical Statistics and Data Analysis*, 2007 (used in MIT course)

Comparison of qualifications according to mathematical content areas

Please refer to the Annex I charts – “Comparison of Qualifications: Mathematical Content Areas” and Annex II charts – “Qualifications by Breadth of Mathematical Content” for the detailed information analyzed in this section. Annex III describes the mathematical content included for each topic, and the specific syllabus items of each qualification that come under each.

Pre-calculus: The three basic areas of “pre-calculus” generally covered in high school and expected for university entrance – algebra and functions, geometry and trigonometry – are either a pre-requisite for, or are covered by, all of these qualifications (in lesser or greater depth).

- **Algebra and Functions:** All of the 5 main areas of algebra and functions are covered by all of the qualifications, either as a pre-requisite or as a requirement of the course itself.
- **Geometry:** Some knowledge of solid geometry and basic coordinate geometry is either a pre-requisite for, or requirement of, all of these qualifications. More complex areas (Parametric Equations of Curves and Conic Sections and Polar Coordinates, which are studied less often at the pre-calculus level) are covered by fewer qualifications.
- The 6 qualifications that include Parametric Equations of Curves and Conic Sections are Singapore Math H2, Singapore Math H3, A Level Math, A Level Further Math, **IB Further Math HL**, and the Chinese Gaokao.
- Polar Coordinates is covered by only one: A Level Further Mathematics (Unit Further Pure Mathematics 2).
- **Trigonometry:** Within this area, Inverse Trigonometric Functions and Identities, as well as Double-Angle and Half Angle Formulas and Addition and Subtraction Formulas were covered less frequently.
- Alberta Math 30-1, Alberta Math 31 and **IB Math Studies SL** introduce only one or two new areas in trigonometry (in addition to requiring one or more areas as prerequisites). Singapore Math H2 and Singapore Math H3 require all of the areas of trigonometry as pre-requisites, but do not cover any area as part of the qualification itself. **IB Math SL** requires four areas of trigonometry as part of the course, while A Level Math, A Level Further Math, **IB Math HL** and **IB Further Math HL** cover all five areas of trigonometry included in this chart (either as a required area or as a prerequisite from the linked higher level course).

Other pure mathematics areas: Between 2 and 6 other areas of pure mathematics are also required by 11 of these qualifications.

- The 4 qualifications that do not require the study of any other pure mathematics topics are: Alberta Math 30-2, AP Calculus AB, AP Calculus BC, and Singapore Math H1.
- Alberta Math 30-1, Alberta 31, **IB Math Studies SL**, and **IB Math SL** require two or fewer additional areas of pure mathematics.
- The qualifications that cover the largest number of other pure mathematics areas (6 each) are **IB Math HL**, **IB Further Math HL** and A Level Further Math.
- The most commonly covered areas of pure mathematics are Basic Sequences and Series (required by 10 qualifications), followed by Vectors (required by 8 qualifications).
- The least commonly covered areas of pure mathematics (required by 3 or less qualifications) are Matrices and Linear Algebra; Numerical Methods; Sets and Combinatorics; and Proof. **IB Math HL** and **IB Further Math HL** both cover Matrices and Linear Algebra and Sets and Combinatorics; they do not cover Numerical Methods or Proof, however.

Calculus: Calculus topics were covered by all of these qualifications except for Alberta Math 30-2 and Alberta Math 30-1.

- The basic areas of calculus related to Definition of Derivatives, Derivative Rules, Integrals and Techniques of Integration were covered by all of the qualifications that require the study of calculus except for **IB Math Studies SL**, which only covers the Definition of Derivatives area.
- Of those qualifications that do include calculus, the ones that cover the smallest number of areas within this topic are IB Math Studies (2) and Singapore H1 (6).
- The qualifications that cover most (11 or 12) of the 13 calculus areas defined in this chart are Alberta Math 31, **IB Math HL**, **IB Further Math HL**, AP Calculus AB and AP Calculus BC.
- Between 8 and 10 qualifications include the study of various applications of calculus (to economics, problems of area and volume, problems of growth and decay in biology or physics) as part of their curriculum, either as a requirement or as an option. A smaller number (6) study the applications of calculus to the problems of motion (kinematics): **IB Math SL**, **IB Math HL**, **IB Further Math HL**, Alberta Math 31, AP Calculus AB and AP Calculus BC.

- Calculus involving vector-valued and parametric functions is covered by a relatively fewer number of qualifications (6 - none of which are IB qualifications), and hyperbolic functions are only covered by A Level Math (Unit Further Mathematics 3).

Applied Mathematics: Applied mathematics topics are neither required nor offered as an option by 4 qualifications that concentrate only on pure mathematics and calculus: Alberta Math 31, AP Calculus AB, AP Calculus BC, and Singapore Math H3. Some area of applied mathematics is required by the other 12 of these qualifications (including AP statistics). Of these, A Level Math and A Level Further Math offer options among different areas of applied mathematics, and at least two units of these are a required part of the overall qualification.

- **Statistics and Probability:** This area is included in 12 qualifications, either as a requirement (in 10 cases), or as an option (in 2 cases- A Level Math and A Level Further Math). All 12 of these qualifications cover Basic Probability. The areas of Experimental Design and Sampling; Descriptive Statistics; Discrete and Continuous Random Variables; and Covariance, Correlation and Regression are also covered by most (10) of these qualifications.
- Alberta 30-2 and Alberta 30-1 only cover Basic Probability. **IB Math Studies SL, IB Math SL** and A Level Math cover 6 of the 8 areas of Statistics and Probability. The qualifications that provide the fullest coverage (all 8 areas) are **IB Math HL, IB Further Math HL**, AP Statistics and A Level Further Math.
- **Discrete or Decision Mathematics** is only required by two qualifications (Gaokao and **IB Further Math HL**), and is optional for another three (**IB Math HL**, A Level Math and A Level Further Math).
- **Mechanics** is only included as an option by A Level Math and Further Math.

CATEGORIZATION OF QUALIFICATIONS ACCORDING TO A PRELIMINARY ASSESSMENT OF LEVEL OF DEMAND

Methodology

As stated in the Introduction, the level of demand can be defined as “the level of knowledge, skills and competence required by the typical learner” in a given subject course or examination.¹⁰¹ The level of demand of a qualification depends on several of its interconnected features, including those related to its overall structure and content, analyzed in the previous section (level of preparation of entering students, cognitive skills required, type of assessment, type of mathematical content, etc.).

¹⁰¹ Elliot, 2011, p.11, cited in Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 20.

With regard to the specific mathematical content of any given qualification, the ICOSSA study considers the following parameters to be especially important¹⁰²:

- **Scope of mathematical content areas:** the extent to which a qualification includes sufficient study of trigonometry and calculus, as well as abstract and technically difficult mathematics (complex numbers, advanced algebraic techniques, proofs, and other areas of more advanced mathematical analysis, for example) raises its level of demand
- **Depth of study:** the pursuit of complex and deep study of the topics covered by the qualification and the degree of in-depth knowledge required were considered essential for raising the level of demand
- **Breadth of study:** the number of topics covered also increases the level of demand, particularly when these provide a broader grasp of areas from both pure and applied mathematics. According to the ICOSSA report, “the ability to adapt one’s approach and mind-set to different applications of mathematics” increases demand and broadens the pathways that students can follow in higher education.¹⁰³

These parameters are useful for making a preliminary assessment of the overall level of demand of each of the qualifications analyzed in this report. The first comparison of these qualifications’ structure and content, together with the second comparison of specific mathematical content according to an external framework of topics, provide basic information related to scope of content areas and breadth of study that can contribute to an assessment of level of demand. ***However, depth of study cannot be assessed in any way in this report; this would naturally require the expertise of mathematicians who would determine the level of complexity with which each topic is treated (the details of each qualification’s curricular content are included in Annex III).***

Categorization of qualifications

Qualifications are listed from lowest to highest level of demand:

1. Alberta Math 30-2

- **Scope of content areas:** Does not cover trigonometry or calculus; in applied math, only covers basic probability
- **Breadth of study:** Covers the least number of topics in Algebra and Functions; does not require any other areas of Pure Math; no coverage of Applied Math except for one unit on probability
- **Other factors:** Explicitly states that its purpose is to prepare students for programs that *do not* require the study of calculus; very basic level of math prerequisites.

¹⁰² Ofqual, International Comparisons in Senior Secondary Assessment, 2012, pp. 102-106.

¹⁰³ Ofqual, International Comparisons in Senior Secondary Assessment, 2012, p. 105.

2. Alberta Math 30-1

- **Scope of content areas:** Does not cover calculus but does cover two basic areas of trigonometry
- **Breadth of study:** Like Alberta Math 30-2, covers the least number of topics in Algebra and Functions; does not require any other areas of Pure Math; no coverage of Applied Math except for one unit on probability
- **Other factors:** Meant for students at a slightly higher level (preparation for calculus); includes trigonometry as a pre-requisite, increasing its level of demand with regard to Math 30-2

3. IB Math Studies SL

- **Scope of content areas:** Requires basic trigonometry topics (2 out of 5) and introduces differentiation in Calculus (2 out of 13 topics)
- **Breadth of study:** Covers a moderate number of topics in Algebra and Functions, some Geometry; requires two other areas of Pure Mathematics, and has a good coverage of Statistics topics (6 out of 8)
- **Other factors:** Requires an independent project, increasing breadth in terms of applicability of mathematics; covers many topics in a lower number of hours however. Has basic pre-requisites

4. Singapore H1 Mathematics

- **Scope of content areas:** Does not cover trigonometry (expects some basic knowledge of this area as a prerequisite), but does cover 6 out of 13 calculus topics (including parametric functions)
- **Breadth of study:** Covers a moderate number of topics in Algebra and Functions, does not require any other areas of Pure Mathematics, but has a very complete coverage of Statistics topics (7 out of 8)
- **Other factors:** Oriented towards students who will require some mathematics in higher education, particularly applied areas

5. IB Mathematics SL

- **Scope of content areas:** Covers 4 out of 5 areas of trigonometry, and 8 out of 13 calculus topics (including applications to kinematics)
- **Breadth of study:** Covers all topics in Algebra and Functions; requires two additional areas of Pure Mathematics, and has a good coverage of Statistics topics (5 out of 8)
- **Other factors:** Oriented towards students who will require some mathematics in higher education, particularly applied areas; requires an independent project, increasing

breadth in terms of applicability of mathematics; includes some trigonometry as a prerequisite

6. A Level Mathematics

- **Scope of content areas:** Covers all topics in trigonometry and 8 out of 13 calculus topics (including parametric functions and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions; includes study of parametric equations and curves in Geometry; requires 3 other areas of Pure Mathematics, has a good coverage of Statistics topics (6 out of 8); also includes Decision Mathematics and Mechanics
- **Other factors:** Oriented towards students requiring a solid base in math, with more emphasis on applied mathematics areas; covers topics in a substantial number of hours; does not require calculus as a prerequisite

7. IB Mathematics HL

- **Scope of content areas:** Covers all topics in trigonometry; covers 11 out of 13 calculus topics, including infinite sequences and series (two of these topics are optional however; also does not include parametric functions)
- **Breadth of study:** Covers all topics in Algebra and Functions; includes study of parametric equations and curves in Geometry; requires 5 other areas of Pure Mathematics (1 as a prerequisite), and makes one more optional; covers all Statistics topics (8 out of 8, but 4 of these are optional); also provides Discrete Mathematics as an option
- **Other factors:** Oriented towards students requiring a solid base in math, for science and engineering areas; covers topics in less hours than A Level or Singapore H2, but requires an individual exploration, increasing breadth and applicability; does not require calculus as a prerequisite

8. Singapore H2 Mathematics

- **Scope of content areas:** Does not cover trigonometry, but expects all trigonometry topics to have been covered as a prerequisite; covers 9 out of 13 calculus topics (including parametric functions and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions (some as prerequisites); includes study of parametric equations and curves in Geometry, requires 4 other areas of Pure Mathematics, has a quite complete coverage of Statistics topics (7 out of 8)

- **Other factors:** Oriented towards students requiring a solid base in math, for science and engineering areas; covers topics in a substantial number of hours; requires some calculus as a prerequisite

9. Singapore H3 Mathematics

- **Scope of content areas:** Does not cover trigonometry, but expects all trigonometry topics to have been covered as a prerequisite; covers 10 out of 13 calculus topics (including parametric functions and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions (some as prerequisites); includes study of parametric equations and curves in Geometry as a prerequisite; requires 5 other areas of Pure Mathematics (4 as prerequisites from H2); does not require any Applied Mathematics topics
- **Other factors:** Oriented towards students preparing for highly mathematical disciplines; requires strong prerequisites that include calculus (H2 course)

10. A Level Further Math

- **Scope of content areas:** Does not cover trigonometry, but expects all trigonometry topics to have been covered as a prerequisite; covers 9 out of 13 calculus topics (including parametric functions, infinite sequences and series, and hyperbolic functions)
- **Breadth of study:** Covers all topics in Algebra and Functions (most as prerequisites); includes study of parametric equations and curves, as well as polar coordinates, in Geometry; requires 6 out of 8 other areas of Pure Mathematics; requires all 8 topics in Statistics, and provides 2 additional areas of Applied Mathematics as options – Decision Mathematics and Mechanics
- **Other factors:** Oriented towards students preparing for highly mathematical disciplines; requires strong prerequisites that include calculus (A Level Mathematics course)

11. IB Further Math HL

- **Scope of content areas:** Covers one new area in trigonometry, expects all other trigonometry topics to have been covered as a prerequisite; covers 12 out of 13 calculus topics (including applications to kinematics and infinite sequences and series)
- **Breadth of study:** Covers all topics in Algebra and Functions (all as prerequisites); includes study of parametric equations and curves in Geometry; requires 6 out of 8 other areas of Pure Mathematics; requires all 8 topics in Statistics; requires one additional area of Applied Mathematics – Decision Mathematics

- **Other factors:** Oriented towards students preparing for highly mathematical disciplines; requires strong prerequisites that include calculus (IB Mathematics HL course)

Specialized Qualifications – Calculus

1. Alberta Mathematics 31

- **Scope of content areas:** Covers 11 out of 13 Calculus topics, but 2 of these are optional (does not cover parametric functions); also covers some pre-calculus topics (Algebra and Functions, Trigonometry)
- **Other factors:** Includes elective components, making this qualification more flexible than AP qualifications; but oriented towards students who want an introduction to Calculus, rather than a more thorough coverage of the topic

2. AP Calculus AB

- **Scope of content areas:** Covers 11 out of 13 Calculus topics, all are required (does not cover parametric functions)
- **Other factors:** Is equivalent to a first semester of college calculus; students need strong pre-requisites

3. AP Calculus BC

- **Scope of content areas:** Covers 12 out of 13 Calculus topics, all are required (includes parametric functions)
- **Other factors:** Is equivalent to a first year of college calculus; students need strong pre-requisites

ANNEX I

COMPARISON OF QUALIFICATIONS: MATHEMATICAL CONTENT AREAS

PURE MATHEMATICS

CONTENT	ALBERTA			AP		GAO-KAO*	A LEVEL (EDEXCEL)		SINGAPORE			INTERNATIONAL BACCALAUREATE			
	30-2	30-1	31	Calc AB	Calc BC		Math	Further Math	H1	H2	H3	Math Studies	Math SL	Math HL	Further Math
ALGEBRA AND FUNCTIONS	Req	Req	Req	Pre-req	Pre-req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Pre-req
1. Basic Skills: Numbers, Operations, Logic	X	Pre-req (20-1)	Pre-req (30-1)			X	X (C1)	Pre-req (C1)	Pre-req (O-Level)	Pre-req (O-Level)	Pre-req (O-Level)	X	Pre-req	Pre-req	Pre-req (HL)
2. Functions & Graphs	X	X	X			X	X (C1, C3, C4)	Pre-req (C1, C3, C4)	X	X	X	X	X	X	Pre-req (HL)
3. Linear, Quadratic, Polynomial & Rational Functions	Pre-req (20-2)	X	X			X	X (C1, C2, C3)	Pre-req (C1, C2, C3)	Pre-req (O-Level)	Pre-req (O-Lev Add. Math)	Pre-req (O-Lev Add. Math)	X	X	X	Pre-req (HL)
4. Solving Equations & Inequalities	X	Pre-req (20-1)	X			X	X (C1)	X (FP2)	X	X	X	X	X	X	Pre-req (HL)
5. Exponential Functions, Logarithms & E	X	X	Pre-req (30-1)				X (C2, C3)	Pre-req (C2, C3)	X	Pre-req (O-Lev Add. Math)	Pre-req (O-Lev Add. Math)	X	X	X	Pre-req (HL)

GEOMETRY	Pre-req	Pre-req	Pre-req	Pre-req	Pre-req	Req	Req	Req	Pre-req	Req	Pre-req	Req	Pre-req	Pre-req	Req
6. Solid Geometry & Basic Coordinate Geometry	Pre-req (20-2)	Pre-req (10C)	Pre-req (10C)			X	X (C1, C2)	Pre-req (C1, C2)	Pre-req (O-Level)	Pre-req (O-Lev Add. Math)	Pre-req (O-Lev Add. Math)	X			Pre-req
7. Parametric Equations of Curves & Conic Sections						X	X (C4)	X (FP1, FP3)		X	Pre-req (H2)			X	Pre-req (HL)
8. Polar Coordinates								X (FP2)							
TRIGONOMETRY	Pre-req	Req	Req	Pre-req	Pre-req	Req	Req	Pre-Req	Pre-req	Pre-req	Pre-req	Req	Req	Req	Pre-req
9. Trig: Unit Circle & Radians		X	Pre-req (30-1)				X (C2, C3)	Pre-req (C2, C3)	Pre-req (O-Level)	Pre-req (O-Level)	Pre-req (O-Level)		X	X	Pre-req (HL)
10. Using Trig. to compute Area; Law of Sines & Cosines	Pre-req (20-2)	Pre-req (20-1)	Pre-req (20-1)			X	X (C2)	Pre-req (C2)	Pre-req (O-Level)	Pre-req (O-Level)	Pre-req (O-Level)	X	X	X	Pre-req (HL)
11. Trig. Functions, Ratios, Identities & Transform.		X	X			X	X (C2, C3)	Pre-req (C2, C3)		Pre-req (O-Lev Add. Math)	Pre-req (O-Lev Add. Math)	X	X	X	Pre-req (HL)
12. Inverse Trig. Functions & Inverse Trig. Identities							X (C3)	Pre-req (C3)		Pre-req (O-Lev Add. Math)	Pre-req (O-Lev Add. Math)			X	Pre-req (HL)

13. Double-Angle & Half-Angle Formulas; Addition & Subtraction Formulas		X	X				X (C3)	Pre-req (C3)					X	X	Pre-req (HL)
OTHER AREAS	Not req	Pre req	Req	Not req	Not req	Req	Req	Req	Not req	Req	Req	Req	Req	Req	Req
14. Basic Sequences & Series		Pre-req (20-1)				X	X (C1, C2, C4)	X (FP1, FP2)		X	Pre-req (H2)	X	X	X	Pre-req (HL)
15. Vectors						X	X (C4)	X (FP3)		X	Pre-req (H2)		X	X	Pre-req (HL)
16. Systems of Linear Equations			X							X	Pre-req (H2)	X		Pre-req	Pre-req
17. Matrices & Linear Algebra								X (FP1, FP3)						X	X
18. Complex Numbers						X		X (FP1, FP2)		X	Pre-req (H2)			X	Pre-req (HL)
19. Numerical Methods			X (Op)				X (C3)	X (FP1)							
20. Sets, Relations & Groups; Combinatorics											X			X (Op)	X
21. Proof						X		X (FP1)							

CALCULUS

CONTENT	ALBERTA			AP		GAO-KAO*	A LEVEL (EDEXCEL)		SINGAPORE			INTERNATIONAL BACCALAUREATE			
	30-2	30-1	31	Calc AB	Calc BC		Math	Further Maths	H1	H2	H3	Maths Studies	Math SL	Math HL	Further Maths
CALCULUS	Not req	Not req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req
1. Limits and Continuity			X	X	X								X	X	Pre-req (HL)
2. Defn of Derivatives, Derivatives of Functions, Higher Derivatives			X	X	X		X (C1, C2)	Pre-req (C1, C2)	X	X	Pre-req (H2)	X	X	X (Core & Op)	X
3. Derivative Rules & Derivatives of Exponential, Logarithmic & Trigonometric Functions			X (Req & Op)	X	X		X (C1, C3, C4)	Pre-req (C1, C3, C4)	X	X	Pre-req (H2)		X	X	Pre-req (HL)
4. Derivative Theorems, Optimization, Related Rates & Implicit Differentiation			X (Req & Op)	X	X					Pre-req (O-Lev Add. Math)	Pre-req (O-Lev Add. Math)		X	X (Core & Op)	X
5. Applications of Differentiation – Rates of Change, Economics			X (Op)	X	X				X	X	Pre-req (H2)	X	X	X	Pre-req (HL)

6. Integrals, Techniques of Integration & Fundamental Theorem of Calculus			X (Req & Op)	X	X		X (C1, C2, C4)	X (FP3)	X	X	Pre-req (H2)		X	X (Core & Op)	X
7. Applications of Integration to Problems of Area & Volume			X (Op)	X	X		X (C4)	X (FP3)	X	X	Pre-req (H2)		X	X	Pre-req (HL)
8. Applications of Integration to Problems of Motion (Kinematics)			X	X	X								X	X	Pre-req (HL)
9. Derivatives & Integrals of Vector-valued & Parametric Functions; Polar Coordinates					X		X (C4)	Pre-req (C4)	X	X	Pre-req (H2)				
10. Hyperbolic Functions								X (FP3)							
11. Differential Equations			X	X	X		X (C4)	X (FP2)		X	X			X (Op)	X
12. Applications of Differential Equations: Growth & Decay, Springs & Circuits			X	X	X		X (C4)	Pre-req (C4)			X			X (Op)	X

13. Infinite Sequences & Series, Power Series Maclaurin & Taylor Series			X	X	X		X (C2)	X (FP2)		X	Pre-req (H2)			X (Op)	X
--	--	--	---	---	---	--	--------	---------	--	---	--------------	--	--	--------	---

APPLIED MATHEMATICS

CONTENT	ALBERTA			AP	GAO-KAO*	A LEVEL (EDEXCEL)		SINGAPORE			INTERNATIONAL BACCALAUREATE			
	30-2	30-1	31	Stat		Math	Further Maths	H1	H2	H3	Maths Studies	Math SL	Math HL	Further Maths
STATISTICS & PROBABILITY	Req	Req	Not req	Req	Req	Option	Option	Req	Req	Not req	Req	Req	Req	Req
1. Experimental Design & Sampling				X	X	X (S2)	X (S2, S3)	X	X		X	X	X	Pre-req (HL)
2. Descriptive Statistics				X	X	X (S1)	X (S1)	X	X		X	X	X	Pre-req (HL)
3. Basic Probability	X	X		X	X	X (S1)	X (S1)	X	X		X	X	X	Pre-req (HL)
4. Discrete and Continuous Random Variables, Normal Distribution, Other Prob. Distributions				X	X	X (S1, S2)	X (S1, S2)	X	X		X	X	X (Core & Op)	X

5. Central Limit Theorem & Law of Large Numbers				X			X (S3)	X	X				X (Op)	X
6. Joint Distributions, Independence				X	X		X (S3)						X (Op)	X
7. Hypothesis Tests & Confidence Intervals				X		X (S2)	X (S2, S3, S4)	X	X		X		X (Op)	X
8. Covariance, Correlation & Regression				X	X	X (S1)	X (S3)	X	X		X	X	X (Op)	X
DISCRETE/ DECISION MATHEMATICS					X	Option (D1, D2)	Option (D1, D2)						Option	Req
MECHANICS						Option (M1, M2)	Option (M1, M2, M3, M4, M5)							

* Please note that data for the Chinese Gaokao is incomplete. The categories in the chart marked as covered in the Gaokao are based on data from the ICOSSA study; however, it cannot be assumed that categories that are *not* marked in the chart are *not* covered by the Gaokao, since a detailed revision of the actual curriculum documents for this qualification could not be carried out.

ANNEX II

QUALIFICATIONS BY BREADTH OF MATHEMATICAL CONTENT

PURE MATHEMATICS

	AP Calc AB	AP Calc BC	Alb 30-2	Sing H1	Alb 30-1	IB Math Studies SL	Alb 31	IB Math SL	Sing H2	A Level Math	Sing H3	IB Math HL	IB Further Math HL	A Level Further Math	Gao-kao*
ALGEBRA AND FUNCTIONS	Pre-req	Pre-req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Pre-req	Req	Req
1. Basic Skills: Numbers, Operations, Logic			X	Pre-req (O-Level)	Pre-req (20-1)	X	Pre-req (30-1)	Pre-req	Pre-req (O-Level)	X (C1)	Pre-req (O-Level)	Pre-req	Pre-req (HL)	Pre-req (C1)	X
2. Functions & Graphs			X	X	X	X	X	X	X	X (C1, C3, C4)	X	X	Pre-req (HL)	Pre-req (C1, C3, C4)	X
3. Linear, Quadratic, Polynomial & Rational Functions			Pre-req (20-2)	Pre-req (O-Level)	X	X	X	X	Pre-req (O-Lev Add. Math)	X (C1, C2, C3)	Pre-req (O-Lev Add. Math)	X	Pre-req (HL)	Pre-req (C1, C2, C3)	X
4. Solving Equations & Inequalities			X	X	Pre-req (20-1)	X	X	X	X	X (C1)	X	X	Pre-req (HL)	X (FP2)	X
5. Exponential Functions, Logarithms & E			X	X	X	X	Pre-req (30-1)	X	Pre-req (O-Lev Add. Math)	X (C2, C3)	Pre-req (O-Lev Add. Math)	X	Pre-req (HL)	Pre-req (C2, C3)	

GEOMETRY	Pre-req	Pre-req	Pre-req	Pre-req	Pre-req	Req	Pre-req	Pre-req	Req	Req	Pre-req	Pre-req	Req	Req	Req
6. Solid Geometry & Basic Coordinate Geometry			Pre-req (20-2)	Pre-req (O-Level)	Pre-req (10C)	X	Pre-req (10C)		Pre-req (O-Lev Add. Math)	X (C1, C2)	Pre-req (O-Lev Add. Math)		Pre-req	Pre-req (C1, C2)	X
7. Parametric Equations of Curves & Conic Sections									X	X (C4)	Pre-req (H2)	X	X	X (FP1, FP3)	X
8. Polar Coordinates														X (FP2)	
TRIGONOMETRY	Pre-req	Pre-req	Pre-req	Pre-req	Req	Req	Req	Req	Pre-req	Req	Pre-req	Req	Pre-req	Pre-Req	Req
9. Trig: Unit Circle & Radians				Pre-req (O-Level)	X		Pre-req (30-1)	X	Pre-req (O-Level)	X (C1, C3)	Pre-req (O-Level)	X	Pre-req (HL)	Pre-req (C1, C3)	
10. Using Trig. to compute Area; Law of Sines & Cosines			Pre-req (20-2)	Pre-req (O-Level)	Pre-req (20-1)	X	Pre-req (20-1)	X	Pre-req (O-Level)	X (C2)	Pre-req (O-Level)	X	Pre-req (HL)	Pre-req (C2)	X
11. Trig. Functions, Ratios, Identities & Transform.					X	X	X	X	Pre-req (O-Lev Add. Math)	X (C2, C3)	Pre-req (O-Lev Add. Math)	X	Pre-req (HL)	Pre-req (C2, C3)	X
12. Inverse Trig. Functions & Inverse Trig. Identities									Pre-req (O-Lev Add. Math)	X (C3)	Pre-req (O-Lev Add. Math)	X	Pre-req (HL)	Pre-req (C3)	

13. Double-Angle & Half-Angle Formulas; Addition & Subtraction Formulas					X		X	X		X (C3)		X	Pre-req (HL)	Pre-req (C3)	
OTHER AREAS	Not req	Not req	Not req	Not req	Pre req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req
14. Basic Sequences & Series					Pre-req (20-1)	X		X	X	X (C1, C2, C4)	Pre-req (H2)	X	Pre-req (HL)	X (FP1, FP2)	X
15. Vectors								X	X	X (C4)	Pre-req (H2)	X	Pre-req (HL)	X (FP3)	X
16. Systems of Linear Equations						X	X		X		Pre-req (H2)	Pre-req	Pre-req		
17. Matrices & Linear Algebra												X	X	X (FP1, FP3)	
18. Complex Numbers									X		Pre-req (H2)	X	Pre-req (HL)	X (FP1, FP2)	X
19. Numerical Methods							X (Op)			X (C3)				X (FP1)	
20. Sets, Relations & Groups; Combinatorics											X	X (Op)	X		
21. Proof														X (FP1)	X

CALCULUS

	Alb 30-2	Alb 30-1	IB Math Studies SL	Sing H1	IB Math SL	A Level Math	Sing H2	A Level Further Math	Sing H3	Alb 31	IB Math HL	AP Calc AB	IB Further Math	AP Calc BC	Gao-kao*
CALCULUS	Not req	Not req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req	Req
1. Limits and Continuity					X					X	X	X	Pre-req (HL)	X	
2. Defn of Derivatives, Derivatives of Functions, Higher Derivatives			X	X	X	X (C1, C2)	X	Pre-req (C1, C2)	Pre-req (H2)	X	X (Core & Op)	X	X	X	
3. Derivative Rules & Derivatives of Exponential, Logarithmic & Trigonometric Functions				X	X	X (C1, C3, C4)	X	Pre-req (C1, C3, C4)	Pre-req (H2)	X (Req & Op)	X	X	Pre-req (HL)	X	
4. Derivative Theorems, Optimization, Related Rates & Implicit Differentiation					X		Pre-req (O-Lev Add. Math)		Pre-req (O-Lev Add. Math)	X (Req & Op)	X (Core & Op)	X	X	X	
5. Applications of Differentiation – Rates of Change, Economics			X	X	X		X		Pre-req (H2)	X (Op)	X	X	Pre-req (HL)	X	

6. Integrals, Techniques of Integration & Fundamental Theorem of Calculus				X	X	X (C1, C2, C4)	X	X (FP3)	Pre-req (H2)	X (Req & Op)	X (Core & Op)	X	X	X	
7. Applications of Integration to Problems of Area & Volume				X	X	X (C4)	X	X (FP3)	Pre-req (H2)	X (Op)	X	X	Pre-req (HL)	X	
8. Applications of Integration to Problems of Motion (Kinematics)					X					X	X	X	Pre-req (HL)	X	
9. Derivatives & Integrals of Vector-valued & Parametric Functions; Polar Coordinates				X		X (C4)	X	Pre-req (C4)	Pre-req (H2)					X	
10. Hyperbolic Functions								X (FP3)							
11. Differential Equations						X (C4)	X	X (FP2)	X	X	X (Op)	X	X	X	
12. Applications of Differential Equations: Growth & Decay, Springs & Circuits						X (C4)		Pre-req (C4)	X	X	X (Op)	X	X	X	

13. Infinite Sequences & Series, Power Series Maclaurin & Taylor Series						X (C2)	X	X (FP2)	Pre-req (H2)	X	X (Core & Op)	X	X	X	
--	--	--	--	--	--	--------	---	---------	--------------	---	---------------	---	---	---	--

APPLIED MATHEMATICS

	Sing H3	Alb 31	Alb 30-2	Alb 30-1	IB Math Studies SL	IB Math SL	Sing H1	Sing H2	A Level Math	IB Math HL	AP Stat	A Level Further Math	IB Further Math	Gaokao*
STATISTICS & PROBABILITY	Not req	Not req	Req	Req	Req	Req	Req	Req	Option	Req	Req	Option	Req	Req
1. Experimental Design & Sampling					X	X	X	X	X (S2)	X	X	X (S2, S3)	Pre-req (HL)	X
2. Descriptive Statistics					X	X	X	X	X (S1)	X	X	X (S1)	Pre-req (HL)	X
3. Basic Probability			X	X	X	X	X	X	X (S1)	X	X	X (S1)	Pre-req (HL)	X
4. Discrete and Continuous Random Variables, Normal Distribution, Other Prob. Distributions					X	X	X	X	X (S1, S2)	X (Core & Op)	X	X (S1, S2)	X	X

5. Central Limit Theorem & Law of Large Numbers							X	X		X (Op)	X	X (S3)	X	
6. Joint Distributions, Independence										X (Op)	X	X (S3)	X	X
7. Hypothesis Tests & Confidence Intervals					X		X	X	X (S2)	X (Op)	X	X (S2, S3, S4)	X	
8. Covariance, Correlation & Regression					X	X	X	X	X (S1)	X (Op)	X	X (S3)	X	X
DISCRETE/ DECISION MATHEMATICS									Option (D1, D2)	Option		Option (D1, D2)	Req	Req
MECHANICS									Option (M1, M2)			Option (M1, M2, M3, M4, M5)		

* Please note that data for the Chinese Gaokao is incomplete. The categories in the chart marked as covered in the Gaokao are based on data from the ICOSA study; however, it cannot be assumed that categories that are *not* marked in the chart are *not* covered by the Gaokao, since a detailed revision of the actual curriculum documents for this qualification could not be carried out.

ANNEX III

CLASSIFICATIONS: PURE MATHEMATICS TOPICS, CALCULUS TOPICS, APPLIED MATHEMATICS TOPICS

A. PURE MATHEMATICS

CATEGORIES ELABORATED FROM THE FOLLOWING TEXTBOOKS/COURSES:

- Sheldon Axler, *Pre-calculus: A Prelude to Calculus*, John Wiley & Sons, 2009 (used in Princeton course MAT100: Precalculus/Prestatistics):
<http://precalculus.axler.net/Contents2.pdf>
- *ALEKS Pre-calculus Course Syllabus*, 2013-2014 (Stanford's Pre-calculus study tool):
https://precalculus.stanford.edu/sites/default/files/CourseSyllabus_1.pdf

ALGEBRA AND FUNCTIONS

1. BASIC SKILLS: NUMBERS, INEQUALITIES, OPERATIONS, LOGIC

(AXLER: THE REAL NUMBER LINE: INTEGERS, RATIONAL NUMBERS, IRRATIONAL NUMBERS; ALGEBRA OF REAL NUMBERS: SIMPLIFYING ALGEBRAIC EXPRESSIONS USING COMMUTATIVE, ASSOCIATIVE AND DISTRIBUTIVE PROPERTIES, ORDER OF ALGEBRAIC OPERATIONS, ALGEBRAIC IDENTITIES INVOLVING ADDITIVE INVERSES AND SUBTRACTION, MULTIPLICATIVE INVERSES AND DIVISION; INEQUALITIES: POSITIVE AND NEGATIVE NUMBERS, LESSER AND GREATER, TRANSITIVITY, ADDITIVE INVERSE, MULTIPLICATIVE INVERSE AND INEQUALITIES; INTERVALS: SETS, NOTATION FOR OPEN, CLOSED AND HALF-OPEN INTERVALS, UNION OF SETS; ABSOLUTE VALUE; ALEKS: OPERATIONS WITH ABSOLUTE VALUE, EXPONENTS AND INTEGERS, ORDER OF OPERATIONS; RATIONAL EXPRESSIONS: ADDING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS AND BINOMIAL NUMERATORS, AND WITH DIFFERENT DENOMINATORS, MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS; RADICAL EXPRESSIONS: SIMPLIFYING THE SQUARE ROOT OF A WHOLE NUMBER, SIMPLIFYING A RADICAL EXPRESSION; SQUARE ROOT ADDITION, SUBTRACTION AND MULTIPLICATION)

ALBERTA MATHS 30-2: Relations and Functions - Determine equivalent forms of rational expressions, perform operations on rational expressions.

ALBERTA MATH 30-2: Logical reasoning - Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. Solve problems that involve the application of set theory.

A LEVEL UNIT C1: Algebra and Functions – Laws of indices for all rational exponents; use and manipulation of surds.

IB MATHS STUDIES SL: Number and Algebra – Natural numbers, integers, rational numbers and real numbers; Expressing numbers in the form $a \times 10^k$, where $1 \leq a < 10$ and k is an integer. Operations with numbers in this form.

IB MATHS STUDIES SL: Number and algebra – Approximation: decimal places, significant figures. Percentage errors. Estimation. Students should be aware of the errors that can result from premature rounding. Students should be able to recognize whether the results of calculations are reasonable, including reasonable values of, for example, lengths, angles and areas. SI (*Système International*) and other basic units of measurement. Students should be able to convert between different units. Currency conversions. Students should be able to perform currency transactions involving commission.

IB MATHS STUDIES SL: Logic, sets and probability – Basic concepts of symbolic logic: definition of a proposition; symbolic notation of propositions. Compound statements: implication \Rightarrow ; equivalence \Leftrightarrow ; negation \neg ; conjunction \wedge ; disjunction \vee ; exclusive disjunction ∇ . Translation between verbal statements and symbolic form. Truth tables: concepts of logical contradiction and tautology. A maximum of three propositions will be used in truth tables. Truth tables can be used to illustrate the associative and distributive properties of connectives, and for variations of implication and equivalence statements, for example, $\neg q \Rightarrow \neg p$. Converse, inverse, contrapositive. Logical equivalence. Testing the validity of simple arguments through the use of truth tables. The topic may be extended to include syllogisms. In examinations these will not be tested.

2. FUNCTIONS AND GRAPHS

(AXLER: DEFINITION: EQUALITY OF FUNCTIONS, DOMAIN AND RANGE OF A FUNCTION, FUNCTIONS VIA TABLES; THE COORDINATE PLANE AND GRAPHS: COORDINATE AXES, THE GRAPH OF A FUNCTION, DETERMINING A FUNCTION FROM ITS GRAPH, VERTICAL LINE TEST; FUNCTION TRANSFORMATIONS AND GRAPHS: SHIFTING A GRAPH UP OR DOWN, SHIFTING A GRAPH RIGHT OR LEFT, SHIFTING A GRAPH VERTICALLY OR HORIZONTALLY, STRETCHING A GRAPH VERTICALLY OR HORIZONTALLY, REFLECTING A GRAPH IN THE HORIZONTAL OR VERTICAL AXIS, EVEN AND ODD FUNCTIONS; COMPOSITION OF FUNCTIONS: DEFINITION OF COMPOSITION, ORDER MATTERS IN COMPOSITION, IDENTITY FUNCTION, DECOMPOSING FUNCTIONS, COMPOSING MORE THAN TWO FUNCTIONS; INVERSE FUNCTIONS: ONE-TO-ONE FUNCTIONS, DEFINITION OF f^{-1} , RELATIONSHIP BETWEEN f AND f^{-1} , FINDING A FORMULA FOR AN INVERSE FUNCTION, DOMAIN AND RANGE OF AN INVERSE FUNCTION; A GRAPHICAL APPROACH TO INVERSE FUNCTIONS: GRAPH OF AN INVERSE FUNCTION, GRAPHICAL INTERPRETATION OF ONE-TO-ONE, HORIZONTAL LINE TEST, INCREASING AND DECREASING FUNCTIONS; ALEKS: FUNCTIONS: DOMAIN AND RANGE FROM ORDERED PAIRS, IDENTIFYING FUNCTIONS FROM RELATIONS, VERTICAL LINE TEST, EVALUATING A PIECEWISE-DEFINED FUNCTION, SUM, DIFFERENCE, AND PRODUCT OF TWO FUNCTIONS, QUOTIENT OF TWO FUNCTIONS, EXPRESSING A FUNCTION AS A COMPOSITION OF TWO FUNCTIONS, HORIZONTAL LINE TEST, DETERMINING WHETHER TWO FUNCTIONS ARE INVERSES OF EACH OTHER; GRAPHING FUNCTIONS: FINDING INPUTS AND OUTPUTS OF A FUNCTION FROM ITS GRAPH, DOMAIN AND RANGE FROM THE GRAPH OF A CONTINUOUS FUNCTION, FINDING WHERE A FUNCTION IS INCREASING, DECREASING OR CONSTANT GIVEN THE GRAPH, FINDING INTERCEPTS OF A NONLINEAR FUNCTION GIVEN ITS GRAPH, FINDING LOCAL MAXIMA AND MINIMA OF A FUNCTION GIVEN THE GRAPH, EVEN AND ODD FUNCTIONS, EQUATIONS FOR FUNCTIONS AFTER VERTICAL AND HORIZONTAL TRANSLATIONS, TRANSLATING THE GRAPH OF A FUNCTION, TRANSFORMING THE GRAPH OF A FUNCTION BY REFLECTING, SHRINKING OR STRETCHING, USING MORE THAN ONE TRANSFORMATION, DOMAIN AND RANGE FROM THE GRAPH OF A PIECEWISE FUNCTION; STEWART: FOUR WAYS TO REPRESENT A FUNCTION: VERBALLY, NUMERICALLY, GRAPHICALLY AND ALGEBRAICALLY, THE VERTICAL LINE TEST, PIECEWISE DEFINED FUNCTIONS, SYMMETRY, INCREASING AND DECREASING FUNCTIONS; NEW FUNCTIONS FROM OLD FUNCTIONS: TRANSFORMATIONS OF FUNCTIONS, VERTICAL AND HORIZONTAL SHIFTS, STRETCHING AND REFLECTING, COMBINATIONS OF FUNCTIONS, COMPOSING AND DECOMPOSING A FUNCTION; GRAPHING CALCULATORS AND COMPUTERS)

ALBERTA MATHS 30-2: Relations and Functions - Represent data, using polynomial functions (of degree ≤ 3), to solve problems. Represent data, using sinusoidal functions, to solve problems.

ALBERTA MATHS 30-1: Relations and Functions - Demonstrate an understanding of operations on, and compositions of, functions. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. Demonstrate an understanding of the effects of horizontal and vertical stretches on

the graphs of functions and their related equations. Apply translations and stretches to the graphs and equations of functions. Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: x -axis, y -axis, line $y = x$.

ALBERTA MATHS 31: Precalculus and Limits - Students will demonstrate conceptual understanding of the algebra of functions, by: illustrating different notations that describe functions and intervals, expressing, in interval notation, the domain and range of functions; expressing the sum, product, difference and quotient, algebraically and graphically, given any two functions; expressing, algebraically and graphically, the composition of two or more functions.

A LEVEL UNIT C1: Algebra and Functions – Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations. Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

A LEVEL UNIT C3: Algebra and Functions – Definition of a function; domain and range of functions; composition of functions; inverse functions and their graphs. The modulus function. Combinations of the transformations $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

A LEVEL UNIT C4: Algebra and Functions – Rational functions. Partial fractions (denominators not more complicated than repeated linear terms). Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions. Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.

SINGAPORE H1: Functions and graphs - Exponential and logarithmic functions and graphing techniques: Include: concept of function, use of notation such as $f(x) = x^2 + 5$; use of a graphic calculator to graph a given function; characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes

SINGAPORE H2: Functions and graphs – Functions, Inverse Functions and Composite Functions – Include: concepts of function, domain and range; use of notations such as $f(x) = x^2 + 5$; $f: x \mapsto x^2 + 5$; $f^{-1}(x)$, $fg(x)$ and $f^2(x)$; finding inverse functions and composite functions; conditions for the existence of inverse functions and composite functions; domain restriction to obtain an inverse function; relationship between a function and its inverse as reflection in the line $y = x$. Graphing techniques- Include: use of a graphing calculator

to graph a given function; relating the following equations with their graphs: $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$, $y = \frac{ax + b}{cx + d}$, $y = \frac{ax^2 + bx + c}{dx + e}$

Characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes; determining the equations of asymptotes, axes of symmetry, and restrictions on the possible values of x and/or y ; effect of transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$ and $y = f(ax)$, and

combinations of these transformations relating the graphs of $y = |f(x)|$, $y = f(|x|)$, $y = \frac{1}{f(x)}$ and $y^2 = f(x)$ to the graph of $y = f(x)$

SINGAPORE H3: Functions and graphs – Functions, inverse functions and composite functions (such as properties of functions and graphs, piecewise defined functions, step functions, even and odd functions, greatest integer, increasing and decreasing functions); Graphing techniques

IB MATHS STUDIES SL: Mathematical models – Concept of a function, domain, range and graph. Function notation, eg $f(x)$, $v(t)$, $C(n)$. Concept of a function as a mathematical model.

IB MATHS SL: Functions and Equations – Concept of function $f: x \mapsto f(x)$. Domain, range; image (value). Composite functions. Identity function. Inverse function f^{-1} . Example:

for $x \mapsto \sqrt{2-x}$, domain is $x \leq 2$, range is $y \geq 0$. A graph is helpful in visualizing the range. $(f \circ g)(x) = f(g(x))$. $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$ On examination papers, students will only be asked to find the inverse of a *one-to-one* function. *Not required*: domain restriction. The graph of a function; its equation $y = f(x)$. Function graphing skills.

Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range. Use of technology to graph a variety of functions, including ones not specifically mentioned. The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$. Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p : $y = pf(x)$. Stretch in the x -direction with scale factor $1/q$: $y = f(qx)$. Composite transformations. Technology should be used to investigate these transformations. Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down. Example: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y -direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

IB MATHS HL: Functions and Equations (Core) - Concept of function $f: x \mapsto f(x)$: Domain, range; image (value). Odd and even functions. Composite functions $f \circ g$ Identity function. One-to-one and many-to-one functions. Inverse function f^{-1} , including domain restriction. Self-inverse functions. $(f \circ g)(x) = f(g(x))$. The graph of a function; its equation $y = f(x)$. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes and symmetry, and consideration of domain and range. Use of technology to graph a variety of functions. The graphs of the functions $y = |f(x)|$ and $y = f(|x|)$. The graph of $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$. Use of technology to graph a variety of functions. Transformations of graphs: translations; stretches; reflections in the axes. The graph of the inverse function as a reflection in $y = x$. Students are expected to be aware of the effect of transformations on both the algebraic expression and the graph of a function.

Solutions of $g(x) \geq f(x)$. Graphical or algebraic methods, for simple polynomials up to degree 3. Use of technology for these and other functions.

3. LINEAR, QUADRATIC, POLYNOMIAL AND RATIONAL FUNCTIONS

(AXLER: LINEAR FUNCTIONS AND LINES: SLOPE, THE EQUATION OF A LINE, CONSTANT FUNCTIONS, PARALLEL LINES, PERPENDICULAR LINES; QUADRATIC FUNCTIONS AND PARABOLAS: THE VERTEX OF A PARABOLA, COMPLETING THE SQUARE, VERTEX AT A MAXIMUM OR MINIMUM VALUE, THE QUADRATIC FORMULA; INTEGER EXPONENTS: EXPONENTIATION BY POSITIVE INTEGERS, PROPERTIES OF EXPONENTIATION, DEFINING x^0 , EXPONENTIATION BY NEGATIVE INTEGERS, MANIPULATIONS WITH POWERS, ALGEBRAIC PROPERTIES OF EXPONENTS; POLYNOMIALS: THE DEGREE OF A POLYNOMIAL, THE ALGEBRA OF POLYNOMIALS, DEGREE OF THE SUM AND DIFFERENCE OF TWO POLYNOMIALS, DEGREE OF THE PRODUCT OF TWO POLYNOMIALS, ZEROS AND FACTORIZATION OF POLYNOMIALS, CUBIC FORMULA FOR POLYNOMIALS OF DEGREE 3- NOT COVERED, THE BEHAVIOR OF A POLYNOMIAL NEAR +/- INFINITY, ZEROS FOR POLYNOMIALS WITH ODD DEGREE, GRAPHS OF POLYNOMIALS; RATIONAL FUNCTIONS: RATIOS OF POLYNOMIALS, ALGEBRA OF RATIONAL FUNCTIONS, DIVISION OF POLYNOMIALS, FACTOR THEOREM, REMAINDER THEOREM, FACTORIZATION DUE TO A ZERO, THE BEHAVIOR OF A RATIONAL FUNCTION NEAR +/- INFINITY, ASYMPTOTE, GRAPHS OF RATIONAL FUNCTIONS; ALEKS: LINES: GRAPHING A LINE GIVEN ITS EQUATION IN SLOPE-INTERCEPT FORM, FINDING SLOPE, WRITING THE EQUATIONS OF VERTICAL AND HORIZONTAL LINES THROUGH A GIVEN POINT, FINDING SLOPES OF LINES PARALLEL AND PERPENDICULAR TO A LINE; POLYNOMIALS: FACTORING A QUADRATIC WITH LEADING COEFFICIENT 1 OR GREATER THAN 1, FACTORING A DIFFERENCE OF SQUARES, COMPLETING THE SQUARE, POLYNOMIAL LONG DIVISION; RATIONAL EXPRESSIONS: SIMPLIFYING A RATIO OF POLYNOMIALS)

ALBERTA MATHS 30-1: Relations and Functions - Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5). Graph and analyze radical functions (limited to functions involving one radical). Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials).

ALBERTA MATHS 31: Precalculus and Limits – Students will demonstrate procedural competence associated with the transformation of functions, by sketching the graph of, and describing algebraically, the effects of any translation, reflection or dilatation on any of the following functions or their inverses: linear, quadratic or cubic polynomial; absolute value; reciprocal; exponential; step. [Also] by describing the relationship between parallel and perpendicular lines; describing the condition for tangent, normal and secant lines to a curve; finding the equation of a line given any two conditions that serve to define it.

A LEVEL UNIT C1: Algebra and Functions – Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorization; Quadratic functions and their graphs; the discriminant of a quadratic function; Completing the square

A LEVEL UNIT C2: Algebra and Functions – Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.

A LEVEL UNIT C3: Algebra and Functions – Simplification of rational expressions including factorising and cancelling, and algebraic division.

IB MATHS STUDIES SL: Mathematical models – Linear models. Linear functions and their graphs, $f(x) = mx + c$. Quadratic models. Quadratic functions and their graphs (parabolas): $f(x) = ax^2 + bx + c$; $a \neq 0$. Properties of a parabola: symmetry; vertex; intercepts on the x -axis and y -axis. Equation of the axis of symmetry, $x = -b/2a$. Functions with zero, one or two real roots are included. The form of the equation of the axis of symmetry may initially be found by investigation. Properties should be illustrated with a GDC or graphical software. Drawing accurate graphs. Creating a sketch from information given. Transferring a graph from GDC to paper. Reading, interpreting and making predictions using graphs. Included all the functions above and additions and subtractions. Students should be aware of the difference between the command terms “draw” and “sketch”. All graphs should be labelled and have some indication of scale. Examples: $f(x) = x^3 + 5 - \frac{2}{x}$, $g(x) = 3^{-x} + x$.

IB MATHS SL: Functions and Equations – The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y -intercept $(0, c)$. Axis of symmetry. The form $x \mapsto a(x-p)(x-q)$, x -intercepts $(p, 0)$ and $(q, 0)$. The form $x \mapsto a(x-h)^2 + k$, vertex (h, k) . Candidates are expected to be able to change from one form to another.

IB MATHS HL: Functions and Equations (Core) - Polynomial functions and their graphs. The factor and remainder theorems. The graphical significance of repeated factors. The relationship between the degree of a polynomial function and the possible numbers of x -intercepts.

4. SOLVING EQUATIONS AND INEQUALITIES

(ALEKS: SOLVING A LINEAR EQUATION, SOLVING EQUATIONS WITH ZERO, ONE OR INFINITELY MANY SOLUTIONS, SOLVING AN ABSOLUTE VALUE EQUATION, FINDING THE ROOTS OF A QUADRATIC EQUATION, SOLVING A QUADRATIC EQUATION NEEDING SIMPLIFICATION, USING THE SQUARE ROOT PROPERTY, OR BY COMPLETING THE SQUARE, APPLYING THE QUADRATIC FORMULA, SOLVING A RATIONAL EQUATION THAT SIMPLIFIES TO LINEAR, SOLVING A RADICAL EQUATION THAT SIMPLIFIES TO A LINEAR OR QUADRATIC EQUATION, SOLVING A LINEAR INEQUALITY, SOLVING A COMPOUND LINEAR INEQUALITY, SOLVING AN ABSOLUTE VALUE INEQUALITY, SOLVING A QUADRATIC INEQUALITY, SOLVING A POLYNOMIAL INEQUALITY, SOLVING A RATIONAL INEQUALITY)

ALBERTA MATHS 30-2: Relations and Functions - Solve problems that involve rational equations (all of these limited to numerators and denominators that are monomials and binomials).

ALBERTA MATHS 31: Precalculus and Limits - Students will demonstrate conceptual understanding of the algebra of functions, by: illustrating the solution sets for linear, quadratic and absolute value inequalities $|P(x)| \geq a$, $|P(x)| \leq a$, $ax^2 + bx + c \geq d$. Conceptual understanding of equivalent forms, by describing what it means for two algebraic

or trigonometric expressions to be equivalent; demonstrate procedural competence by factoring expressions with integral and rational exponents, using a variety of techniques; rationalizing expressions containing a numerator or a denominator that contains a radical; simplifying rational expressions, using any of the four basic operations.

A LEVEL UNIT C1: Algebra and Functions – Solution of quadratic equations; solution of linear and quadratic inequalities. Simultaneous equations: analytical solution by substitution. For example, where one equation is linear and one equation is quadratic.

A LEVEL UNIT FP2: Inequalities – The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.

SINGAPORE H1: Functions and graphs – Equations and Inequalities – Include: solving simultaneous equations, one linear and one quadratic, by substitution; conditions for a quadratic equation to have real or equal roots; solving quadratic inequalities; conditions for $ax^2 + bx + c$ to be always positive (or always negative); solving inequalities by graphical methods; formulating an equation from a problem situation; finding the numerical solution of an equation using a graphic calculator

SINGAPORE H2: Functions and graphs – Equations and Inequalities - solving inequalities of the form $f(x)/g(x) > 0$ where $f(x)$ and $g(x)$ are quadratic expressions that are either factorisable or always positive; solving inequalities by graphical methods; formulating an equation or a system of linear equations from a problem situation; finding the numerical solution of equations (including system of linear equations) using a graphing calculator

SINGAPORE H3: Functions and graphs – Equations and inequalities (such as Triangle inequality, AM-GM inequality)

IB MATHS STUDIES SL: Number and Algebra – Use of a GDC to solve quadratic equations. **Mathematical models** – Use of a GDC to solve equations involving combinations of the functions above [linear, quadratic, exponential]. *Examples: $x + 2 = 2x^3 + 3x - 1$, $5x = 3^x$.*

IB MATHS SL: Functions and Equations – Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Solving $ax^2 + bx + c = 0$, $a \neq 0$. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. Solutions may be referred to as roots of equations or zeros of functions. Examples: $e^x = \sin x$, $x^4 + 5x - 6 = 0$. Example: find k given that the

equation $3kx^2 + 2x + k = 0$ has two equal real roots. Examples: $2^{x-1} = 10$, $\left(\frac{1}{3}\right)^x = 9^{x+1}$. Applications of graphing skills and solving equations that relate to real-life situations.

IB MATHS HL: Functions and Equations (Core) - Solving quadratic equations using the quadratic formula. Use of the discriminant $\Delta = b^2 - 4ac$ to determine the nature of the roots. May be referred to as roots of equations or zeros of functions. Solving polynomial equations both graphically and algebraically. Sum and product of the roots of polynomial equations.

5. EXPONENTIAL FUNCTIONS, LOGARITHMS AND E

(AXLER: RATIONAL AND REAL EXPONENTS: ROOTS, ROOTS AS INVERSE FUNCTIONS, EXPONENTIATION BY A RATIONAL AND IRRATIONAL NUMBER, ALGEBRAIC PROPERTIES OF EXPONENTS; LOGARITHMS AS INVERSES OF EXPONENTIATION: LOGARITHMS BASE 2, LOGARITHMS WITH ARBITRARY BASE, INVERSE PROPERTIES OF LOGARITHMS, CHANGE OF BASE; ALGEBRAIC PROPERTIES OF LOGARITHMS: LOGARITHM OF A PRODUCT – PRODUCT RULE, LOGARITHM OF A QUOTIENT – QUOTIENT RULE, COMMON LOGARITHMS AND THE NUMBER OF DIGITS, LOGARITHMS OF A POWER, POWER RULE; EXPONENTIAL GROWTH: POPULATION GROWTH, EXPONENTIAL GROWTH AND DOUBLING, SIMPLE AND COMPOUND INTEREST; ADDITIONAL APPLICATIONS OF EXPONENTS AND LOGARITHMS: RADIOACTIVE DECAY AND HALF-LIFE, EARTHQUAKES AND THE RICHTER SCALE, SOUND INTENSITY AND DECIBELS, STAR BRIGHTNESS AND APPARENT MAGNITUDE; e AND THE NATURAL LOGARITHM: ESTIMATING AREA USING RECTANGLES, DEFINING e, AN AREA FORMULA, NATURAL LOGARITHM, PROPERTIES OF THE EXPONENTIAL FUNCTION AND ln, CONNECTIONS BETWEEN THE EXPONENTIAL FUNCTION AND THE NATURAL LOGARITHM; APPROXIMATIONS WITH e AND ln: APPROXIMATIONS, INEQUALITIES WITH THE NATURAL

LOGARITHM, APPROXIMATIONS WITH THE EXPONENTIAL FUNCTION, AN AREA FORMULA; **EXPONENTIAL GROWTH REVISITED**: CONTINUOUSLY COMPOUNDED INTEREST, CONTINUOUS GROWTH RATE; **ALEKS: EXPONENTIALS AND LOGS**: SOLVING AN EXPONENTIAL EQUATION BY FINDING COMMON BASES, CONVERTING BETWEEN LOGARITHMIC AND EXPONENTIAL EQUATIONS, EVALUATING A LOGARITHMIC EXPRESSION, SOLVING LOGARITHMIC EQUATIONS, SOLVING A MULTI-STEP EQUATION INVOLVING A SINGLE NATURAL LOGARITHM, SOLVING EXPONENTIAL EQUATIONS BY USING LOGARITHMS AND NATURAL LOGARITHM; **STEWART CALCULUS: EXPONENTIAL FUNCTIONS**: LAWS OF EXPONENTS, APPLICATIONS OF EXPONENTIAL FUNCTIONS, THE NUMBER e , THE NATURAL EXPONENTIAL FUNCTION; **INVERSE FUNCTIONS AND LOGARITHMS**: ONE-TO-ONE FUNCTIONS, LOGARITHMIC FUNCTIONS, LAWS OF LOGARITHMS, NATURAL LOGARITHMS, CHANGE OF BASE FORMULA, GRAPH AND GROWTH OF THE NATURAL LOGARITHM)

ALBERTA MATHS 30-2: Relations and Functions - Demonstrate an understanding of logarithms and the laws of logarithms; solve problems that involve exponential equations; represent data, using exponential and logarithmic functions, to solve problems.

ALBERTA MATHS 30-1: Relations and Functions - Demonstrate an understanding of logarithms. Demonstrate an understanding of the product, quotient and power laws of logarithms. Graph and analyze exponential and logarithmic functions. Solve problems that involve exponential and logarithmic equations.

A LEVEL UNIT C2: Exponentials and Logarithms – $y = ax$ and its graph; laws of logarithms; the solution of equations of the form $ax = b$.

A LEVEL UNIT C3: Exponentials and Logarithms – The function e^x and its graph. To include the graph of $y = e^{ax+b} + c$. The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x .

SINGAPORE H1: Functions and graphs - Exponential and logarithmic functions and graphing techniques: Include: functions e^x and $\ln x$ and their graphs; laws of logarithms; equivalence of $y = e^x$ and $x = \ln y$; use of a graphic calculator to graph a given function; characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes

IB MATHS STUDIES SL: Mathematical models – Exponential models. Exponential functions and their graphs: $f(x) = ka^x + c$; $a \in \mathbb{Q}^+$, $a \neq 1$, $k \neq 0$.

$f(x) = ka^{-x} + c$; $a \in \mathbb{Q}^+$, $a \neq 1$, $k \neq 0$. Concept and equation of a horizontal asymptote. Models using functions of the form $f(x) = ax^m + bx^n + \dots$; $m, n \in \mathbb{Z}$. In examinations, students will be expected to use graphical methods, including GDCs, to solve problems. Functions of this type and their graphs. The y -axis as a vertical asymptote.

Examples: $f(x) = 3x^4 - 5x + 3$, $g(x) = 3x^2 - \frac{4}{x}$.

IB MATHS SL: Algebra – Elementary treatment of exponents and logarithms. Laws of exponents, laws of logarithms. Change of base. Examples: $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$;

$\log 32 = 5 \log 2$; $(2^3)^{-4} = 2^{-12}$. Examples: $\log_4 7 = \frac{\ln 7}{\ln 4}$, $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} \left(= \frac{3}{2} \right)$.

IB MATHS SL: Functions and Equations – Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$. Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$.

Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$. Solving exponential equations. The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.

The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph. Vertical and horizontal asymptotes. Examples: $h(x) = \frac{4}{3x-2}$, $x \neq \frac{2}{3}$; $y = \frac{x+7}{2x-5}$, $x \neq \frac{5}{2}$. Diagrams should include all asymptotes and intercepts.

IB MATHS HL: Algebra (Core) - Exponents and logarithms. Laws of exponents; laws of logarithms. Change of base. The rational function $x \mapsto \frac{ax+b}{cx+d}$, and its graph. The function $x \mapsto a^x$, $a > 0$ and its graph. The function $x \mapsto \log_a x$, $x > 0$ and its graph. The reciprocal function is a particular case. Graphs should include both asymptotes and any intercepts with axes. Exponential and logarithmic functions as inverses of each other.

IB MATHS HL: Functions and Equations (Core) - Solution of $a^x = b$ using logarithms. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. For the polynomial equation $\sum_{r=0}^n a_r x^r = 0$, the sum is $\frac{-a_{n-1}}{a_n}$, the product is $\frac{(-1)^n a_0}{a_n}$.

GEOMETRY

6. SOLID GEOMETRY AND BASIC COORDINATE/ANALYTIC GEOMETRY

(AXLER: DISTANCE, LENGTH AND CIRCLE: DISTANCE BETWEEN TWO POINTS, MIDPOINTS, DISTANCE BETWEEN A POINT AND A LINE, PERIMETER AND CIRCUMFERENCE, PIE- RATIO OF CIRCUMFERENCE TO DIAMETER OF A CIRCLE; AREAS OF SIMPLE REGIONS: SQUARES, RECTANGLES, PARALLELOGRAMS, TRIANGLES, TRAPEZOIDS, STRETCHING, AREA STRETCH THEOREM, AREAS INSIDE A CIRCLE, ELLIPSES; ALEKS: PYTHAGOREAN THEOREM, AREA OF A TRIANGLE, CIRCUMFERENCE AND AREA OF A CIRCLE, VOLUME OF A RECTANGULAR PRISM, PLOTTING A POINT IN THE COORDINATE PLANE, DISTANCE BETWEEN TWO POINTS IN THE PLANE, GRAPHING A CIRCLE GIVEN ITS EQUATION IN STANDARD FORM; STEWART: CARTESIAN COORDINATE SYSTEM: COORDINATE AXES, QUADRANTS, DISTANCE FORMULA; LINES: SLOPE, POINT-SLOPE FORM OF THE EQUATION OF A LINE, SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE; PARALLEL AND PERPENDICULAR LINES)

A LEVEL UNIT C1: Coordinate geometry in the (x,y) plane – Equation of a straight line, including the forms $y-y_1 = m(x-x_1)$ and $ax + by + c = 0$. Conditions for two straight lines to be parallel or perpendicular to each other.

A LEVEL UNIT C2: Coordinate geometry in the (x,y) plane – Coordinate geometry of the circle using the equation of a circle in the form $(x-a)^2 + (y-b)^2 = r^2$ and including use of the following circle properties: i) the angle in a semicircle is a right angle; ii) the perpendicular from the centre to a chord bisects the chord; iii) the perpendicularity of radius and tangent.

IB MATHS STUDIES SL: Geometry and Trigonometry – Equation of a line in two dimensions: the form $y = mx + c$ and $ax + by + d = 0$. Gradient; intercepts. Points of intersection of lines. Lines with gradients m_1 and m_2 . Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.

IB MATHS STUDIES SL: Geometry and Trigonometry – Geometry of three-dimensional solids: cuboid; right prism; right pyramid; right cone; cylinder; sphere; hemisphere; and combinations of these solids. The distance between two points; eg between two vertices or vertices with midpoints or midpoints with midpoints. The size of an angle between two lines or between a line and a plane. Volume and surface areas of the three-dimensional solids defined above. In examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes. *Not required:* angle between two planes.

7. PARAMETRIC EQUATIONS OF CURVES AND CONIC SECTIONS

(AXLER: EQUATION OF A CIRCLE, LENGTH OF A CURVE; STEWART: CIRCLES: EQUATION OF A CURVE, EQUATION OF A CIRCLE – CENTER AND RADIUS; CONIC SECTIONS: PARABOLAS – VERTEX, AXIS, EQUATION OF THE PARABOLA; ELLIPSES - FOCI, EQUATION OF THE ELLIPSE; HYPERBOLAS - EQUATION OF THE HYPERBOLA;

STEWART CALCULUS: PARAMETRIC EQUATIONS, GRAPHING A PARAMETRIC CURVE, IDENTIFYING A PARAMETRIC CURVE, GRAPHING PARAMETRIC CURVES WITH GRAPHING CALCULATORS, DERIVING PARAMETRIC EQUATIONS FOR A CYCLOID)

A LEVEL UNIT C4: Coordinate geometry in the (x,y) plane – Parametric equations of curves and conversion between Cartesian and parametric forms. Students should be able to find the area under a curve given its parametric equations. Students will *not* be expected to sketch a curve from its parametric equations.

A LEVEL UNIT FP1: Coordinate systems – Idea of parametric equation for parabola and rectangular hyperbola.

A LEVEL UNIT FP1: Coordinate systems – Cartesian equations for the parabola and rectangular hyperbola. The focus-directrix property of the parabola. Concept of focus and directrix and parabola as locus of points equidistant from focus and directrix. Tangents and normal to these curves.

A LEVEL UNIT FP3: Further coordinate systems – Cartesian and parametric equations for the ellipse and hyperbola. The focus-directrix properties of the ellipse and hyperbola, including the eccentricity. Tangents and normal to these curves. The condition for $y = mx + c$ to be a tangent to these curves is expected to be known. Simple loci problems.

SINGAPORE H2: Functions and graphs – Simple parametric equations and their graphs.

IB FURTHER MATH HL: Geometry – Parametric equations. Parametric differentiation. Tangents and normal. The standard parametric equations of the circle, parabola, ellipse, rectangular hyperbola, hyperbola. $x = a \cos \theta$, $y = a \sin \theta$,

$$x = at^2, y = 2at, x = a \cos \theta, y = b \sin \theta, x = ct, y = \frac{c}{t}, x = a \sec \theta, y = b \tan \theta$$

IB FURTHER MATH HL: Geometry – Similar and congruent triangles. Euclid's theorem for proportional segments in a right-angled triangle. Centres of a triangle: orthocenter, incentre, circumcenter and centroid. The terms altitude, angle bisector, perpendicular bisector, median. Proof of concurrency theorems. The terms inscribed and circumscribed.

Circle geometry. Tangents; arcs, chords and secants. In a cyclic quadrilateral, opposite angles are supplementary, and the converse. Angle at centre theorem and corollaries. The tangent-secant and secant-secant theorems and intersecting chord theorems.

Angle bisector theorem; Apollonius' circle theorem, Menelaus' theorem; Ceva's theorem; Ptolemy's theorem for cyclic quadrilaterals. Proofs of these theorems and converses. The use of these theorems to prove further results.

Finding equations of loci. Coordinate geometry of the circle. Tangents to a circle. The equations $(x - h)^2 + (y - k)^2 = r^2$ and $x^2 + y^2 + dx + ey + f = 0$. Conic sections. The

parabola, ellipse and hyperbola, including rectangular hyperbola. Focus-directrix definitions. Tangents and normal. The standard forms $y^2 = 4ax$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and their translations.

The general conic $ax^2 + 2bxy + cy^2 + dx + ey + f = 0$ and the quadratic form $\mathbf{x}^T \mathbf{A} \mathbf{x} = ax^2 + 2bxy + cy^2$. Diagonalizing the matrix A with the rotation matrix P and reducing the general conic to standard form. The general conic can be rotated to give the form $\lambda_1 x^2 + \lambda_2 y^2 + dx + ey + f = 0$ where λ_1 and λ_2 are the eigenvalues of the matrix A in the quadratic form $\mathbf{x}^T \mathbf{A} \mathbf{x}$.

8. POLAR COORDINATES

(AXLER: DEFINING POLAR COORDINATES - The polar coordinates r and θ of a point in the coordinate plane are characterized as follows:

The polar coordinate r is the distance from the origin to the point. The polar coordinate θ is the angle between the positive horizontal axis and the line segment from the origin to the point.

, CONVERTING FROM POLAR TO RECTANGULAR COORDINATES AND VICE-VERSA; Choosing the polar coordinate θ in $(-\pi, \pi]$, GRAPHS OF POLAR EQUATIONS)

The sketching of curves such as

$$\theta = a, r = p \sec(\alpha - \theta), r = a,$$

$$r = 2a \cos \theta, r = k\theta, r = a(1 \pm \cos \theta),$$

$$r = a(3 + 2 \cos \theta), r = a \cos 2\theta \text{ and}$$

A LEVEL UNIT FP2: Polar coordinates - Polar coordinates (r, θ) , $r \geq 0$. $r^2 = a^2 \cos 2\theta$ may be set.

Use of the formula $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$ for area.

The ability to find tangents parallel to, or at right angles to, the initial line is expected.

TRIGONOMETRY

9. THE UNIT CIRCLE AND RADIAN

(AXLER: EQUATION OF THE UNIT CIRCLE, ANGLES CORRESPONDING TO RADIUS OF THE UNIT CIRCLE, NEGATIVE ANGLES, ANGLES GREATER THAN 360, LENGTH OF A CIRCULAR ARC, SPECIAL POINTS ON THE UNIT CIRCLE – FIND COORDINATES OF THE ENDPOINT OF THE RADIUS OF UNIT CIRCLE CORRESPONDING TO ANY MULTIPLE OF 30 OR 45 DEGREES; RADIAN AS NATURAL UNIT OF MEASUREMENT FOR ANGLES, CONVERTING FROM RADIAN TO DEGREE AND VICE-VERSA, NEGATIVE ANGLES, ANGLES GREATER THAN 2PI, LENGTH OF A CIRCULAR ARC DESCRIBED BY RADIAN, AREA OF A SLICE, SPECIAL POINTS ON THE UNIT CIRCLE USING RADIAN RATHER THAN DEGREE; ALEKS: FINDING COORDINATES ON THE UNIT CIRCLE FOR SPECIAL ANGLES, FINDING A POINT ON THE UNIT CIRCLE GIVEN ONE COORDINATE, FINDING TRIGONOMETRIC RATIOS FROM A POINT ON THE UNIT CIRCLE, CONVERTING BETWEEN DEGREE AND RADIAN MEASURES)

ALBERTA MATH 30-1: Trigonometry – Develop and apply the equation of the unit circle.

Demonstrate an understanding of angles in standard position, expressed in degrees and radians.

A LEVEL UNIT C2: Trigonometry – Radian measure, including use for arc length and area of sector.

A LEVEL UNIT C3: Trigonometry – Angles measured in both degrees and radians.

IB MATH SL: Circular Functions and Trigonometry – The circle: radian measures of angles; length of an arc; area of a sector. Radian measure may be expressed as exact multiples of Pi, or decimals.

IB MATH HL: Circular Functions and Trigonometry (Core) – The circle: radian measures of angles; length of an arc; area of a sector. . Radian measure may be expressed as multiples of Pi, or decimals.

10. USING TRIGONOMETRY TO COMPUTE AREA, LAW OF SINES AND COSINES

(AXLER: THE AREA OF A TRIANGLE VIA TRIGONOMETRY; AMBIGUOUS ANGLES; AREA OF A PARALLELOGRAM VIA TRIGONOMETRY; AREA OF A POLYGON; AXLER:

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

; USING THE LAW OF SINES;

Law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

; USING THE LAW OF COSINES; WHEN TO

USE WHICH LAW)

A LEVEL UNIT C2: Trigonometry – The area of a triangle in the form $\frac{1}{2} ab \sin C$.

IB MATH STUDIES SL: Geometry and Trigonometry – Use of area of a triangle $\frac{1}{2} ab \sin C$. Construction of labelled diagrams from verbal statements. The ambiguous case could be taught, but will not be examined.

IB MATH SL: Circular Functions and Trigonometry – Solution of triangles. Pythagoras' theorem is a special case of the cosine rule. Area of a triangle $\frac{1}{2} ab \sin C$. Applications.

IB MATH HL: Circular Functions and Trigonometry (Core) – Area of a triangle $\frac{1}{2} ab \sin C$. Applications.

A LEVEL UNIT C2: Trigonometry – The sine and cosine rules.

IB MATH STUDIES SL: Geometry and Trigonometry – Use of the sine rule and cosine rule.

IB MATH SL: Circular Functions and Trigonometry – The cosine rule. The sine rule, including the ambiguous case.

IB MATH HL: Circular Functions and Trigonometry (Core) –

The cosine rule

The sine rule including the ambiguous case.

11. TRIGONOMETRIC FUNCTIONS, RATIOS, IDENTITIES AND TRANSFORMATIONS

(AXLER: DEFINITION OF COSINE AND SINE, COSINE AND SINE OF SPECIAL ANGLES, SIGNS OF COSINE AND SINE, KEY EQUATION CONNECTING COSINE AND SINE -

$$(\cos \theta)^2 + (\sin \theta)^2 = 1$$

, GRAPHS OF COSINE AND SINE, DOMAIN AND RANGE OF COSINE AND SINE; DEFINITION OF TANGENT -

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

TANGENT AS SLOPE,

TANGENT OF SPECIAL ANGLES, SIGN OF TANGENT, CONNECTIONS AMONG COSINE, SINE AND TANGENT, GRAPH OF TANGENT, MULTIPLICATIVE INVERSES OF

COSINE, SINE AND TANGENT FUNCTIONS – SECANT, COSECANT, AND COTANGENT FUNCTIONS; TRIGONOMETRIC FUNCTIONS VIA RIGHT TRIANGLES, TWO SIDES OF A RIGHT TRIANGLE, ONE SIDE AND ONE ANGLE OF A RIGHT TRIANGLE; BASIC TRIGONOMETRIC IDENTITIES FOR SIMPLIFYING TRIG EXPRESSIONS, RELATIONSHIP BETWEEN COSINE AND SINE IN NEW NOTATION - $\cos^2\theta + \sin^2\theta = 1$, TRIGONOMETRIC IDENTITIES FOR THE NEGATIVE OF AN ANGLE, *Trigonometric identities with $-\theta$* - $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$, AND $\tan(-\theta) = -\tan\theta$, *Trigonometric Identities with $\frac{\pi}{2}$* , *Trigonometric identities with $\frac{\pi}{2} - \theta$* , *Trigonometric identities with $(90 - \theta)^\circ$* , *Trigonometric Identities Involving a Multiple of π* , *Trigonometric identities with $\theta + \pi$* , *Trigonometric identities with $\theta + 2\pi$* , *Trigonometric identities with $\theta + n\pi$* AMPLITUDE OF A FUNCTION, PERIOD OF A FUNCTION, PHASE SHIFT **ALEKS: TRIGONOMETRIC FUNCTIONS AND SPECIAL ANGLES; SINE, COSINE AND TANGENT RATIOS; FINDING TRIGONOMETRIC RATIOS GIVEN A RIGHT TRIANGLE, SOLVING A RIGHT TRIANGLE, FINDING VALUES OF TRIGONOMETRIC FUNCTIONS GIVEN INFORMATION ABOUT AN ANGLE; SIMPLIFYING TRIGONOMETRIC EXPRESSIONS)**

ALBERTA MATH 30-1: Trigonometry – Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems. Prove trigonometric identities, using: reciprocal identities, quotient identities, Pythagorean identities, sum and difference identities (restricted to sine, cosine and tangent), double-angle identities (restricted to sine, cosine and tangent).

ALBERTA MATH 30-1: Trigonometry – Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

ALBERTA MATHS 31: Precalculus and Limits – Students will demonstrate procedural competence associated with the transformation of functions, by sketching and describing, algebraically, the effects of any combination of translation, reflection or dilatation on the following functions: $f(x) = a \sin [b(x+c)]+d$, $f(x) = a \cos [b(x+c)]+d$, $f(x) = a \tan [b(x+c)]$.

A LEVEL UNIT C2: Trigonometry – Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.

Knowledge and use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta = 1$.

Solution of simple trigonometric equations in a given interval

Students should be able to solve equations such as

$$\sin\left(x + \frac{\pi}{2}\right) = \frac{3}{4} \text{ for } 0 < x < 2\pi,$$

$$\cos(x + 30^\circ) = \frac{1}{2} \text{ for } -180^\circ < x < 180^\circ,$$

$$\tan 2x = 1 \text{ for } 90^\circ < x < 270^\circ,$$

$$6 \cos^2 x + \sin x - 5 = 0, 0^\circ \leq x < 360,$$

$$\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2} \text{ for } -\pi \leq x < \pi.$$

A LEVEL UNIT C3: Trigonometry – Knowledge of secant, cosecant and cotangent [...] their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.

Knowledge and use of $\sec^2 \theta = 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$.

IB MATH STUDIES SL: Geometry and Trigonometry – Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. Angles of elevation and depression. Problems may incorporate Pythagoras' Theorem. In examinations, questions will only be set in degrees.

IB MATH SL: Circular Functions and Trigonometry –

Definition of $\cos \theta$ and $\sin \theta$ in terms of the unit circle.

Definition of $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$.

Exact values of trigonometric ratios of

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.

The equation of a straight line through the origin is $y = x \tan \theta$.

Examples:

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}, \quad \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}, \quad \tan 210^\circ = \frac{\sqrt{3}}{3}.$$

The Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$.

Relationship between trigonometric ratios.

Examples:

Given $\sin \theta$, finding possible values of $\tan \theta$ without finding θ .

Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x .

Transformations.

Example: $y = \sin x$ used to obtain $y = 3 \sin 2x$ by a stretch of scale factor 3 in the y -direction and a stretch of scale factor $\frac{1}{2}$ in the x -direction.

Link to 2.3, transformation of graphs.

Applications.

Examples include height of tide, motion of a Ferris wheel.

IB MATH HL: Circular Functions and Trigonometry (Core) –

Definition of $\cos \theta$ and $\sin \theta$ in terms of the unit circle.

Definition of $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$.

Exact values of trigonometric ratios of

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.

Definition of the reciprocal trigonometric ratios $\sec \theta$, $\csc \theta$ and $\cot \theta$.

Pythagorean identities: $\cos^2 \theta + \sin^2 \theta = 1$;

$1 + \tan^2 \theta = \sec^2 \theta$; $1 + \cot^2 \theta = \csc^2 \theta$.

IB MATH SL: Circular Functions and Trigonometry –

The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs.

Composite functions of the form

$$f(x) = a \sin(b(x+c)) + d.$$

Examples:

$$f(x) = \tan\left(x - \frac{\pi}{4}\right), f(x) = 2 \cos(3(x-4)) + 1.$$

Solving trigonometric equations in a finite interval, both graphically and analytically.

Examples: $2 \sin x = 1, 0 \leq x \leq 2\pi,$

$2 \sin 2x = 3 \cos x, 0^\circ \leq x \leq 180^\circ,$

$2 \tan(3(x-4)) = 1, -\pi \leq x \leq 3\pi.$

Equations leading to quadratic equations in $\sin x, \cos x$ or $\tan x$.

Examples:

$2 \sin^2 x + 5 \cos x + 1 = 0$ for $0 \leq x < 4\pi,$

$2 \sin x = \cos 2x, -\pi \leq x \leq \pi.$

Not required:

the general solution of trigonometric equations.

IB MATH HL: Circular Functions and Trigonometry (Core) –

Composite functions of the form

$$f(x) = a \sin(b(x+c)) + d.$$

Algebraic and graphical methods of solving trigonometric equations in a finite interval, including the use of trigonometric identities and factorization.

Not required:

The general solution of trigonometric equations.

12. INVERSE TRIGONOMETRIC FUNCTIONS AND INVERSE TRIGONOMETRIC IDENTITIES

(AXLER: THE ARCCOSINE FUNCTION, DOMAIN AND RANGE OF ARCCOSINE, THE ARCSINE FUNCTION, THE ARCTANGENT FUNCTION, THE ARCCOSINE, ARCSINE AND ARCTANGENT OF $-t$: GRAPHICAL APPROACH; THE ARCCOSINE, ARCSINE AND ARCTANGENT OF $-t$: ALGEBRAIC APPROACH; ARCCOSINE PLUS ARCSINE; COMPOSITION OF TRIGONOMETRIC FUNCTIONS AND THEIR INVERSES)

A LEVEL UNIT C3: Trigonometry – Knowledge of \arcsin, \arccos and \arctan [...] their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.

IB MATH HL: Circular Functions and Trigonometry (Core) –

The inverse functions $x \mapsto \arcsin x,$

$x \mapsto \arccos x, x \mapsto \arctan x;$ their domains and ranges; their graphs.

13. DOUBLE-ANGLE AND HALF-ANGLE FORMULAS; ADDITION AND SUBTRACTION FORMULAS

(**AXLER:** The Cosine of 2θ , DOUBLE-ANGLE FORMULAS FOR COSINE, The Sine of 2θ , DOUBLE-ANGLE FORMULA FOR SINE, The Tangent of 2θ , DOUBLE-ANGLE FORMULA FOR TANGENT, The Cosine and Sine of $\frac{\theta}{2}$, HALF-ANGLE FORMULA FOR COSINE, HALF-ANGLE FORMULA FOR SINE, The Tangent of $\frac{\theta}{2}$, HALF-ANGLE FORMULAS FOR TANGENT; THE COSINE OF A SUM AND DIFFERENCE; ADDITION AND SUBTRACTION FORMULAS FOR COSINE; THE SINE OF A SUM AND DIFFERENCE; ADDITION AND SUBTRACTION FORMULA FOR SINE; THE TANGENT OF A SUM AND DIFFERENCE; ADDITION AND SUBTRACTION FORMULA FOR TANGENT)

ALBERTA MATH 30-1: Trigonometry – Prove trigonometric identities, using: double-angle identities (restricted to sine, cosine and tangent)

ALBERTA MATH 31: Precalculus and Limits – Illustrating the difference between the concepts of equation and identity in trigonometric contexts; using the following trigonometric identities: primary and reciprocal ratio, double and half-angle, sum and difference, Pythagorean to simplify expressions and solve equations, express sum and difference as products, and rewrite expressions in a variety of equivalent forms.

A LEVEL UNIT C3: Trigonometry –

Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos(\theta \pm a)$ or $r \sin(\theta \pm a)$.

To include application to half angles. Knowledge of the $r \tan \frac{1}{2}\theta$ formulae will *not* be required.

IB MATH SL: Circular Functions and Trigonometry – Double angle identities for sine and cosine. Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities).

IB MATH HL: Circular Functions and Trigonometry (Core) – Compound angle identities. Double angle identities. Derivation of double angle identities from compound angle identities. Not required: proof of compound angle identities.

Finding possible values of trigonometric ratios without finding θ , for example, finding $\sin 2\theta$ given $\sin \theta$.

A LEVEL UNIT C3: Trigonometry –

Students should be able to solve equations such as $a \cos \theta + b \sin \theta = c$ in a given interval, and to prove simple identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.

ALBERTA MATH 30-1: Trigonometry – Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

Prove trigonometric identities, using: sum and difference identities (restricted to sine, cosine and tangent)

OTHER AREAS OF PURE MATHEMATICS

14. BASIC SEQUENCES & SERIES, BINOMIAL THEOREM

(AXLER: FINITE AND INFINITE SEQUENCES; ARITHMETIC SEQUENCES; FORMULA FOR AN ARITHMETIC SEQUENCE; GEOMETRIC SEQUENCES; FORMULA FOR AN ARITHMETIC SEQUENCE; QUADRATIC SEQUENCES; RECURSIVE SEQUENCES; FIBONACCI SEQUENCE; SERIES AS A SUM OF SEQUENCES; ARITHMETIC SERIES; GEOMETRIC SERIES; SUMMATION/SIGMA NOTATION; SUMS OF FINITE ARITHMETIC AND GEOMETRIC SEQUENCES; PASCAL'S TRIANGLE; ALEKS: FINDING THE FIRST TERMS OF A SEQUENCE USING AN EXPLICIT RULE WITH MULTIPLE OCCURANCES OF N; ARITHMETIC AND GEOMETRIC SEQUENCES – IDENTIFYING AND WRITING AN EXPLICIT RULE; FINDING A SPECIFIED TERM OF AN ARITHMETIC SEQUENCE GIVEN TWO TERMS OF THE SEQUENCE; FINDING A SPECIFIED TERM OF A GEOMETRIC SEQUENCE GIVEN TWO TERMS OF THE SEQUENCE; SUM OF THE FIRST N TERMS OF AN ARITHMETIC SEQUENCE; SUM OF THE FIRST N TERMS OF A GEOMETRIC SEQUENCE; (AXLER: BINOMIAL THEOREM; ALEKS: SUM OF FINITE ARITHMETIC AND GEOMETRIC SERIES)

A LEVEL UNIT C1: Sequences and Series – Sequences, including those given by a formula for the n th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$. Arithmetic series, including the formula for the sum of the first n natural numbers.

A LEVEL UNIT C2: Sequences and Series – The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$. The general term and the sum to n terms are required. The proof of the sum formula should be known. Binomial expansion of $(1+x)^n$ for positive integer n . The notation $n!$ and (n/r) .

A LEVEL UNIT C4: Sequences and Series – Binomial series for any rational n .

A LEVEL UNIT FP1: Sequences and Series – Summation of simple finite series.

A LEVEL UNIT FP2: Sequences and Series – Summation of simple finite series using the method of differences.

SINGAPORE H2: Sequences and Series – Concepts of sequence and series; relationship between u_n (the n th term) and S_n (the sum to n terms); sequence given by a formula for the n th term; sequence generated by a simple recurrence relation of the form $x_{n+1} = f(x_n)$; use of sigma notation.

SINGAPORE H2: Sequences and Series – Summation of series by the method of differences; convergence of a series and the sum to infinity; binomial expansion of $(1+x)^n$ for any rational n ; condition for convergence of a binomial series; proof by the method of mathematical induction; formula for the n th term and the sum of a finite arithmetic series; formula for the n th term and the sum of a finite geometric series

IB MATH STUDIES SL: Number and Algebra – Arithmetic sequences and series, and their applications. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Students may use a GDC for calculations, but they will be expected to identify the first term and the common difference. Geometric sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Financial applications of geometric sequences and series: compound interest, annual depreciation. Not required: formal proofs of formulae. Not required: use of logarithms to find n , given the sum of the first n terms; sums to infinity.

IB MATH SL: Algebra – Arithmetic sequences and series; geometric sequences and series; Sigma notation. Technology may be used to generate and display sequences in several ways.

IB MATH SL: Algebra – Sum of finite arithmetic series; sum of finite and infinite geometric series. The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$. Counting principles may be used in the development of the theorem. $\binom{n}{r}$ should be found using both the formula and technology. Not required: formal treatment of permutations and formula for ${}^n P_r$.

IB MATH HL: Algebra (Core) – Sum of finite arithmetic series; sum of finite and infinite geometric series.

IB MATH HL: Algebra (Core) – Arithmetic sequences and series; geometric sequences and series; Sigma notation. Applications. Sequences can be generated and displayed in several ways, including recursive functions.

15. VECTORS

(AXLER: ALGEBRAIC AND GEOMETRIC INTRODUCTION TO VECTORS; NOTATION FOR VECTORS WITH INITIAL POINT AT THE ORIGIN; COMPUTING THE MAGNITUDE AND DIRECTION OF A VECTOR; VECTOR ADDITION AND SUBTRACTION; SCALAR MULTIPLICATION; THE DOT PRODUCT; ALGEBRAIC PROPERTIES OF THE DOT PRODUCT; COMPUTING THE DOT PRODUCT GEOMETRICALLY)

A LEVEL UNIT C4: Vectors – Vectors in two and three dimensions; magnitude of a vector; algebraic operations of vector addition and multiplication by scalars, and their geometric interpretations; position vectors; the distance between two points; vector equations of lines; the scalar product and its use for calculating the angle between two lines.

A LEVEL UNIT FP3: Vectors –

The vector product $\mathbf{a} \times \mathbf{b}$ and the triple scalar product $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$.

The interpretation of $|\mathbf{a} \times \mathbf{b}|$ as an area and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ as a volume.

Use of vectors in problems involving points, lines and planes.

Students may be required to use equivalent cartesian forms also.

The equation of a line in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$.

Applications to include

- (i) distance from a point to a plane,
 - (ii) line of intersection of two planes,
 - (iii) shortest distance between two skew lines.
-

The equation of a plane in the forms
 $\mathbf{r} \cdot \mathbf{n} = p$, $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$.

Students may be required to use equivalent cartesian forms also.

SINGAPORE MATH H2: Vectors – Vectors in two and three dimensions –

Include:

- addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations
- use of notations such as $\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$,
 \overrightarrow{AB} , \mathbf{a}
- position vectors and displacement vectors
- magnitude of a vector
- unit vectors
- distance between two points
- angle between a vector and the x-, y- or z-axis
- use of the ratio theorem in geometrical applications

The scalar and vector products of vectors –

Include:

- concepts of scalar product and vector product of vectors
- calculation of the magnitude of a vector and the angle between two directions
- calculation of the area of triangle or parallelogram
- geometrical meanings of $|\mathbf{a} \cdot \mathbf{b}|$ and $|\mathbf{a} \times \mathbf{b}|$, where \mathbf{b} is a unit vector

Exclude triple products $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} \times \mathbf{c}$

Three-dimensional geometry –

Include:

- vector and cartesian equations of lines and planes
- finding the distance from a point to a line or to a plane
- finding the angle between two lines, between a line and a plane, or between two planes
- relationships between
 - two lines (coplanar or skew)
 - a line and a plane
 - two planes
 - three planes
- finding the intersections of lines and planes

Exclude:

- finding the shortest distance between two skew lines
- finding an equation for the common perpendicular to two skew lines

IB MATH SL: Vectors –

Vectors as displacements in the plane and in three dimensions.

Components of a vector; column

$$\text{representation; } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k} .$$

Algebraic and geometric approaches to the following:

- the sum and difference of two vectors; the zero vector, the vector $-\mathbf{v}$;
- multiplication by a scalar, $k\mathbf{v}$; parallel vectors;
- magnitude of a vector, $|\mathbf{v}|$;
- unit vectors; base vectors; \mathbf{i} , \mathbf{j} and \mathbf{k} ;
- position vectors $\vec{OA} = \mathbf{a}$;
- $\vec{AB} = \vec{OB} - \vec{OA} = \mathbf{b} - \mathbf{a}$.

Link to three-dimensional geometry, x , y and z -axes.

Components are with respect to the unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} (standard basis).

Applications to simple geometric figures are essential.

The difference of \mathbf{v} and \mathbf{w} is $\mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w})$. Vector sums and differences can be represented by the diagonals of a parallelogram.

Multiplication by a scalar can be illustrated by enlargement.

Distance between points A and B is the magnitude of \vec{AB} .

The scalar product of two vectors.

The scalar product is also known as the “dot product”.

Link to 3.6, cosine rule.

Perpendicular vectors; parallel vectors.

For non-zero vectors, $\mathbf{v} \cdot \mathbf{w} = 0$ is equivalent to the vectors being perpendicular.

For parallel vectors, $\mathbf{w} = k\mathbf{v}$, $|\mathbf{v} \cdot \mathbf{w}| = |\mathbf{v}||\mathbf{w}|$.

The angle between two vectors.

Vector equation of a line in two and three dimensions: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

Relevance of \mathbf{a} (position) and \mathbf{b} (direction).

Interpretation of t as time and \mathbf{b} as velocity, with $|\mathbf{b}|$ representing speed.

The angle between two lines.

Distinguishing between coincident and parallel lines.

Finding the point of intersection of two lines.

Determining whether two lines intersect.

IB MATH HL: Vectors (Core) –

Concept of a vector.

Representation of vectors using directed line segments.

Unit vectors; base vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

Components of a vector:

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}.$$

Algebraic and geometric approaches to the following:

- the sum and difference of two vectors;
- the zero vector $\mathbf{0}$, the vector $-\mathbf{v}$;
- multiplication by a scalar, $k\mathbf{v}$;
- magnitude of a vector, $|\mathbf{v}|$;
- position vectors $\vec{OA} = \mathbf{a}$.

$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

Proofs of geometrical properties using vectors.

Distance between points A and B is the magnitude of \vec{AB} .

The definition of the scalar product of two vectors.

Properties of the scalar product:

$$\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v};$$

$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w};$$

$$(k\mathbf{v}) \cdot \mathbf{w} = k(\mathbf{v} \cdot \mathbf{w});$$

$$\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2.$$

The angle between two vectors.

Perpendicular vectors; parallel vectors.

Vector equation of a line in two and three dimensions: $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$.

Simple applications to kinematics.

The angle between two lines.

Coincident, parallel, intersecting and skew lines; distinguishing between these cases.

Points of intersection.

$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}|\cos\theta$, where θ is the angle between \mathbf{v} and \mathbf{w} .

Link to 3.6.

For non-zero vectors, $\mathbf{v} \cdot \mathbf{w} = 0$ is equivalent to the vectors being perpendicular.

For parallel vectors, $|\mathbf{v} \cdot \mathbf{w}| = |\mathbf{v}||\mathbf{w}|$.

Knowledge of the following forms for equations of lines.

Parametric form:

$$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n.$$

Cartesian form:

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}.$$

The definition of the vector product of two vectors.

Properties of the vector product:

$$\mathbf{v} \times \mathbf{w} = -\mathbf{w} \times \mathbf{v};$$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w};$$

$$(k\mathbf{v}) \times \mathbf{w} = k(\mathbf{v} \times \mathbf{w});$$

$$\mathbf{v} \times \mathbf{v} = \mathbf{0}.$$

Geometric interpretation of $|\mathbf{v} \times \mathbf{w}|$.

Vector equation of a plane $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$.

Use of normal vector to obtain the form

$$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}.$$

Cartesian equation of a plane $ax + by + cz = d$.

Intersections of: a line with a plane; two planes; three planes.

Angle between: a line and a plane; two planes.

$\mathbf{v} \times \mathbf{w} = |\mathbf{v}||\mathbf{w}|\sin\theta \mathbf{n}$, where θ is the angle between \mathbf{v} and \mathbf{w} and \mathbf{n} is the unit normal vector whose direction is given by the right-hand screw rule.

Areas of triangles and parallelograms.

Link to 1.9.

Geometrical interpretation of solutions.

16. SYSTEMS OF LINEAR EQUATIONS

(AXLER: GRAPHICALLY SOLVING A SYSTEM OF EQUATIONS WITH TWO VARIABLES, SOLVING A SYSTEM A EQUATIONS BY SUBSTITUTION, LINEAR EQUATION IN TWO VARIABLES, LINEAR EQUATION IN THREE VARIABLES, GAUSSIAN ELIMINATION FOR A SYSTEM OF LINEAR EQUATIONS; ALEKS: CLASSIFYING SYSTEMS OF LINEAR EQUATIONS FROM GRAPHS, SOLVING A SYSTEM OF LINEAR EQUATIONS THAT IS INCONSISTENT OR CONSISTENT DEPENDENT, SOLVING A SYSTEM OF THREE LINEAR EQUATIONS IN THREE UNKNOWNNS, CONSISTENCY AND INDEPENDENCE OF A SYSTEM OF LINEAR EQUATIONS)

ALBERTA MATHS 31: Precalculus and Limits - Students will demonstrate conceptual understanding of the transformation of functions, by: linking two problem conditions to a system of two equations for two unknowns; solving systems of linear–linear, linear quadratic or quadratic–quadratic equations.

SINGAPORE H2: Functions and graphs – Equations and Inequalities - formulating an equation or a system of linear equations from a problem situation; finding the numerical solution of equations (including system of linear equations) using a graphing calculator

IB MATHS STUDIES SL: Number and Algebra – Use of a GDC to solve pairs of linear equations in two variables. In examinations, no specific method of solution will be required. Standard terminology, such as zeros and roots, should be taught.

17. MATRICES AND LINEAR ALGEBRA

(AXLER (PRECALCULUS): MATRICES – ROWS AND COLUMNS, SOLVING A SYSTEM OF LINEAR EQUATIONS WITH ELEMENTRY ROW OPERATIONS, GAUSSIAN ELIMINATION PRODUCING NO SOLUTIONS AND INFINITELY MANY SOLUTIONS; ALEKS (PRECALCULUS): GAUSS-JORDAN ELIMINATION WITH A 2X2 MATRIX, SOLVING A SYSTEM OF LINEAR EQUATIONS GIVEN ITS AUGMENTED MATRIX, SCALAR MULTIPLICATION OF A MATRIX, ADDITION OR SUBTRACTION OF MATRICES, LINEAR COMBINATION OF MATRICES, MULTIPLICATION OF MATRICES, FINDING THE DETERMINANT OF A 2X2 MATRIX AND A 3X3 MATRIX, FINDING THE INVERSE OF A 2X2 MATRIX AND A 3X3 MATRIX, USING THE INVERSE OF A MATRIX TO SOLVE A SYSTEM OF LINEAR EQUATIONS, USING CRAMER’S RULE TO SOLVE A 2X2 SYSTEM OF LINEAR EQUATIONS; STRANG (CALCULUS): TWO EQUATIONS IN TWO UNKNOWNNS, SOLUTION BY DETERMINANTS, CRAMER’S RULE, MATRIX MULTIPLICATION A^{-1} , MATRIX-VECTOR RULE, THE INVERSE OF A MATRIX, THE IDENTITY MATRIX I , PROJECTION ONTO A PLANE – LEAST SQUARES FITTING BY A LINE; LINEAR ALGEBRA – THREE DIMENSIONS, THREE UNKNOWNNS AND THREE EQUATIONS, SOLUTIONS USING ALGORITHMS: THE ROW PICTURE: INTERSECTING PLANES, COLUMN PICTURE: COMBINATION OF COLUMN VECTORS, THE DETERMINANT AND THE INVERSE MATRIX, THE SOLUTION $u = A^{-1}d$, THE SINGULAR CASE, THE ELIMINATION ALGORITHM)

A LEVEL UNIT FP1: Matrix algebra – Linear transformations of column vectors in two dimensions and their matrix representation. Addition and subtraction of matrices. Multiplication of a matrix by a scalar. Products of matrices. Evaluation of 2×2 determinants. Singular and non-singular matrices. Inverse of 2×2 matrices. Combinations of transformations. The inverse (when it exists) of a given transformation or combination of transformations.

A LEVEL UNIT FP3: Matrix algebra – Linear transformations of column vectors in two and three dimensions and their matrix representation. Combination of transformations. Products of matrices. Transpose of a matrix. Evaluation of 3×3 determinants. Singular and non-singular matrices. Inverse of 3×3 matrices. The inverse (when it exists) of a

given transformation or combination of transformations. Eigenvalues and eigenvectors of 2×2 and 3×3 matrices. Normalised vectors may be required. Reduction of symmetric matrices to diagonal form.

IB MATHS HL: Algebra (Core) - Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinity of solutions or no solution. These systems should be solved using both algebraic and technological methods, eg. row reduction. Systems that have solution(s) may be referred to as consistent. When a system has an infinity of solutions, a general solution may be required.

IB FURTHER MATHS HL: Linear algebra - Definition of a matrix: the terms element, row, column and order for $m \times n$ matrices. Algebra of matrices: equality; addition; subtraction; multiplication by a scalar for $m \times n$ matrices. Including use of the GDC. Multiplication of matrices. Properties of matrix multiplication: associativity, distributivity.

Identity and zero matrices. Transpose of a matrix including **AT** notation. $(AB)^T = B^T A^T$. Students should be familiar with the notation **I** and **0**. Students should be familiar with the definition of symmetric and skew-symmetric matrices.

Definition and properties of the inverse of a square matrix:

$$(AB)^{-1} = B^{-1}A^{-1},$$

$$(A^T)^{-1} = (A^{-1})^T,$$

$$(A^n)^{-1} = (A^{-1})^n.$$

Calculation of A^{-1} .

Use of elementary row operations to find A^{-1} . Formulae for the inverse and determinant of a 2×2 matrix and the determinant of a 3×3 matrix. Elementary row and column operations for matrices. Scaling, swapping and pivoting. Corresponding elementary matrices. Row reduced echelon form. Row space, column space and null space. Row rank and column rank and their equality.

Solutions of m linear equations in n unknowns: both augmented matrix method, leading to reduced row echelon form method, and inverse matrix method, when applicable.

The vector space R^n . Linear combinations of vectors. Spanning set. Linear independence of vectors. Basis and dimension for a vector space. Subspaces. Include linear dependence and use of determinants. Students should be familiar with the term orthogonal.

Linear transformations: $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$, $T(k\mathbf{u}) = kT(\mathbf{u})$. Composition of linear transformations. Domain, range, codomain and kernel. The kernel of a linear transformation is the null space of its matrix representation. Result and proof that the kernel is a subspace of the domain. Result and proof that the range is a subspace of the codomain. Rank-nullity theorem (proof not required).

Result that any linear transformation can be represented by a matrix, and the converse of this result. Result that the numbers of linearly independent rows and columns are equal, and this is the dimension of the range of the transformation (proof not required). Application of linear transformations to solutions of system of equations. Solution of $A\mathbf{x} = \mathbf{b}$.

Geometric transformations represented by 2×2 matrices include general rotation, general reflection in $y = (\tan \alpha)x$: stretches parallel to axes, shears parallel to axes, and projection onto $y = (\tan \alpha)x$. Compositions of the above transformations. Geometric interpretation of determinant. New area = $|\det A| \times$ old area.

Eigenvalues and eigenvectors of 2×2 matrices. Characteristic polynomial of 2×2 matrices. Diagonalization of 2×2 matrices (restricted to the case where there are distinct real eigenvalues). Applications to powers of 2×2 matrices.

18. COMPLEX NUMBERS

(AXLER: COMPLEX NUMBERS: THE COMPLEX NUMBER SYSTEM, ADDITION AND SUBTRACTION OF COMPLEX NUMBERS, MULTIPLICATION OF COMPLEX NUMBERS, COMPLEX CONJUGATES AND DIVISION OF COMPLEX NUMBERS, PROPERTIES OF COMPLEX CONJUGATION; COMPLEX PLANE: COMPLEX NUMBERS AS POINTS IN THE PLANE, GEOMETRIC INTERPRETATION OF COMPLEX MULTIPLICATION AND DIVISION, DE MOIVRE'S THEOREM, FINDING COMPLEX ROOTS; ALEKS: USING i TO REWRITE SQUARE ROOTS OF NEGATIVE NUMBERS, SIMPLIFYING A PRODUCT AND QUOTIENT INVOLVING SQUARE ROOTS OF NEGATIVE NUMBERS, ADDING OR SUBTRACTING COMPLEX NUMBERS, MULTIPLYING AND DIVIDING COMPLEX NUMBERS, SIMPLIFYING A POWER OF i , SOLVING A QUADRIC EQUATION WITH COMPLEX ROOTS, MULTIPLYING EXPRESSIONS INVOLVING COMPLEX CONJUGATES, LINEAR FACTORS THEOREM AND CONJUGATE ZEROS THEOREM, WRITING A COMPLEX NUMBER IN TRIGONOMETRIC FORM, DE MOIVRE'S THEOREM, FINDING THE N TH ROOTS OF A NUMBER; STEWART (CALCULUS): REAL AND IMAGINARY PARTS OF COMPLEX NUMBERS, COMPLEX PLANE IS KNOWN AS ARGAND PLANE, PROPERTIES OF CONJUGATES, MODULUS OR ABSOLUTE VALUE OF A COMPLEX NUMBER, POLAR FORM OF COMPLEX NUMBERS, DE MOIVRE'S THEOREM, ROOTS OF A COMPLEX NUMBER, COMPLEX EXPONENTIALS, EULER'S FORMULA)

A LEVEL UNIT FP1: Complex numbers – Definition of complex numbers in the form $a + ib$ and $r \cos \theta + i r \sin \theta$. The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known. Sum, product and quotient of complex numbers. Geometrical representation of complex numbers in the Argand diagram. Geometrical representation of sums, products and quotients of complex numbers. Complex solutions of quadratic equations with real coefficients. Conjugate complex roots of polynomial equations with real coefficients.

A LEVEL UNIT FP2: Further Complex Numbers – Euler's relation $e^{i\theta} = \cos \theta + i \sin \theta$. De Moivre's theorem and its application to trigonometric identities and to roots of a complex number. Loci and regions in the Argand diagram. Elementary transformations from the z -plane to the w -plane.

SINGAPORE H2: Complex numbers – Complex numbers expressed in cartesian form - Include: extension of the number system from real numbers to complex numbers; complex roots of quadratic equations; four operations of complex numbers expressed in the form $(x + iy)$; equating real parts and imaginary parts; conjugate roots of a polynomial equation with real coefficients. **Complex numbers expressed in polar form** – Include: complex numbers expressed in the form $r(\cos \theta + i \sin \theta)$ or $r e^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Calculation of modulus (r) and argument (θ) of a complex number. Multiplication and division of complex numbers expressed in polar form; representation of complex numbers in the Argand diagram; geometrical effects of conjugating a complex number and of adding, subtracting, multiplying, dividing two complex numbers. Loci such as $|z - c| \leq r$, $|z - a| = |z - b|$ and $\arg(z - a) = \alpha$. Use of de Moivre's theorem to find the powers and n th roots of a complex number.

IB MATHS HL: Algebra (Core) – Complex numbers: the number $i = \sqrt{-1}$ the terms real part, imaginary part, conjugate, modulus and argument. Cartesian form $z = a + ib$. Sums, products and quotients of complex numbers. Modulus–argument (polar) form; The complex plane. $r e^{i\theta}$ is also known as Euler's form. The ability to convert between forms is expected. The complex plane is also known as the Argand diagram. Powers of complex numbers: de Moivre's theorem. n th roots of a complex number. Proof by mathematical induction for $n \in \mathbb{Z}^+$ Conjugate roots of polynomial equations with real coefficients.

19. NUMERICAL METHODS

ALBERTA MATHS 31: Numerical Methods (Option) – Students will demonstrate conceptual understanding of the principles of numerical analysis, by: describing the difference between an exact solution and an approximate solution; identifying when a particular numerical method is likely to give poor results; explaining the difference between iterative and noniterative procedures; explaining the basis of the Newton–Raphson procedure for determining the roots of $f(x) = 0$

A LEVEL UNIT C3: Numerical methods – Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous. Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.

A LEVEL UNIT FP1: Numerical solution of equations – Equations of the form $f(x) = 0$ solved numerically by: i) interval bisection, ii) linear interpolation, iii) the Newton-Raphson process.

20. SETS, RELATIONS AND GROUPS; COMBINATORICS

SINGAPORE H3: Combinatorics – Distribution problems: -identical objects into distinct boxes; distinct objects into distinct boxes; distinct objects into identical boxes; identical objects into identical boxes. Counting using: -bijection principle; inclusion-exclusion principle. Formulating recurrence relations in the context of combinatorial problems.

IB MATH HL: Sets, relations and groups (Option) – Finite and infinite sets. Subsets. Operations on sets: union; intersection; complement; set difference; symmetric difference. De Morgan’s laws: distributive, associative and commutative laws (for union and intersection). Illustration of these laws using Venn diagrams. Students may be asked to prove that two sets are the same by establishing that $A \subseteq B$ and $B \subseteq A$.

Ordered pairs: the Cartesian product of two sets. Relations: equivalence relations; equivalence classes. An equivalence relation on a set forms a partition of the set. Functions: injections; surjections; bijections. The term codomain. Composition of functions and inverse functions. Knowledge that the function composition is not a commutative operation and that if f is a bijection from set A to set B then f^{-1} exists and is a bijection from set B to set A .

Binary operations. A binary operation $*$ on a non-empty set S is a rule for combining any two elements $a, b \in S$ to give a unique element c . That is, in this definition, a binary operation on a set is not necessarily closed. Operation tables (Cayley tables). Binary operations: associative, distributive and commutative properties. The arithmetic operations on \mathbb{R} and \mathbb{C} . Examples of distributivity could include the fact that, on \mathbb{R} , multiplication is distributive over addition but addition is not distributive over multiplication.

The identity element e . The inverse a^{-1} of an element a . Proof that left-cancellation and right-cancellation by an element a hold, provided that a has an inverse. Proofs of the uniqueness of the identity and inverse elements. Both the right-identity $a * e = a$ and left-identity $e * a = a$ must hold if e is an identity element. Both $a * a^{-1} = e$ and $a^{-1} * a = e$ must hold.

The definition of a group $\{G, *\}$. The operation table of a group is a Latin square, but the converse is false. For the set G under a given operation $*$: G is closed under $*$; $*$ is associative; G contains an identity element; each element in G has an inverse in G . Abelian groups. $a * b = b * a$, for all $a, b \in G$.

Examples of groups: \mathbb{R} , \mathbb{Q} , \mathbb{Z} and \mathbb{C} under addition; integers under addition modulo n ; • non-zero integers under multiplication, modulo p , where p is prime; symmetries of plane figures, including equilateral triangles and rectangles; invertible functions under composition of functions. The composition $T_2 \circ T_1$ denotes T_1 followed by T_2 .

The order of a group. The order of a group element. Cyclic groups. Generators. Proof that all cyclic groups are Abelian. Permutations under composition of permutations. Cycle notation for permutations. Result that every permutation can be written as a composition of disjoint cycles. The order of a combination of cycles.

Subgroups, proper subgroups. A proper subgroup is neither the group itself nor the subgroup containing only the identity element. Use and proof of subgroup tests. Suppose that $\{G, *\}$ is a group and H is a non-empty subset of G . Then $\{H, *\}$ is a subgroup of $\{G, *\}$ if $a * b^{-1} \in H$ whenever $a, b \in H$. Suppose that $\{G, *\}$ is a finite group and H is a non-empty subset of G . Then $\{H, *\}$ is a subgroup of $\{G, *\}$ if H is closed under $*$. Definition and examples of left and right cosets of a subgroup of a group. Lagrange's theorem. Use and proof of the result that the order of a finite group is divisible by the order of any element. (Corollary to Lagrange's theorem.)

Definition of a group homomorphism. Definition of the kernel of a homomorphism. Proof that the kernel and range of a homomorphism are subgroups. Proof of homomorphism properties for identities and inverses. Isomorphism of groups. The order of an element is unchanged by an isomorphism.

21. PROOF

A LEVEL UNIT FP1: Proof – Proof by mathematical induction.

CALCULUS TOPICS

CATEGORIES ELABORATED FROM THE FOLLOWING TEXTBOOKS/COURSES:

- James Stewart, *Single Variable Calculus: Concepts and Contexts*, Brooks/Cole, 2010 (used in Stanford course Math 19: Introductory Calculus)
- Gilbert Strang, *Calculus*, Wellesley-Cambridge Press, 1991 (used in MIT open courseware courses) <http://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/>
- Tom M. Apostol, *Calculus Volume I: One Variable Calculus with an Introduction to Linear Algebra*, John Wiley & Sons, 1967 (used in MIT course 18.014 Calculus with Theory (2010); Caltech course Math 1A: Calculus of One and Several Variables)

I. LIMITS AND CONTINUITY

(AXLER: LIMIT OF A SEQUENCE; LIMIT NOTATION; ALEKS: ESTIMATING A LIMIT NUMERICALLY; FINDING LIMITS FOR A PIECEWISE-DEFINED FUNCTION; LIMIT LAWS; INFINITE LIMITS AND RATIONAL FUNCTIONS; FINDING LIMITS FROM A GRAPH; DETERMINING POINTS OF DISCONTINUITY FROM A GRAPH; LIMITS AT INFINITY AND GRAPHS; SQUEEZE THEOREM; DETERMINING A PARAMETER TO MAKE A FUNCTION CONTINUOUS; FINDING A LIMIT OF A TRIGONOMETRIC FUNCTION BY USING CONTINUITY; FINDING A LIMIT BY USING SPECIAL TRIGONOMETRIC LIMITS; STEWART: LIMIT OF A FUNCTION; ONE-SIDED LIMITS; CALCULATING LIMITS USING LIMIT LAWS, INCLUDING THE SQUEEZE THEOREM; CONTINUITY, INCLUDING INTERMEDIATE VALUE THEOREM; LIMITS INVOLVING INFINITY; STRANG: RULES FOR LIMITS;

ONE-SIDED LIMITS; THE SQUEEZE THEOREM; CONTINUOUS FUNCTIONS; DIFFERENTIABLE FUNCTIONS; CONTINUOUS FUNCTION ON A CLOSED INTERVAL – EXTREME VALUE PROPERTY, INTERMEDIATE VALUE PROPERTY; APOSTOL: LIMIT AND CONTINUITY OF A FUNCTION; BASIC LIMIT THEOREMS; PROOFS OF BASIC LIMIT THEOREMS; COMPOSITE FUNCTIONS AND CONTINUITY; BOLZANO’S THEOREM FOR CONTINUOUS FUNCTIONS; INTERMEDIATE VALUE THEOREM; INVERSES OF PIECEWISE MONOTONIC FUNCTIONS; EXTREME-VALUE THEOREM; SMALL-SPAN THEOREM)

ALBERTA MATH 31: Pre-calculus and Limits – Students will demonstrate *conceptual understanding* of limits and limit theorems, by: explaining the concept of limit; giving examples of functions with limits, with left-hand or right-hand limits, or with no limit; giving examples of bounded and unbounded functions, and of bounded functions with no limit; explaining, and giving examples of, continuous and discontinuous functions; explaining the limit theorems for sum, difference, multiple, product, quotient and power; illustrating, using suitable examples, the limit theorems for sum, difference, multiple, product, quotient and power. Students will demonstrate competence in the procedures associated with limits and limit theorems, by: determining the limit of any algebraic function as the independent variable approaches finite or infinite values for continuous and discontinuous functions; sketching continuous and discontinuous functions, using limits, intercepts and symmetry; using definitions and limit theorems to determine the limit of any algebraic function as the independent variable approaches a fixed value; using definitions and limit theorems to determine the limit of any algebraic function as the independent variable approaches +/- infinity.

AP CALCULUS AB: Limits – Express limits symbolically using correct notation. Interpret limits expressed symbolically: Given a function f , the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \rightarrow c} f(x) = R$ [See AP Curriculum Framework for proper notation]. The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits. A limit might not exist for some functions at particular values of x . Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near the value, or if the limit from the left does not equal the limit from the right. **Estimate limits of functions:** Numerical and graphical information can be used to estimate limits. **Determine limits of functions:** limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules. The limits of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem. **Deduce and interpret behavior of functions using limits:** Asymptotic and unbounded behavior of functions can be explained and described using limits. Relative magnitudes of functions and their rates of change can be compared using limits. **Analyze functions for intervals of continuity or points of discontinuity:** A function f is continuous at $x=c$ provided that $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists and $\lim_{x \rightarrow c} f(x) = f(c)$ [See AP Curriculum Framework for proper notation]. Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains. Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes. **Determine the applicability of important calculus theorems using continuity:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

IB MATH SL: Calculus – Informal ideas of limit and convergence. Technology should be used to explore ideas of limits, numerically and graphically. Limit notation;

Example: $\lim_{x \rightarrow \infty} \left(\frac{2x+3}{x-1} \right)$

IB MATH HL: Calculus (Core) – Informal ideas of limit, continuity and convergence. Include result $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$.

II. DEFINITION OF DERIVATIVES AS RATES OF CHANGE AND TYPE OF LIMIT, DERIVATIVES OF FUNCTIONS, HIGHER DERIVATIVES

(STEWART: CHANGE AND MOTION; TANGENT AND VELOCITY PROBLEMS; ESTIMATING THE SLOPE OF A TANGENT LINE; VELOCITY OF A FALLING BALL; REPRESENTATIONS OF FUNCTIONS – PIECEWISE DEFINED FUNCTIONS; DERIVATIVES AS RATES OF CHANGE; TANGENT LINE TO THE CURVE; VELOCITIES; DERIVATIVES AS A TYPE OF LIMIT; DERIVATIVE OF A COST FUNCTION AND OF A TABULAR FUNCTION; THE DERIVATIVE AS A FUNCTION – GIVEN BY A GRAPH, TABLE OR FORMULA;

DIFFERENTIATION AS THE PROCESS OF CALCULATING A DERIVATIVE; NON-DIFFERENTIABLE FUNCTIONS; HIGHER DERIVATIVES- SECOND DERIVATIVE, THIRD DERIVATIVE; WHAT f' SAYS ABOUT f – INCREASING OR DECREASING FUNCTIONS, CONCAVE UPWARD AND DOWNWARD, SKETCHING f , GIVEN f' AND f'' ; LOCAL MAXIMUM AND LOCAL MINIMUM OF FUNCTION f ; **STRANG**: CONSTANT VELOCITY; VELOCITY VS DISTANCE: SLOPE VS AREA; FUNCTIONS AND RATE OF CHANGE; CALCULUS WITHOUT LIMITS – PIECEWISE LINEAR AND PIECEWISE CONSTANT GRAPHS; EXPONENTIAL VELOCITY AND DISTANCE; OSCILLATING VELOCITY AND DISTANCE; INSTANTANEOUS VELOCITY; FUNCTIONS ACROSS TIME; CIRCULAR MOTION; THE DERIVATIVE OF A FUNCTION; POWERS AND POLYNOMIALS; THE DERIVATIVE OF $1/t$; DIFFERENTIAL EQUATIONS – EQUATIONS WITH DERIVATIVES IN THEM; THE SLOPE AND THE TANGENT LINE – THE EQUATION OF A LINE; **APOSTOL**: PROBLEMS INVOLVING VELOCITY; THE DERIVATIVE OF A FUNCTION; THE ALGEBRA OF DERIVATIVES; GEOMETRIC INTERPRETATION OF THE DERIVATIVE AS A SLOPE)

ALBERTA MATHS 31: Derivatives and Derivative Theorems – Students will demonstrate conceptual understanding of derivatives, by: showing that the slope of a tangent line is a limit; explaining how the derivative of a polynomial function can be approximated, using a sequence of secant lines; explaining how the derivative is connected to the slope of

the tangent line; recognizing that $f(x) = x^n$ can be differentiated where $n \in \mathbb{R}$; identifying the notations $f'(x)$, y' , and $\frac{dy}{dx}$ as alternative notations for the first derivative of a function.

AP CALCULUS AB: Derivatives – Identify the derivative of a function as the limit of a difference quotient: The difference quotients $\frac{f(a+h) - f(a)}{h}$ and $\frac{f(x) - f(a)}{x-a}$ [See AP Curriculum Framework p.10 for proper notation] express the average rate of change of a function over an interval. The instantaneous rate of change of a function at a point can be expressed by $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ and $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x-a}$ [See AP Curriculum Framework p.10 for proper notation], provided that the limit exists. These are common forms of

the definition of the derivative and are denoted $f'(a)$. The derivative of f is the function whose value at x is $f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$, provided this limit exists. For $y = f(x)$, notations for the derivative include dy/dx , $f'(x)$, and y' . The derivative can be represented graphically, numerically, analytically, and verbally. **Estimate derivatives:** The derivative of a point can be estimated from information given in tables or graphs. **Determine higher order derivatives:** Differentiating f' produces the second derivative f'' , provided the derivative of f' exists; repeating this process produces higher order derivatives of f . Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include d^2y/dx^2 , $f''(x)$, and y'' . Higher order derivatives can be denoted $d^n y/dx^n$ or $f^{(n)}(x)$. **Use derivatives to analyze properties of a function:** First and second order derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection. Key features of functions and their derivatives can be identified and related to their graphical, numerical and analytical representations. Key features of the graphs of f , f' and f'' are related to one another. **Recognize the connection between differentiability and continuity:** A continuous function may fail to be differentiable at a point in its domain. If a function is differentiable at a point, then it is continuous at that point. **Interpret the meaning of a derivative within a problem:** The unit for $f'(x)$ is the unit for f divided by the unit for x . The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable. **Solve problems involving the slope of a tangent line:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph. The tangent line is the graph of a locally linear approximation of the function near the point of tangency.

A LEVEL UNIT C1: Differentiation – The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; Second-order derivatives; applications of differentiation to gradients, tangents and normals.

A LEVEL UNIT C2: Differentiation – Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.

SINGAPORE H1: Calculus – Differentiation: Include derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; use of standard notations $f'(x)$ and dy/dx ; graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$; stationary points (local maximum and minimum points and points of inflection); finding the numerical value of a

derivative at a given point using a graphic calculator; finding equations of tangents and normal to curves. *Exclude:* Finding non-stationary points of inflection; problems involving small increments and approximation; relating the graph of $y = f'(x)$ to the graph of $y = f(x)$.

SINGAPORE H2: Calculus – Differentiation: Include graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$; $f''(x) > 0$ and $f''(x) < 0$; relating the graph of $y = f'(x)$ to the graph of $y = f(x)$; finding the numerical value of a derivative at a given point using a graphic calculator; finding equations of tangents and normal to curves. *Exclude:* finding non-stationary points of inflection; problems involving small increments and approximation.

IB MATH STUDIES SL: Introduction to Differential Calculus – Concept of the derivative as a rate of change; tangent to a curve. Teachers are encouraged to introduce differentiation through a graphical approach, rather than a formal treatment. Emphasis is placed on interpretation of the concept in different contexts. In examinations, questions on differentiation from first principles will not be set. Not required: Formal treatment of limits.

Gradients of curves for given values of x . Values of x where $f'(x)$ is given. Equation of the tangent at a given point. Equation of the line perpendicular to the tangent at a given point (normal). The use of technology to find the gradient at a point, and to draw tangent and normal lines, is also encouraged. Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$.

The principle that

$$f(x) = ax^n \Rightarrow f'(x) = anx^{n-1}.$$

The derivative of functions of the form

$f(x) = ax^n + bx^{n-1} + \dots$, where all exponents are integers.

Students should be familiar with the alternative notation for derivatives $\frac{dy}{dx}$ or $\frac{dV}{dr}$.

In examinations, knowledge of the second derivative will not be assumed.

Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$. The use of technology to display $f(x)$ and $f'(x)$, and find the solutions of $f'(x) = 0$ is also encouraged. Stationary points. Local maximum and minimum points. Awareness that a local maximum/minimum will not necessarily be the greatest/least value of the function in the given domain. Awareness of points of inflection with zero gradient is to be encouraged, but will not be examined.

IB MATH SL: Calculus – Definition of derivative from first principles as $f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$. Use of this definition for derivatives of simple polynomial functions only. Technology could be used to illustrate other derivatives. Use of both forms of notation, dy/dx and $f'(x)$ for the first derivative. Derivative interpreted as gradient function and as rate of change. Identifying intervals on which functions are increasing or decreasing. Tangents and normals, and their equations. Use of both analytic approaches and technology. Not required: analytic methods of calculating limits. Technology can be used to explore graphs and their derivatives.

Local maximum and minimum points; testing for maximum and minimum. Using change of sign of the first derivative and using sign of the second derivative. Use of the terms “concave-up” for $f''(x) > 0$, and “concave-down” for $f''(x) < 0$. Points of inflection with zero and non-zero gradients. At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$. Graphical behavior of functions, including the relationship between the graphs of f , f' and f'' . Not required: points of inflection where $f''(x)$ is not defined.

The second derivative. Use of both forms of notation $\frac{d^2y}{dx^2}$ and $f''(x)$. Extension to higher derivatives. $\frac{d^n y}{dx^n}$ and $f^{(n)}(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

IB MATH HL: Calculus (Core) – Definition of derivative from first principles

Use of this definition for polynomials only. The derivative interpreted as a gradient function and as a rate of change. Finding equations of tangents and normal. Identifying increasing and decreasing functions. Both forms of notation, dy/dx and $f'(x)$, for the first derivative. The second derivative. Use of both algebra and technology. Higher derivatives. Both forms of notation, d^2y/dx^2 and $f''(x)$, for the second derivative. Familiarity with the notation $d^n y/dx^n$ and $f^{(n)}(x)$.

IB MATH HL: Calculus (Option) – Continuity and differentiability of a function at a point. Test for continuity: $\lim_{x \rightarrow a^-} f(x) = f(a) = \lim_{x \rightarrow a^+} f(x)$. Continuous functions and

differentiable functions. Test for differentiability: f is continuous at a and $\lim_{h \rightarrow 0^-} \frac{f(a+h) - f(a)}{h}$ and $\lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h}$ exist and are equal. Students should be aware that a function may be continuous but not differentiable at a point, eg. $f(x) = |x|$ and simple piecewise functions.

III. DERIVATIVE RULES & DERIVATIVES OF EXPONENTIAL, LOGARITHMIC AND TRIGONOMETRIC FUNCTIONS

(STEWART: DERIVATIVES OF POLYNOMIALS AND EXPONENTIAL FUNCTIONS; THE POWER RULE, CONSTANT MULTIPLE RULE, SUM RULE, DIFFERENCE RULE; EXPONENTIAL FUNCTIONS – THE NUMBER e ; DERIVATIVE OF THE NATURAL EXPONENTIAL FUNCTION; PRODUCT AND QUOTIENT RULES; DERIVATIVES OF TRIGONOMETRIC FUNCTIONS; THE CHAIN RULE; TANGENTS TO PARAMETRIC CURVES; PROVING THE CHAIN RULE; IMPLICIT DIFFERENTIATION; INVERSE TRIGONOMETRIC FUNCTIONS AND THEIR DERIVATIVES; DERIVATIVES OF LOGARITHMIC FUNCTIONS; THE NUMBER e AS A LIMIT; STRANG: SECOND DERIVATIVES OF SINE AND COSINE; DERIVATIVE RULES: SUM RULE, RULE OF LINEARITY, PRODUCT RULE, RECIPROCAL RULE, QUOTIENT RULE, POWER RULE; THE CHAIN RULE; DERIVATIVES BY THE CHAIN RULE; INVERSE FUNCTIONS AND THEIR DERIVATIVES; EXPONENTIALS AND LOGARITHMS; INVERSE OF A CHAIN; INVERSES OF TRIGONOMETRIC FUNCTIONS – DERIVATIVE OF THE INVERSE SINE, INVERSE COSINE AND ITS DERIVATIVE, INVERSE TANGENT AND ITS DERIVATIVE, INVERSE COTANGENT, SECANT AND COSECANT; APOSTOL: THE CHAIN RULE FOR DIFFERENTIATING COMPOSITE FUNCTIONS; APPLICATIONS OF THE CHAIN RULE)

ALBERTA MATHS 31: Derivatives and Derivative Theorems - Students will demonstrate conceptual understanding of derivatives, by: explaining the derivative theorems for sum and difference $(f \pm g)'(x) = f'(x) \pm g'(x)$ explaining the sense of the derivative theorems for sum and difference, using practical examples; demonstrating that the chain, power, product and quotient rules are aids to differentiate complicated functions; identifying implicit differentiation as a tool to differentiate functions where one variable is difficult to isolate; explaining the relationship between implicit differentiation and the chain rule comparing the sum, difference, product and quotient theorems for limits and

derivatives; explaining the derivation of the derivative theorems for product and quotient $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$, $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ explaining the derivative theorems for product and quotient, using practical examples. Students will demonstrate conceptual understanding of the derivatives of trigonometric functions, by: demonstrating that the three primary trigonometric functions have derivatives at all points where the functions are defined; explaining how the derivative of a trigonometric function can be approximated, using a sequence of secant lines.

ALBERTA MATHS 31: Calculus of Logarithmic and Exponential Functions (Option): Students will demonstrate conceptual understanding of the calculus of exponential and logarithmic functions, by: defining exponential and logarithmic functions as inverse functions; explaining the special properties of the number e , together with a definition of e as a limit; illustrating that the derivative of an exponential or logarithmic function may be derived from the definition of the derivative; illustrating that base- e exponential and logarithmic functions form a convenient framework within which the calculus of similar functions in any base may be developed.

ALBERTA MATHS 31: Calculus Theorems (Option): Students will demonstrate conceptual understanding of the nature of proof in the context of limit, derivative and integral theorems, by: proving the equivalence of the product and the quotient rule for derivatives.

AP CALCULUS AB: Derivatives – Calculate derivatives: Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions. Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric. Sums, differences, products, and quotients of functions can be differentiated using derivative rules. The chain rule provides a way to differentiate composite functions. The chain rule is the basis for implicit differentiation. The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.

A LEVEL UNIT C1: Differentiation – differentiation of x^n , and related sums and differences.

A LEVEL UNIT C3: Differentiation – Differentiation of e^x , $\ln x$, $\sin x$, $\tan x$ and their sums and differences; differentiation using the product rule, the quotient rule and the chain rule; the use of $dy/dx = 1/dx/dy$.

A LEVEL UNIT C4: Differentiation – Differentiation of simple functions defined implicitly or parametrically.

SINGAPORE H1: Calculus – Differentiation: Derivatives of x^n for any rational n , e^x , $\ln x$, together with constant multiples, sums and differences; use of chain rule. *Exclude:* differentiation from first principles; derivatives of products and quotients of functions; use of $dy/dx = 1/dx/dy$, differentiation of functions defined implicitly or parametrically.

SINGAPORE H2: Calculus – Differentiation: Differentiation of simple functions defined implicitly or parametrically

IB MATH SL: Calculus – Derivative of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$. Differentiation of a sum and a real multiple of these functions. The chain rule for composite functions. Technology may be used to identify the chain rule. The product and quotient rules.

IB MATH HL: Calculus (Core) – Derivatives of x^n , $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$. Differentiation of sums and multiples of functions. The product and quotient rules. The chain rule for composite functions. Related rates of change. Implicit differentiation. Derivatives of $\sec x$, $\csc x$, $\cot x$, a^x , $\log_a x$, $\arcsin x$, $\arccos x$ and $\arctan x$.

IV. DERIVATIVE THEOREMS, OPTIMIZATION, RELATED RATES & IMPLICIT DIFFERENTIATION

(STEWART): LINEAR APPROXIMATIONS AND DIFFERENTIALS; RELATED RATES; OPTIMIZATION; MAXIMUM AND MINIMUM VALUES - EXTREME-VALUE THEOREM, FERMAT'S THEOREM, THE CLOSED-INTERVAL METHOD; DERIVATIVES AND THE SHAPES OF CURVES – THE MEAN-VALUE THEOREM; INCREASING AND DECREASING FUNCTIONS – THE FIRST DERIVATIVE TEST; CONCAVITY – SECOND DERIVATIVE TEST; GRAPHING - CALCULATORS; INDETERMINATE FORMS AND L'HOPITAL'S RULE; INDETERMINATE DIFFERENCES; INDETERMINATE POWERS; OPTIMIZATION PROBLEMS – FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES; NEWTON'S METHOD; **STRANG):** LINEAR APPROXIMATION AND DIFFERENTIALS – ABSOLUTE CHANGE, RELATIVE CHANGE AND PERCENTAGE CHANGE; SECOND DERIVATIVES – INFLECTION POINTS, CENTERED DIFFERENCES AND SECOND DIFFERENCES, LINEAR APPROXIMATION VS QUADRATIC APPROXIMATION; GRAPHING – THE MECHANICS OF GRAPHS, GRAPHS BY COMPUTERS AND CALCULATORS, THE CENTERING TRANSFORM AND ZOOM TRANSFORM; NEWTON'S METHOD AND CHAOS – CHAOS FROM A PARABOLA, CANTOR SETS AND FRACTALS, NEWTON'S METHOD VS SECANT METHOD; THE MEAN-VALUE THEOREM AND L'HOPITAL'S RULE – ROLLE'S THEOREM; IMPLICIT DIFFERENTIATION AND RELATED RATES; **APOSTOL):** RELATED RATES AND IMPLICIT DIFFERENTIATION; THE MEAN-VALUE THEOREM FOR DERIVATIVES; APPLICATIONS OF MEAN-VALUE THEOREM TO GEOMETRIC PROPERTIES OF FUNCTIONS; SECOND-DERIVATIVE TEST FOR EXTREMA; L'HOPITAL'S RULE)

ALBERTA MATHS 31: Derivatives and Derivative Theorems - Students will demonstrate conceptual understanding of derivative theorems, by: illustrating second, third and higher derivatives of algebraic functions; describing the second derivative geometrically. **Applications of derivatives** - Students will demonstrate conceptual understanding of maxima and minima, by: identifying, from a graph sketch, locations at which the first and second derivative are zero; illustrating under what conditions symmetry about the x -axis, y -axis or the origin will occur; explaining how the sign of the first derivative indicates whether or not a curve is rising or falling; and by explaining how the sign of the second derivative indicates the concavity of the graph; illustrating, by examples, that a first derivative of zero is one possible condition for a maximum or a minimum to occur; explaining circumstances wherein maximum and minimum values occur when $f'(x)$ is not zero; illustrating, by examples, that a second derivative of zero is one possible condition for an inflection point to occur; explaining the differences between local maxima and minima and absolute maxima and minima in an interval; explaining when finding a maximum value is appropriate and when finding a minimum value is appropriate. Students will demonstrate conceptual understanding of related rates, by: illustrating how the chain rule can be used to represent the relationship between two or more rates of change; explaining the clarity that Leibnitz's notation gives to expressing related rates.

ALBERTA MATHS 31: Calculus Theorems (Option): Students will demonstrate conceptual understanding of the nature of proof in the context of limit, derivative and integral theorems, by: illustrating the intermediate value theorem, Rolle's theorem, the mean value theorem and the fundamental theorem of calculus, by examples and counterexamples, using both graphical and algebraic formulations.

AP CALCULUS AB: Derivatives – Solve problems involving related rates, optimization, rectilinear motion: The derivative can be used to solve rectilinear motion problems including position, speed, velocity, and acceleration. The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known. The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval. **Apply the Mean Value Theorem to describe the behavior of a function over an interval:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b) , the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

IB MATH SL: Calculus – Optimization. Use of the first or second derivative test to justify maximum and/or minimum values.

IB MATH HL: Calculus (Option) – Rolle's theorem; mean-value theorem.

IB MATH HL: Calculus (Core) Local maximum and minimum values. Optimization problems. Points of inflexion with zero and non-zero gradients. Graphical behavior of functions, including the relationship between the graphs of f , f' and f'' . Testing for the maximum and minimum using the change of sign of the first derivative and using the sign of the second derivative. Use of the terms "concave up" for $f''(x) > 0$, "concave down" for $f''(x) < 0$. At the point of inflexion, $f''(x) = 0$ and changes sign (concavity change).

V. APPLICATIONS OF DIFFERENTIATION: RATES OF CHANGE, ECONOMICS

(STEWART: OPTIMIZATION PROBLEMS – APPLICATIONS TO BUSINESS AND ECONOMICS; STRANG: LINEAR APPROXIMATION AND DIFFERENTIALS – ABSOLUTE CHANGE, RELATIVE CHANGE AND PERCENTAGE CHANGE; MAXIMUM AND MINIMUM PROBLEMS – APPLICATIONS, MARGINAL COST; SECOND DERIVATIVES – BENDING AND ACCELERATION)

ALBERTA MATHS 31: Applications of calculus to business and economics (Option) - Students will demonstrate conceptual understanding of the links among calculus, business and economics, by: explaining how calculus procedures frequently arise in models used in business and economics; explaining how both maximum and minimum values are important in the making of business and economic decisions.

AP CALCULUS AB: Derivatives – Solve problems involving rates of change in applied contexts: The derivative can be used to express information about rates of change in applied contexts.

AP CALCULUS BC: Derivatives – Solve problems involving related rates, optimization, rectilinear motion and planar motion: Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions.

SINGAPORE H1: Calculus – Differentiation: Solving practical problems involving differentiation.

SINGAPORE H2: Calculus – Differentiation: Solving practical problems involving differentiation.

IB MATH STUDIES SL: Introduction to Differential Calculus –Optimization problems. Examples: Maximizing profit, minimizing cost, maximizing value for given surface area. In examinations, questions on kinematics will not be set.

IB MATH SL: Calculus – Optimization. Applications. Examples include profit, area, volume.

IB MATH HL: Calculus (Core) – Related rates of change; Physics applications; mathematics and the real world

VI. INTEGRALS, TECHNIQUES OF INTEGRATION & THE FUNDAMENTAL THEOREM OF CALCULUS

(STEWART: AREAS AND DISTANCES; THE DEFINITE INTEGRAL; EVALUATING INTEGRALS – AS A LIMIT OF RIEMANN SUMS, USING GEOMETRY TO EVALUATE INTEGRALS; THE MIDPOINT RULE; PROPERTIES OF THE DEFINITE INTEGRAL; EVALUATING DEFINITE INTEGRALS; EVALUATION THEOREM; INDEFINITE INTEGRALS; APPLICATIONS – NET CHANGE THEOREM; THE FUNDAMENTAL THEOREM OF CALCULUS – CONNECTION BETWEEN DIFFERENTIAL AND INTEGRAL CALCULUS; DIFFERENTIATION AND INTEGRATION AS INVERSE PROCESSES; PROOF OF THE FUNDAMENTAL THEOREM OF CALCULUS; THE SUBSTITUTION RULE; INTEGRALS OF SYMMETRIC FUNCTIONS; INTEGRATION BY PARTS; ADDITIONAL TECHNIQUES OF INTEGRATION – TRIGONOMETRIC INTEGRALS, TRIGONOMETRIC SUBSTITUTION, PARTIAL FRACTIONS; INTEGRATION USING TABLES AND COMPUTER ALGEBRA SYSTEMS; APPROXIMATE INTEGRATION – MIDPOINT RULE, TRAPEZOIDAL RULE, SIMPSON’S RULE, ERROR BOUND FOR SIMPSON’S RULE; IMPROPER INTEGRALS – INFINITE INTEGRALS, DISCONTINUOUS INTEGRANDS, A COMPARISON TEST FOR IMPROPER INTEGRALS, COMPARISON THEOREM; STRANG: IDEA OF THE INTEGRAL – SUMS AND DIFFERENCES, SUMS APPROACH INTEGRALS; ANTIDERIVATIVES – AREA UNDER A STRAIGHT LINE, INDEFINITE AND DEFINITE INTEGRALS; SUMMATION VS INTEGRATION – SIGMA NOTATION, SPECIAL SUMMATION FORMULA; INDEFINITE INTEGRALS AND SUBSTITUTIONS – RULES FOR INTEGRALS, LINEARITY OF INTEGRALS, INTEGRALS BY SUBSTITUTION; THE DEFINITE INTEGRAL – THE CONSTANT OF INTEGRATION, INTEGRALS AS LIMITS OF REIMANN SUMS; PROPERTIES OF THE INTEGRAL AND AVERAGE VALUE – PREDICTABLE AVERAGES FROM RANDOM EVENTS; THE FUNDAMENTAL THEOREM OF CALCULUS AND ITS APPLICATIONS; NUMERICAL INTEGRATION – TRAPEZOIDAL AND MIDPOINT RULES, FOURTH ORDER RULE: SIMPSON, GAUSS RULE, DEFINITE INTEGRALS ON A CALCULATOR; INTEGRATION BY PARTS - THE DELTA FUNCTION; TRIGONOMETRIC INTEGRALS – SECANTS AND TANGENTS; TRIGONOMETRIC SUBSTITUTIONS- COMPLETING THE SQUARE; PARTIAL FRACTIONS; IMPROPER INTEGRALS; APOSTOL: SUM AND PRODUCT OF STEP FUNCTIONS; INTEGRAL OF A STEP FUNCTION; UPPER AND LOWER INTEGRALS; AREA OF AN ORDINATE SET EXPRESSED AS AN INTEGRAL; PIECEWISE MONOTONIC FUNCTIONS; INTEGRAL OF A BOUNDED MONOTONIC FUNCTION; INTEGRATION OF POLYNOMIALS; TRIGONOMETRIC FUNCTIONS – INTEGRATION FORMULAS FOR SINE AND COSINE; INTEGRABILITY THEOREM; MEAN-VALUE THEOREMS FOR INTEGRALS OF CONTINUOUS FUNCTIONS; DERIVATIVE OF AN INDEFINITE INTEGRAL; FUNDAMENTAL THEOREM OF CALCULUS; ZERO-DERIVATIVE THEOREM; INTEGRATION BY SUBSTITUTION; INTEGRATION BY PARTS)

ALBERTA MATHS 31: Integrals, Integral Theorems and Integral Applications: Students will demonstrate conceptual understanding of antiderivatives, by: explaining how differentiation can have an inverse operation; showing that many different functions can have the same derivative; representing, on the same grid, a family of curves that form a

sequence of functions, all having the same derivative. Students will demonstrate competence in the procedures associated with antiderivatives, by: finding the antiderivatives of polynomials, rational algebraic functions and trigonometric functions.

Students will demonstrate conceptual understanding of area limits, by: defining the area under a curve as a limit of the sums of the areas of rectangles; establishing the existence of upper and lower bounds for the area under a curve. Students will demonstrate conceptual understanding of definite integrals, by: identifying the indefinite integral $\int f(x)dx$ as a sum of an antiderivative $F(x)$ and a constant c ; explaining how the definite integral between fixed limits a and b is a number whose value is $F(b) - F(a)$. Explaining the connection between the numerical values of the area and the definite integral for functions f of a constant sign and a variable sign; illustrating the following properties of integrals $\int_a^b cf(x)dx = c \int_a^b f(x)dx$, $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$. Describing the sense of the fundamental theorem of calculus; explaining how the fundamental theorem of calculus relates the area limit to the antiderivative of a function describing a curve.

ALBERTA MATHS 31: Numerical Methods (Option) – Students will demonstrate conceptual understanding of the principles of numerical analysis, by: connecting the number of subdivisions of the range of integration with the accuracy of the estimate for the integral; showing that all numerical integration formulas are procedures that interpolate between the lower and upper Riemann sums for the integral. Students will demonstrate competence in the procedures associated with numerical methods, by: calculating the upper and lower Riemann sums for a definite integral; calculating the value of a definite integral, using the midpoint rule; calculating the value of a definite integral, using the trapezoidal rule; calculating the value of a definite integral, using Simpson’s rule.

ALBERTA MATHS 31: Further methods of integration (Option): *Students will* demonstrate conceptual understanding of the methods of integration, by: identifying integrals that cannot be evaluated, using the antiderivatives of polynomial or trigonometric functions; showing how substitutions into a definite integral change both the function to be integrated and the limits of integration; recognizing when it would be appropriate to use integration by substitution, by parts or by partial fractions.

AP CALCULUS AB: Integrals and the Fundamental Theorem of Calculus – Recognize antiderivatives as basic functions: An antiderivative of a function f is a function g whose derivative is f . Differentiation rules provide the foundation for finding antiderivatives. **Interpret the definite integral as the limit of a Riemann sum:** A Riemann sum, which requires a partition of an interval I , is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition. **Express the limit of a Riemann sum in integral notation:** The definite integral of a continuous function f over the interval $[a,b]$, denoted by $\int_a^b f(x)dx$ [See AP Curriculum Framework p.14 for proper notation], is the limit of Riemann sums as the widths of the subintervals approach 0. That is $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x_i$ [See AP Curriculum Framework p.14 for proper notation], where x_i^* is a value in the i th subinterval, Δx_i is the width of the i th subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of definition is $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \frac{b-a}{n}$ [See AP Curriculum Framework p.14 for proper notation], where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value of the i th interval. The definition in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral. **Approximate a definite integral:** Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally. Definite integrals can be approximated using a left Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions. **Calculate a definite integral using areas and properties of definite integrals:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area. Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals. The definition of the definite integral may be extended to functions with removable or jump discontinuities. **Analyze functions defined by an integral:** The definite integral can be used to define new functions; for example $F(x) = \int_a^x f(t)dt$ [See AP Curriculum Framework p.15 for proper notation]. If f is a continuous function on the interval $[a,b]$, then $F(x)$ is a function on the interval $[a,b]$ [See AP Curriculum Framework p.15 for proper notation], where x is between a and b . Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$ [See AP Curriculum Framework p.15 for proper notation]. **Calculate antiderivatives:** The function defined by $F(x) = \int_a^x f(t)dt$ [See AP Curriculum Framework p.15 for proper notation], is an antiderivative of f . **Evaluate definite integrals:** If f is continuous on the interval $[a,b]$ and F is an antiderivative of f , then $\int_a^b f(x)dx = F(b) - F(a)$ [See AP Curriculum Framework p.15 for proper notation]. Many functions do not have closed form antiderivatives. Techniques for finding antiderivatives include algebraic

manipulation such as long division and completing the square, substitution of variables. **Interpret the meaning of a definite integral within a problem:** A function defined as an integral represents an accumulation of a rate of change. The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval. The limit of an approximating Riemann sum can be interpreted as a definite integral.

AP CALCULUS BC: Integrals and the Fundamental Theorem of Calculus – Evaluate an improper integral or show that an improper integral diverges: An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration. Improper integrals can be determined using limits of definite integrals. **Evaluate definite integrals:** Integration by parts, and nonrepeating linear partial fractions.

A LEVEL UNIT C1: Integration – Indefinite integration as the reverse of differentiation; students should know that a constant of integration is required; integration of x^n .

A LEVEL UNIT C2: Integration – Evaluation of definite integrals; interpretation of the definite integral as the area under a curve; approximation of the area under a curve using the trapezium rule.

A LEVEL UNIT C4: Integration – Integration of e^x , $1/x$, $\sin x$, $\cos x$; simple cases of integration by substitution and integration by parts; these methods as the reverse processes of the chain and product rules respectively; simple cases of integration using partial fractions.

A LEVEL UNIT FP3: Integration – Use of substitution for integrals involving quadratic surds. In more complicated cases, substitutions will be given. The derivation and use of simple reduction formulae.

SINGAPORE H1: Calculus – Integration: Include integration as the reverse of differentiation; integration of x^n , for any rational n , and e^x , together with constant multiples, sums and differences; integration of $(ax + b)^n$, for any rational n , and $e^{(ax+b)}$; definite integral as the area under a curve; evaluation of definite integrals. Finding the numerical value of a definite integral using a graphic calculator. Exclude: definite integral as a limit of sum; approximation of area under a curve using the trapezium rule.

SINGAPORE H2: Calculus – Integration techniques: Include integration of $f'(x)/f(x)$; $\sin^2 x$, $\cos^2 x$, $\tan^2 x$; $\frac{1}{a^2 + x^2}$, $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{a^2 - x^2}$ and $\frac{1}{x^2 - a^2}$; integration by a given substitution; integration by parts. Exclude: Reduction formulae. **Definite integrals:** Include definite integral as a limit of sum; definite integral as the area under a curve; evaluation of definite integrals; finding the numerical value of a definite integral using a graphic calculator. Exclude: approximation of area under a curve using the trapezium rule.

IB MATH SL: Calculus – Indefinite integration as anti-differentiation. Indefinite integral of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $1/x$ and e^x . $\int \frac{1}{x} dx = \ln|x| + C$, $x > 0$. The composites of any of these with the linear function $ax + b$. Example: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$. Integration by inspection, or substitution of the form $\int f(g(x))g'(x) dx$. Anti-differentiation with the boundary condition to determine the constant term. Definite integrals, both analytically and using technology. $\int_a^b g'(x) dx = g(b) - g(a)$.

IB MATH HL: Calculus (Core) – Indefinite integration as anti-differentiation. Indefinite integral of x^n , $\sin x$, $\cos x$, and e^x . Other indefinite integrals [...]. The composites of any of these with a linear function. Indefinite integral interpreted as a family of curves. $\int \frac{1}{x} dx = \ln|x| + c$. Examples include $\int (2x-1)^5 dx$, $\int \frac{1}{3x+4} dx$ and $\int \frac{1}{x^2+2x+5} dx$. Anti-differentiation with a boundary condition to determine the constant of integration. Definite integrals. The value of some definite integrals can only be found using technology. Integration by substitution. On examination papers, non-standard substitutions will be provided. Integration by parts. Examples: $\int x \sin x dx$ and $\int \ln x dx$. Repeated integration by parts. Examples $\int x^2 e^x dx$ and $\int e^x \sin x dx$.

IB MATH HL: Calculus (Option) – The integral as a limit of a sum; lower and upper Riemann sums. Fundamental theorem of calculus. $\frac{d}{dx} \left[\int_a^x f(y) dy \right] = f(x)$. Improper integrals of the type $\int_a^{\infty} f(x) dx$.

VII. APPLICATIONS OF INTEGRATION TO PROBLEMS OF AREA AND VOLUME

(STEWART: AREAS BETWEEN CURVES; AREAS ENCLOSED BY PARAMETRIC CURVES; VOLUMES; VOLUMES BY CYLINDRICAL SHELLS; ARC LENGTH, ARC LENGTH FORMULA; AVERAGE VALUE OF A FUNCTION; THE MEAN VALUE THEOREM FOR INTEGRALS; PROBABILITY – AVERAGE VALUES, NORMAL DISTRIBUTION; STRANG: AREAS AND VOLUMES BY SLICES – SOLIDS OF REVOLUTION, VOLUMES BY CYLINDRICAL SHELLS,; LENGTH OF A PLANE CURVE – LENGTH OF A CURVE FROM PARAMETRIC EQUATIONS; AREA OF A SURFACE OF REVOLUTION; PROBABILITY AND CALCULUS – DISCRETE RANDOM VARIABLES, CONTINUOUS RANDOM VARIABLES, MEAN, VARIANCE AND STANDARD DEVIATION, LAW OF AVERAGES AND CENTRAL LIMIT THEOREM; MASSES AND MOMENTS; APOSTOL: AREA OF A REGION BETWEEN TWO GRAPHS EXPRESSED AS AN INTEGRAL; INTEGRAL FOR AREA IN POLAR COORDINATES; APPLICATION OF INTEGRATION TO THE CALCULATION OF VOLUME; INTEGRAL AS A FUNCTION OF THE UPPER LIMIT; AVERAGE VALUE OF A FUNCTION)

ALBERTA MATHS 31: Volumes of revolution (Option) – Students will demonstrate conceptual understanding of volumes of revolution, by: identifying the solid generated by the rotation of the graph of a function, either between two boundary values, or between two graphs; explaining the connection between the volume of revolution and the volume of a cylindrical disc; demonstrating how the formula for the volume of revolution by the disc method could be generated. Students will demonstrate competence in the procedures associated with volumes of revolution, by: using the relationship $V = \pi \int_a^b [f(x)]^2 dx$ to find the volume of revolution between the boundaries of a and b for polynomial and trigonometric functions; finding the volume of revolution between two polynomial or trigonometric functions by first finding the intersection points of the graphs of the two functions.

AP CALCULUS AB: Apply definite integrals to problems involving area and volume: Areas of certain regions in the plane can be calculated with definite integrals. Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals. **Use of the definite integral to solve problems in various contexts:** The definite integral can be used to express information about accumulation and net change in many applied contexts.

AP CALCULUS BC: Integrals and the Fundamental Theorem of Calculus – Apply definite integrals to problems involving area, volume, and length of a curve: Areas bounded by polar curves can be calculated with definite integrals. The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral.

A LEVEL UNIT C4: Integration – Evaluation of volume of revolution.

A LEVEL UNIT FP3: Integration – The calculation of arc length and the area of a surface of revolution.

SINGAPORE H1: Calculus – Integration: Finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves. Exclude: area below the x-axis.

SINGAPORE H2: Calculus – Definite integrals: Include finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between the curve and a line, or between two curves; area below the x-axis; finding the area under the curve defined parametrically; finding the volume of revolution about the x- or y-axis.

IB MATH SL: Calculus – Areas under curves (between the curve and the x-axis). Students are expected to first write a correct expression before calculating the area. Areas between curves. Volumes of revolution about the x-axis. Technology may be used to enhance understanding of area and volume.

IB MATH HL: Calculus (Core) – Area of the region enclosed by a curve and the x-axis or y-axis in a given interval; areas of regions enclosed by curves. Volumes of revolution about the x-axis or y-axis.

VIII. APPLICATIONS OF INTEGRATION TO PROBLEMS OF MOTION (KINEMATICS)

(STEWART: APPLICATIONS TO PHYSICS AND ENGINEERING; APPLICATIONS TO ECONOMICS AND BIOLOGY; STRANG: MASSES AND MOMENTS; FORCE, WORK AND KINETIC ENERGY; APOSTOL: APPLICATION OF INTEGRATION TO THE CONCEPT OF WORK)

ALBERTA MATHS 31: Integrals, Integral Theorems and Integral Applications: Students will demonstrate conceptual understanding of the relationships among displacement, velocity and acceleration, by: describing the motion of a body, using sketches of the first and second derivatives; explaining the difference between a stationary point and a turning point in the context of linear motion; illustrating the concepts of derivative and antiderivative in the context of displacement, velocity and acceleration.

AP CALCULUS AB: Integrals and the Fundamental Theorem of Calculus – Apply definite integrals to problems involving the average value of a function: The average value of a function f over an interval $[a,b]$ is [See AP Curriculum Framework p.16 for proper notation]. **Apply definite integrals to problems involving motion:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle’s displacement over the interval of time, and the definite integral of speed represents the particle’s total distance traveled over the interval of time.

AP CALCULUS BC: Integrals and the Fundamental Theorem of Calculus – Apply definite integrals to problems involving motion: The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.

IB MATH SL: Calculus – Kinematic problems involving displacement s , velocity v and acceleration a . $v = \frac{ds}{dt}$; $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$. Total distance travelled = $\int_a^{t_2} |v| dt$.

IB MATH HL: Calculus (Core) – Kinematic problems involving displacement s , velocity v and acceleration a . Total distance travelled. $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$. Total distance travelled = $\int_a^{t_2} |v| dt$.

IX. DERIVATIVES AND INTEGRALS OF VECTOR-VALUED AND PARAMETRIC FUNCTIONS; POLAR COORDINATES

AP CALCULUS BC: Derivatives – For a curve given by a polar equation $r = f(\theta)$, derivatives of r , x , and y with respect to (θ) and first and second derivatives of y with respect to x can provide information about the curve.

AP CALCULUS BC: Derivatives – Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.

A LEVEL UNIT C4: Differentiation – Differentiation of simple functions defined implicitly or parametrically.

SINGAPORE H1: Calculus – Differentiation: Derivatives of x^n for any rational n , e^x , $\ln x$, together with constant multiples, sums and differences; use of chain rule. *Exclude:* differentiation from first principles; derivatives of products and quotients of functions; use of $dy/dx = 1/dx/dy$, differentiation of functions defined implicitly or parametrically.

SINGAPORE H2: Calculus – Differentiation: Differentiation of simple functions defined implicitly or parametrically

X. HYPERBOLIC FUNCTIONS

(STEWART: SPECIAL FUNCTIONS: FUNCTIONS RELATED TO THE HYPERBOLA – HYPERBOLIC SINE, COSINE, TANGENT AND SECANT, HYPERBOLIC IDENTITIES, DIFFERENTIATION FORMULAS FOR HYPERBOLIC FUNCTIONS AS ANALOGOUS TO THOSE FOR TRIGONOMETRIC FUNCTIONS, INVERSE HYPERBOLIC FUNCTIONS; STRANG: HYPERBOLIC FUNCTIONS AS THE COMBINATION OF e^x AND e^{-x} , COSH AND SINH, INVERSE HYPERBOLIC FUNCTIONS AND THEIR DERIVATIVES, TECHNIQUES OF INTEGRATION – HYPERBOLIC SUBSTITUTIONS)

A LEVEL UNIT FP3: Hyperbolic functions – Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions. Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents.

A LEVEL UNIT FP3: Differentiation – Differentiation of hyperbolic functions and expressions involving them. Differentiation of inverse functions, including trigonometric and hyperbolic functions.

A LEVEL UNIT FP3: Integration – Integration of hyperbolic functions and expressions involving them. Integration of inverse trigonometric and hyperbolic functions. Integration using hyperbolic and trigonometric substitutions.

XI. DIFFERENTIAL EQUATIONS

(STEWART: DIRECTION FIELDS AND EULER’S METHOD; SEPARABLE EQUATIONS – ORTHOGONAL TRAJECTORIES, MIXING PROBLEMS; THE LOGISTIC EQUATION – THE LOGISTIC MODEL, DIRECTION FIELDS, EULER’S METHOD, THE ANALYTIC SOLUTION; STRANG: SECOND-ORDER EQUATIONS; PARTICULAR SOLUTIONS – THE METHOD OF UNDETERMINED COEFFICIENTS; NUMERICAL METHODS; APOSTOL: FIRST-ORDER DIFFERENTIAL EQUATION FOR EXPONENTIAL FUNCTION; FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS; LINEAR EQUATIONS OF SECOND ORDER WITH CONSTANT COEFFICIENTS; NON-LINEAR DIFFERENTIAL EQUATIONS; INTEGRAL CURVES AND DIRECTION FIELDS; GEOMETRICAL AND PHYSICAL PROBLEMS LEADING TO FIRST-ORDER EQUATIONS)

ALBERTA MATHS 31: Integrals, Integral Theorems and Integral Applications: Students will demonstrate competence in the procedures associated with antiderivatives, by: finding the family of curves whose first derivative has been given; solving separable first order differential equations for general and specific solutions.

AP CALCULUS AB: Derivatives – Verify solutions to differential equations: Solutions to differential equations are functions or families of functions. Derivatives can be used to verify that a function is a solution to a given differential equation. **Estimate solutions to differential equations:** Slope fields provide visual clues to the behavior of solutions to first order differential equations.

AP CALCULUS AB: Integrals and the Fundamental Theorem of Calculus – Analyze differential equations to obtain general and specific solutions: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line; exponential growth and decay. Some differential equations can be solved by separation of variables. Solutions to differential equations may be subject to domain restrictions.

AP CALCULUS BC: Derivatives – Estimate solutions to differential equations: For differential equations, Euler’s method provides a procedure for approximating a solution or a point on a solution curve.

A LEVEL UNIT C4: Integration – Analytical solution of simple first-order differential equations with separable variables; general and particular solutions will be required. Numerical integration of functions.

A LEVEL UNIT FP2: First Order Differential Equations – Further solution of first order differential equations with separable variables; first order differential equations of the form $Dy/dx + Py = Q$ where P and Q are functions of x . Differential equations reducible to the above types by means of a given substitution.

A LEVEL UNIT FP2: Second Order Differential Equations – The linear second order differential equation $a d^2y/dx^2 + b dy/dx + cy = f(x)$ where a , b and c are real constants and the particular integral can be found by inspection or trial. Differential equations reducible to the above types by means of a given substitution.

SINGAPORE H2: Calculus – Differential Equations: Include solving differential equations of the forms $dy/dx = f(x)$, $dy/dx = f(y)$, $d^2y/dx^2 = f(x)$; formulating a differential equation from a problem situation; use of a family of solution curves to represent the general solution of a differential equation; use of an initial condition to find a particular solution; interpretation of a solution in terms of the problem situation.

SINGAPORE H3: Calculus – Differential Equations as Mathematical Models: Analytical solution of first and second order differential equations of the form: $\frac{dy}{dx} = g(x)f(y)$,

$\frac{dy}{dx} + p(x)y = q(x)$, $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$, including those that can be reduced to the above by means of a given simple substitution; particular and general solutions; phase lines, slope fields, and sketching solution curves; numerical solution of first order differential equations using Euler’s method (including the use of the improved Euler’s formula)

IB MATH HL: Calculus (Option) – First-order differential equations. Geometric interpretation using slope fields, including identification of isoclines. Numerical solutions of $dy/dx = f(x,y)$ using Euler’s method. $y_{n+1} = y_n + hf(x_n, y_n)$, $x_{n+1} = x_n + h$, where h is a constant. Variables separable. Homogenous differential equation $dy/dx = f(y/x)$ using the substitution $y = vx$. Solution of $y' + P(x)y = Q(x)$, using the integration factor.

XII. APPLICATIONS OF DIFFERENTIAL EQUATIONS: GROWTH & DECAY, SPRINGS & CIRCUITS

(STEWART: MODELLING WITH DIFFERENTIAL EQUATIONS – MODELS OF POPULATION GROWTH, MOTION OF A SPRING; EXPONENTIAL GROWTH AND DECAY – POPULATION GROWTH, RADIOACTIVE DECAY, NEWTON’S LAW OF COOLING, CONTINUOUSLY COMPOUNDED INTEREST; PREDATOR-PRAY SYSTEMS; STRANG: SPRINGS AND CIRCUITS: MECHANICAL AND ELECTRICAL ENGINEERING)

ALBERTA MATHS 31: Calculus of exponential and logarithmic functions (Option): Students will demonstrate conceptual understanding of the calculus of exponential and logarithmic functions, by: illustrating how exponential and logarithmic functions may be used to model certain natural problems involving growth, decay and return to equilibrium.

ALBERTA MATHS 31: Applications of calculus to physical sciences and engineering (Option) – Students will demonstrate conceptual understanding of the links among calculus, the physical sciences and engineering, by: illustrating situations in which differential equations are required to represent problems; developing one or more differential equations in the areas of linear motion, simple harmonic motion, work, hydrostatic force, moments of inertia, radioactive decay or similar situations; showing that the concept of mean value can be applied to situations where the quantity varies with time, or where a range of values exists in a system of multiple bodies.

ALBERTA MATHS 31: Applications of calculus to biological sciences (Option) – Students will demonstrate conceptual understanding of the links between calculus and the biological sciences, by: illustrating how differential equations may be used to model certain biological problems involving growth, decay and movement across a boundary.

AP CALCULUS AB: Integrals and the Fundamental Theorem of Calculus – Interpret, create and solve differential equations from problems in context: The model for exponential growth and decay that arises from the statement “the rate of change of a quantity is proportional to the size of the quantity” is $dy/dt = ky$.

AP CALCULUS BC: Integrals and the Fundamental Theorem of Calculus – Interpret, create and solve differential equations from problems in context: The model for logistic growth that arises from the statement “the rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity” is $dy/dt = ky(a - y)$.

A LEVEL UNIT C4: Differentiation – Exponential growth and decay; formation of simple differential equations.

SINGAPORE H3: Calculus – Differential Equations as Mathematical Models: Mathematical models of population dynamics (including logistic growth equation, equilibrium points and their stability, harvesting and bifurcation) and of vibrating springs (including dampened vibrations, damping constant, over-damping, critical damping and under-damping, frictional force, restoring force and damping force, equilibrium position, Hooke’s Law and Newton’s Second Law of motion). Note: knowledge of conservation of energy that involves elastic potential energy, gravitational potential energy and kinetic energy is excluded.

IB MATH HL: Calculus (Option) – Real-life differential equations, eg. Newton’s law of cooling, population growth, carbon dating.

XIII. INFINITE SEQUENCES AND SERIES, POWER SERIES, MACLAURIN AND TAYLOR SERIES

(STEWART: SEQUENCES: THE FIBONACCI SEQUENCE, ADAPTATION OF LIMIT LAWS AND SQUEEZE THEOREM TO LIMITS OF SEQUENCES, APPLYING L’HOPITAL’S RULE TO A RELATED FUNCTION, BOUNDED SEQUENCES, MONOTONIC SEQUENCE THEOREM; SERIES: INFINITE SERIES, GEOMETRIC SERIES, SERIES WITH VARIABLE TERMS, TEST FOR DIVERGENCE, CONVERGENT AND DIVERGENT SERIES; THE INTEGRAL AND COMPARISON TESTS: USING THE INTEGRAL TEST, CONVERGENCE OF THE p-SERIES, USING THE COMPARISON TEST, THE LIMIT COMPARISON TEST; ESTIMATING THE SUM OF A SERIES: REMAINDER ESTIMATE FOR THE INTEGRAL TEST; OTHER CONVERGENCE TESTS: ALTERNATING SERIES TEST, ALTERNATING SERIES ESTIMATION THEOREM, ABSOLUTE CONVERGENCE, THE RATIO TEST, POWER SERIES; REPRESENTATIONS OF FUNCTIONS AS POWER SERIES: DIFFERENTIATION AND INTEGRATION OF POWER SERIES; TAYLOR AND MCLAURIN SERIES: MCLAURIN SERIES FOR THE EXPONENTIAL FUNCTION, TAYLOR POLYNOMIALS, TAYLOR’S INEQUALITY, TAYLOR REMAINDER THEOREM, OBTAINING A MACLAURIN SERIES BY DIFFERENTIATING A KNOWN SERIES, THE BINOMIAL SERIES, USING A BINOMIAL SERIES TO OBTAIN A MCLAURIN SERIES, MCLAURIN SERIES AND THEIR RADII OF CONVERGENCE, USING A SERIES TO EVALUATE AN INTEGRAL; MULTIPLICATION AND DIVISION OF POWER SERIES; APPLICATION OF TAYLOR POLYNOMIALS: APPROXIMATING FUNCTIONS BY POLYNOMIALS, APPLICATIONS TO PHYSICS; STRANG: GEOMETRIC SERIES, REPEATING DECIMALS; CONVERGENCE TESTS – POSITIVE SERIES: COMPARISON TEST AND INTEGRAL TEST, COMPARISON WITH THE GEOMETRIC SERIES, SUMMARY FOR POSITIVE SERIES; CONVERGENCE TESTS – ALL SERIES: ALTERNATING SERIES, MULTIPLYING AND REARRANGING SERIES; THE TAYLOR SERIES FOR e^x , $\sin x$ AND $\cos x$: DEFINING THE FUNCTION BY ITS SERIES; POWER SERIES: CONVERGENCE TO THE FUNCTION – REMAINDER TERM AND RADIUS r , THE BINOMIAL SERIES; APOSTOL: POLYNOMIAL APPROXIMATIONS TO FUNCTIONS: TAYLOR POLYNOMIALS, TAYLOR’S FORMULA WITH REMAINDER, L’HOPITAL’S RULE FOR THE INDETERMINATE FORM $0/0$, INFINITE LIMITS; SEQUENCES, INFINITE SERIES, IMPROPER INTEGRALS: MONOTONIC SEQUENCES, INFINITE SERIES, CONVERGENT SERIES, TELESCOPING SERIES, GEOMETRIC SERIES, DECIMAL EXPANSIONS, TESTS FOR CONVERGENCE, COMPARISON AND INTEGRAL TESTS, ROOT TEST AND RATIO TEST FOR SERIES OF NON-NEGATIVE TERMS, ALTERNATING SERIES, CONDITIONAL AND ABSOLUTE CONVERGENCE, REARRANGEMENTS OF SERIES)

ALBERTA MATH 31: Pre-calculus and Limits – defining the limit of an infinite sequence and an infinite series; calculating the sum of an infinite convergent geometric series

AP CALCULUS AB: Limits – Limits of the indeterminate forms $0/0$ and infinite/infinite may be evaluated using L'Hopital's Rule.

AP CALCULUS BC: Series – Determine whether a series converges or diverges: The n th partial sum is defined as the sum of the first n terms of a sequence; an infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S . Common series of numbers include geometric series, the harmonic series, and p -series. A series may be absolutely convergent, conditionally convergent, or divergent. If a series converges absolutely, then it converges. In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the n th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test. **Determine or estimate the sum of a series:** If a is a real number and r is a real number such that $|r| < 1$, then the geometric series [See AP Curriculum Framework p.19 for proper notation]. If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series. If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value. **Construct and use Taylor polynomials:** The coefficient of the n th-degree term in a Taylor polynomial centered at $x = a$ for the function f is $f^{(n)}(a)/n!$ Taylor polynomials for a function f centered at $x = a$ can be used to approximate function values of f near $x = a$. In many cases, as the degree of a Taylor polynomial increases, the n th-degree polynomial will converge to the original function over some interval. The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function. In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function. **Write a power series representing a given function:** A power series is a series of the form [See AP Curriculum Framework p.19 for proper notation] where n is a non-negative integer, $\{a_n\}$ is a sequence of real numbers, and r is a real number. The Maclaurin series for $\sin(x)$, $\cos(x)$ and e^x provide the foundation for constructing the Maclaurin series for other functions. The Maclaurin series for $1/1-x$ is a geometric series. A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$. A power series for a given function can be derived by various methods (e.g., algebraic processes, substitutions, using properties of geometric series, and operations on known series such as term-by-term integration or term-by-term differentiation). **Determine the radius and interval of convergence of a power series:** If a power series converges, it either converges at a single point or has an interval of convergence. The ratio test can be used to determine the radius of convergence of a power series. If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval. The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.

SINGAPORE H2: Calculus – Maclaurin's Series: Include derivation of the first few terms of the series expansion of $(1 + x)^n$, e^x , $\sin x$, $\ln(1 + x)$, and other simple functions; finding the first few terms of the series expansions of sums and products of functions, e.g. $e^x \cos 2x$, using standard series; summation of infinite series in terms of standard series; $\sin x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$, $\tan x \approx x$; concepts of "convergence" and "approximation". *Exclude:* derivation of the general term of the series.

A LEVEL UNIT C2: Sequences and Series: The sum to infinity of a convergent geometric series, including the use of $|r| < 1$. The general term and the sum to n terms is required. The proof of the sum formula should be known.

A LEVEL UNIT FP2: Maclaurin and Taylor series – Third and higher order derivatives. Derivation and use of Maclaurin series. Derivation and use of Taylor series. Use of Taylor series method for series solutions of differential equations.

SINGAPORE H2: Sequences and Series – condition for convergence of an infinite geometric series; formula for the sum to infinity of a convergent geometric series

IB MATH HL: Calculus (Option) – Infinite sequences of real numbers and their convergence or divergence. Informal treatment of limits of sum, difference, product, quotient; squeeze theorem. Divergent is taken to mean not convergent.

IB FURTHER MATH HL: Calculus (Option) – Convergence of infinite series. Tests for convergence: comparison test; limit comparison test; ratio test; integral test. The sum of a series is the limit of the sequence of its partial sums. Students should be aware that if $\lim_{n \rightarrow \infty} x_n = 0$ then the series is not necessarily convergent, but if $\lim_{n \rightarrow \infty} x_n \neq 0$, the series diverges. The p-series, $\sum \frac{1}{n^p}$, is convergent for $p > 1$ and divergent otherwise. When $p = 1$, this is the harmonic series. Series that converge absolutely. Series that converge conditionally. Conditions for convergence. Alternating series. Power series: radius of convergence and interval of convergence. Determination of the radius of convergence by the ratio test. The absolute value of the truncation error is less than the next term in the series.

Taylor polynomials; the Lagrange form of the error term. Applications for the approximation of functions; formula for the error term, in terms of the value of the $(n + 1)^{\text{th}}$ derivative at an intermediate point. Maclaurin series for e^x , $\sin x$, $\cos x$, $\ln(1 + x)$, $(1 + x)^p$, $p \in \mathbb{Q}$. Use of substitution, products, integration and differentiation to obtain other series. Taylor series developed from differential equations. Students should be aware of the intervals of convergence.

The evaluation of limits of the form $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ and $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$. Using L'Hopital's Rule or the Taylor series. The indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$. Repeated use of L'Hopital's Rule.

APPLIED MATHEMATICS

STATISTICS & PROBABILITY

CATEGORIES ELABORATED FROM THE FOLLOWING TEXTBOOKS/COURSES:

- Jeremy Orloff and Jonathan Bloom, *MIT Course 18.05: Introduction to Probability and Statistics*, Spring 2014: <http://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/index.htm>
- John A Rice, *Mathematical Statistics and Data Analysis*, 2007 (used in MIT course)

I. EXPERIMENTAL DESIGN AND SAMPLING

(RICE: METHODS OF DATA COLLECTION, POPULATION PARAMETERS, SIMPLE RANDOM SAMPLING, STRATIFIED RANDOM SAMPLING, EXPECTATION AND VARIANCE OF SAMPLE MEAN)

AP STATISTICS: Sampling and Experimentation: Overview of methods of data collection: Census, Sample survey, Experiment, Observational study, Planning and conducting surveys, Characteristics of a well-designed and well-conducted survey. Populations, samples and random selection. Sources of bias in sampling and surveys. Sampling methods, including simple random sampling, stratified random sampling and cluster sampling; Planning and conducting experiments; Characteristics of a well-designed and well-conducted experiment, Treatments, control groups, experimental units, random assignments and replication, Sources of bias and confounding, including placebo effect and blinding, Completely randomized design, Randomized block design, including matched pairs design, Generalizability of results and types of conclusions that can be drawn from observational studies, experiments and surveys.

A LEVEL UNIT S2: Hypothesis Tests: population, census and sample; sampling unit, sampling frame, advantages & disadvantages of census and sample survey.

A LEVEL UNIT S3: Sampling: Methods for collecting data, simple random sampling, other methods of sampling – stratified, systematic, quota.

SINGAPORE H1 AND H2: Sampling: concepts of population and sample, random, stratified, systematic and quota samples, advantages and disadvantages of the various sampling methods.

IB MATH STUDIES SL: Descriptive Statistics: Students should understand the concept of population and of representative and random sampling. Sampling will not be examined but can be used in internal assessment.

IB MATH SL: Statistics and Probability: Concepts of population, sample, random sample.

IB MATH HL: Core: Statistics and Probability: Concepts of population, sample, random sample.

II. DESCRIPTIVE STATISTICS

(RICE: SUMMARY STATISTICS; MEASURES OF LOCATION -MEAN, MEDIAN, INTERQUARTILE RANGE; MEASURES OF DISPERSION; GRAPHICAL REPRESENTATIONS OF DATA – HISTOGRAMS, SCATTERPLOTS, BOXPLOTS)

AP STATISTICS: Exploring Data: Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot): Center and spread, Clusters and gaps, Outliers and other unusual features, Shape; Summarizing distributions of univariate data: Measuring center: median, mean, Measuring spread: range, interquartile range, standard deviation, Measuring position: quartiles, percentiles, standardized scores (z-scores), Using boxplots. The effect of changing units on summary measures; Comparing distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots): Comparing center and spread: within group, between group variation, Comparing clusters and gaps, Comparing outliers and other unusual features, Comparing shapes

A LEVEL UNIT S1: Representation and Summary of Data: histograms, stem and leaf diagrams, box plots; measures of location – mean, median, mode; measures of dispersion – variance, standard deviation, range and interpercentile ranges; skewness, concepts of outliers.

SINGAPORE H1 AND H2: Sampling: distribution of sample means from a normal population, calculation of unbiased estimates of the population mean and variance from a sample, solving problems involving the sampling distribution.

IB MATH STUDIES SL: Descriptive Statistics: classification of data as discrete or continuous; simple discrete data, frequency tables; Grouped discrete or continuous data: frequency tables; mid-interval values; upper and lower boundaries. Frequency histograms. Cumulative frequency tables for grouped discrete data and for grouped continuous data; cumulative frequency curves, median and quartiles. Box-and-whisker diagram. (Not required: treatment of outliers.) Measures of central tendency. For simple discrete data: mean; median; mode. For grouped discrete and continuous data: estimate of a mean; modal class Measures of dispersion: range, interquartile range, standard deviation.

IB MATH SL: Statistics and Probability: Concepts of population, sample, random sample, discrete and continuous data. Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals; box-and-whisker plots; outliers. Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class. (Not required: frequency density histograms.) Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles,

percentiles. Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data. Applications. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.

IB MATH HL: Core: Statistics and Probability: Concepts of population, sample, random sample and frequency distribution of discrete and continuous data. Grouped data: mid-interval values, interval width, upper and lower interval boundaries. Mean, variance, standard deviation. (Not required: Estimation of mean and variance of a population from a sample)

III. **BASIC PROBABILITY: TERMINOLOGY, COUNTING AND SETS, CONDITIONAL PROBABILITY, INDEPENDENCE AND BAYES' THEOREM (TERMINOLOGY – EXPERIMENT, DISCRETE SAMPLE SPACE, EVENT, PROBABILITY FUNCTION; BASIC RULES OF PROBABILITY; COUNTING PRINCIPLES, SETS & NOTATION, VENN DIAGRAMS, PRODUCTS OF SETS, INCLUSION-EXCLUSION PRINCIPLE, PERMUTATIONS & COMBINATIONS; CONDITIONAL PROBABILITY, MULTIPLICATION RULE, LAW OF TOTAL PROBABILITY, USING TREES TO ORGANIZE THE COMPUTATION, INDEPENDENT EVENTS, BAYES' THEOREM, BASE RATE FALLACY)**

ALBERTA MATH 30-2: Probability: Develop critical thinking skills related to uncertainty, interpret and assess the validity of odds and probability statements, solve problems that involve: the probability of mutually-exclusive and non-mutually exclusive events, the probability of two events, the fundamental counting principle, permutations and combinations

ALBERTA MATH 30-1: Permutations, Combinations and Binomial Theorem: Counting principle, permutations and combinations

AP STATISTICS: Anticipating Patterns: Addition rule, multiplication rule, conditional probability and independence

A LEVEL UNIT S1: Probability: elementary probability, sample space, exclusive and complementary events, conditional probability, independence of two events, sum and product laws

SINGAPORE H1 AND H2: Probability: addition and multiplication of probabilities, mutually exclusive events and independent events, use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities, calculation of conditional probabilities in simple cases, use of $P(A') = 1 - P(A)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A/B) = P(A \cap B) / P(B)$

SINGAPORE H2: Permutations and Combinations: addition and multiplication principles for counting, concepts of permutation ($n!$ or nPr) and combination (nCr), arrangements of objects in a line or in a circle, cases involving repetition and restriction

IB MATH STUDIES SL: Logic, Sets and Probability: Basic concepts of set theory: elements $x \in A$, subsets $A \subset B$; intersection $A \cap B$; union $A \cup B$; complement A' . Venn diagrams and simple applications. (Not required: knowledge of de Morgan's laws); Sample space; event A ; complementary event, A' . Probability of an event. Probability of a complementary event. Expected value; Probability of combined events, mutually exclusive events, independent events; Use of tree diagrams, Venn diagrams, sample space diagrams and tables of outcomes. Probability using "with replacement" and "without replacement". Conditional probability.

IB MATH SL: Statistics and Probability: Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A is $P(A) = n(A)/n(U)$, The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams and tables of outcomes. Combined events, $P(A \cup B)$. Mutually exclusive events: $P(A \cap B) = 0$. Conditional probability; the definition $P(A|B) = P(A \cap B)/P(B)$. Independent events; the definition $P(A|B) = P(A) = P(A|B')$ Probabilities with and without replacement.

IB MATH HL: Core-Statistics and Probability: Same content as IB Maths SL, also including Use of Bayes' Theorem for a maximum of three events.

IV. DISCRETE & CONTINUOUS RANDOM VARIABLES, NORMAL DISTRIBUTION, OTHER PROBABILITY DISTRIBUTIONS (DISCRETE RANDOM VARIABLES, PROBABILITY MASS FUNCTION- PMF, CUMULATIVE DISTRIBUTION FUNCTION – CDF, GRAPHS OF $p(a)$ AND $F(a)$; SPECIFIC DISTRIBUTIONS – BERNOULLI DISTRIBUTIONS, BINOMIAL DISTRIBUTIONS, GEOMETRIC DISTRIBUTIONS, UNIFORM DISTRIBUTION, DISCRETE DISTRIBUTIONS, POISSON DISTRIBUTION; ARITHMETIC WITH RANDOM VARIABLES; EXPECTED VALUE – MEAN AND CENTER OR MASS, PROPERTIES OF $E(X)$, FUNCTIONS OF A RANDOM VARIABLE; SPREAD, VARIANCE AND STANDARD DEVIATION OF A RANDOM VARIABLE, VARIANCE OF A BERNOULLI RANDOM VARIABLE, VARIANCE OF BINOMIAL; CONTINUOUS RANDOM VARIABLES AND PROBABILITY DENSITY FUNCTIONS – PDF; EXPONENTIAL DISTRIBUTION, NORMAL DISTRIBUTION, NORMAL PROBABILITIES, SYMMETRY CALCULATIONS; PARETO AND OTHER DISTRIBUTIONS; TRANSFORMATION OF RANDOM VARIABLES, EXPECTATION, VARIANCE AND STANDARD DEVIATION OF CONTINUOUS RANDOM VARIABLES; QUANTILES FOR DISCRETE AND CONTINUOUS RANDOM VARIABLES)

AP STATISTICS: Anticipating patterns: Discrete random variables and their probability distributions, including binomial and geometric. Simulation of random behavior and probability distributions. Mean (expected value) and standard deviation of a random variable, and linear transformation of a random variable. The normal distribution, properties of the normal distribution, using tables of the normal distribution, the normal distribution as a model for measurements

A LEVEL UNIT S1: Discrete Random Variables: Concept of discrete random variable, probability function and cumulative distribution function (cdf) for a discrete random variable, mean and variance of a discrete random variable, discrete uniform distribution. **Normal Distribution:** mean, variance and use of tables of the cumulative distribution function.

A LEVEL UNIT S2: Continuous Random Variables: Concept of continuous random variable, probability density function (pdf) and cumulative distribution function (cdf), relationship between density and distribution functions, mean and variance of continuous random variables, mode, median and quartiles of continuous random variables

A LEVEL UNIT S2: Continuous Distributions: The continuous uniform (rectangular) distribution, use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction. **Binomial and Poisson Distributions:** mean and variance of binomial and Poisson distributions (no derivations will be required), the use of the Poisson distribution as an approximation to the binomial distribution

SINGAPORE H1: Binomial Distribution: knowledge of the binomial expansion of $(a + b)^n$ for positive integer n , use of the notation $n!$ and $(n \text{ above } r)$, concept of binomial distribution $B(n, p)$ and use of $B(n, p)$ as a probability model, use of mean and variance of a binomial distribution (without proof), solving problems using binomial variables (exclude calculation of mean and variance for other probability distributions). **Normal Distribution:** concept of a normal distribution and its mean and variance; use of $N(\mu, \sigma^2)$ as a probability model, standard normal distribution, finding the value of $P(X < x_1)$ given the values of x_1, μ, σ , use of the symmetry of the normal distribution, finding a relationship between $1x, \mu, \sigma$ given the value of $P(X < x_1)$, solving problems involving: normal variables, the use of $E(aX + b)$ and $\text{Var}(aX + b)$, the use of $E(aX + bY)$ and $\text{Var}(aX + bY)$, where X and Y are independent, normal approximation to binomial (Exclude: finding probability density functions and distribution functions, calculation of $E(X)$ and $\text{Var}(X)$ from other probability density functions).

SINGAPORE H2: Binomial and Poisson Distributions: concepts of binomial distribution $B(n, p)$ and Poisson distribution $Po(\mu)$; use of $B(n, p)$ and $Po(\mu)$ as probability models, use of mean and variance of binomial and Poisson distributions (without proof), solving problems involving binomial and Poisson variables, additive property of the Poisson distribution, Poisson approximation to binomial (Exclude calculation of mean and variance for other probability distributions. **Normal Distribution:** same as Singapore H1, also including normal approximation to Poisson.

IB MATH STUDIES SL: Statistical Applications: The normal distribution, The concept of a random variable; of the parameters μ and σ ; of the bell shape; the symmetry about $x = \mu$. Diagrammatic representation. Normal probability calculations. Expected value. Inverse normal calculations. (Not required: Transformation of any normal variable to the standardized normal).

IB MATH SL: Statistics and Probability: Binomial distribution. Mean and variance of the binomial distribution. (Not required: formal proof of mean and variance.) Concept of discrete random variables and their probability distributions. Expected value (mean), $E(X)$ for discrete data. Applications. Normal distributions and curves. Standardization of normal variables (z -values, z -scores). Properties of the normal distribution.

IB MATH HL: Core-Statistics and Probability: Concept of discrete and continuous random variables and their probability distributions. Definition and use of probability density functions (pdf). Expected value (mean), mode, median, variance and standard deviation. Applications. Binomial distribution, its mean and variance. Poisson distribution, its mean and variance. (Not required: Formal proof of means and variances. Properties of the normal distribution. Standardization of normal variables.

IB MATH HL: Option – Statistics and Probability: Probability generating functions for discrete random variables. Using probability generating functions to find mean, variance and the distribution of the sum of n independent random variables. Cumulative distribution functions for both discrete and continuous distributions. Geometric distribution. Negative binomial distribution. Linear transformation of a single random variable. Mean of linear combinations of n random variables.

V. CENTRAL LIMIT THEOREM AND LAW OF LARGE NUMBERS

(THE LAW OF LARGE NUMBERS, HISTOGRAMS, CENTRAL LIMIT THEOREM – STANDARDIZATION, STANDARD NORMAL PROBABILITIES, APPLICATIONS OF CENTRAL LIMIT THEOREM)

AP STATISTICS: Anticipating patterns: Central Limit Theorem; Interpreting probability, including long-run relative frequency interpretation, “Law of Large Numbers” concept.

A LEVEL UNIT S3: Estimation, Confidence Intervals and Tests: Use of Central Limit Theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions.

SINGAPORE H1 AND H2: Sampling: Use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large.

IB MATH HL: Option – Statistics and Probability: The Central Limit Theorem

VI. JOINT DISTRIBUTIONS, INDEPENDENCE

(JOINT DISTRIBUTION - DISCRETE CASE, CONTINUOUS CASE, EVENTS, JOINT CDF, MARGINAL DISTRIBUTIONS, MARGINAL PMF, PDF, CDF; INDEPENDENCE – INDEPENDENT DISCRETE AND CONTINUOUS VARIABLES)

AP STATISTICS: Anticipating patterns: Combining independent random variables: Notion of independence versus dependence, Mean and standard deviation for sums and differences of independent random variables; Sampling distribution of a sample proportion, Sampling distribution of a sample mean, Central Limit Theorem, Sampling distribution of a difference between two independent sample proportions, Sampling distribution of a difference between two independent sample means, Simulation of sampling distributions, t -distribution, Chi-square distribution.

A LEVEL UNIT S3: Combinations of random variables: distribution of linear combinations of independent Normal random variables

IB MATH HL: Option – Statistics and Probability: Variance of linear combinations of n independent random variables. Expectation of the product of independent random variables. A linear combination of independent normal random variables is normally distributed.

VII. HYPOTHESIS TESTS AND CONFIDENCE INTERVALS

(NULL HYPOTHESIS SIGNIFICANCE TESTING, ALTERNATIVE HYPOTHESIS, SIMPLE AND COMPOSITE HYPOTHESIS, SIGNIFICANCE LEVEL, POWER, P-VALUES FOR A NORMAL HYPOTHESIS, CHI-SQUARE TEST FOR GOODNESS OF FIT, TYPE I AND TYPE II ERRORS, ONE AND TWO SAMPLE T-TESTS, Z-TESTS, ONE-WAY ANOVA; Z-CONFIDENCE INTERVALS FOR THE MEAN, T-CONFIDENCE INTERVALS, CHI-SQUARE CONFIDENCE INTERVALS FOR THE VARIANCE)

AP STATISTICS: Statistical Inference: Estimation (point estimators and confidence intervals): Estimating population parameters and margins of error, Properties of point estimators, including unbiasedness and variability, Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals, Large sample confidence interval for a proportion, Large sample confidence interval for a difference between two proportions, Confidence interval for a mean, Confidence interval for a difference between two means (unpaired and paired), Confidence interval for the slope of a least-squares regression line; Tests of significance: Logic of significance testing, null and alternative hypotheses; p -values; one- and two-sided tests; concepts of Type I and Type II errors; concept of power; Large sample test for a proportion, Large sample test for a difference between two proportions, Test for a mean, Test for a difference between two means (unpaired and paired), Chi-square test for goodness of fit, homogeneity of proportions, and independence (one- and two-way tables), Test for the slope of a least-squares regression line.

A LEVEL UNIT S2: Hypothesis Tests: Concept and interpretation of a hypothesis test, null and alternative hypotheses, critical region, one-tailed and two-tailed tests, hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.

A LEVEL UNIT S3: Goodness of fit and contingency tables: The null and alternative hypotheses, degrees of freedom.

A LEVEL UNIT S3: Estimation, Confidence Intervals and Tests: Concepts of standard error, estimator, bias, the distribution of the sample mean \bar{x} , concept of a confidence interval and its interpretation, link with hypothesis tests, confidence intervals for a Normal mean, with variance known, use of Central Limit Theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions, use of large sample results to extend to the case in which the variance is unknown, hypothesis test for the difference between the means of two Normal distributions with variances known, use of large sample results to extend to the case in which the population variances are unknown.

A LEVEL UNIT S4: Quality of Tests and Estimators: Type I and Type II errors, size and power of test, the power test, assessment of the quality of estimators.

A LEVEL UNIT S4: One-Sample Procedures: Hypothesis test and confidence interval for the mean of a Normal distribution with unknown variance, hypothesis test and confidence interval for the variance of a Normal distribution.

A LEVEL UNIT S4: Two-sample procedures: hypothesis test that two independent random samples are from Normal populations with equal variances, use of the pooled estimate of variance, hypothesis test and confidence interval for the difference between two means from independent Normal distributions when the variances are equal but unknown, paired t -test.

SINGAPORE H1: Hypothesis Testing: concepts of null and alternative hypotheses, test statistic, level of significance and p-value, tests for a population mean based on: a sample from a normal population of known variance, a large sample from any population; 1-tail and 2-tail tests, exclude testing the difference between two population means

SINGAPORE H2: Hypothesis Testing: same as Singapore H1, also includes use of t -test.

IB MATH STUDIES SL: Statistical Applications: The χ^2 test for independence: formulation of null and alternative hypotheses; significance levels; contingency tables; expected frequencies; degrees of freedom; p -values.

IB MATH HL: Option – Statistics and Probability: Unbiased estimators and estimates, comparison of unbiased estimators based on variances, Null and alternative hypotheses, H_0 and H_1 . Significance level. Critical regions, critical values, p -values, one-tailed and two-tailed tests. Type I and II errors, including calculations of their probabilities. Testing hypotheses for the mean of a normal population. Confidence intervals for the mean of a normal population. Use of the normal distribution when σ is known and use of the t -distribution when σ is unknown, regardless of sample size.

VIII. COVARIANCE, CORRELATION AND REGRESSION

(PROPERTIES OF COVARIANCE, SUMS AND INTEGRALS FOR COMPUTING COVARIANCE; CORRELATION – BIVARIATE NORMAL DISTRIBUTIONS, OVERLAPPING UNIFORM DISTRIBUTIONS; FITTING A LINE TO BIVARIATE DATA USING LEAST SQUARES, LINEAR REGRESSION FOR FITTING POLYNOMIALS, MULTIPLE LINEAR REGRESSION, REGRESSION TO THE MEAN)

AP STATISTICS: Exploring Data: Exploring bivariate data: Analyzing patterns in scatterplots, Correlation and linearity, Least-squares regression line, Residual plots, outliers and influential points, Transformations to achieve linearity: logarithmic and power transformations.

A LEVEL UNIT S1: Correlation and Regression: Scatter diagrams, linear regression, Independent and dependent variables, applications and interpretations, product moment correlation coefficient, its use, interpretation and limitations.

A LEVEL UNIT S3: Regression and Correlation: Spearman's rank correlation coefficient, its use, interpretation and limitations, testing the hypothesis that a correlation is zero.

SINGAPORE H1: Correlation Coefficient and Linear Regression: concepts of scatter diagram, correlation coefficient and linear regression, calculation and interpretation of the product moment correlation, coefficient and of the equation of the least squares regression line, concepts of interpolation and extrapolation, Exclude: derivation of formulae, hypothesis tests, use of a square, reciprocal or logarithmic transformation to achieve linearity.

SINGAPORE H2: Correlation Coefficient and Linear Regression: same as Singapore H1, but also includes use of a square, reciprocal or logarithmic transformation to achieve linearity, excludes derivation of formulae, hypothesis tests.

IB MATH STUDIES SL: Statistical Applications: Bivariate data: the concept of correlation. Scatter diagrams; line of best fit, by eye, passing through the mean point. Pearson's product-moment correlation coefficient, r . Interpretation of positive, zero and negative, strong or weak correlations. The regression line for y on x . Use of the regression line for prediction purposes.

IB MATH SL: Statistics and Probability: Linear correlation of bivariate data. Pearson's product-moment correlation coefficient r . Scatter diagrams; lines of best fit. Equation of the regression line of y on x . Use of the equation for prediction purposes. Mathematical and contextual interpretation. (Not required: the coefficient of determination R^2 .)

IB MATH HL: Option – Statistics and Probability: Introduction to bivariate distributions. Covariance and (population) product moment correlation coefficient ρ . Proof that $\rho = 0$ in the case of independence and ± 1 in the case of a linear relationship between X and Y . Definition of the (sample) product moment correlation coefficient R in terms of n paired observations on X and Y . Its application to the estimation of ρ . Informal interpretation of r , the observed value of R . Scatter diagrams. The following topics are based on the assumption of bivariate normality. Use of the t -statistic to test the null hypothesis $\rho = 0$. Knowledge of the facts that the regression of X on Y and Y on X are linear. Least-squares estimates of these regression lines (proof not required). The use of these regression lines to predict the value of one of the variables given the value of the other.

DISCRETE MATHEMATICS

(CALTECH MATH 6A COURSE: INTRODUCTION TO NUMBER THEORY, CONGRUENCES, THE RSA ALGORITHM, PRIMALITY TESTING, BASIC COUNTING, GRAPHS AND THE BFS ALGORITHM, EULERIAN CYCLES, COLORING, SPANNING TREES, MATCHINGS, NETWORK FLOW, FLOWS AND BIPARTITE GRAPHS, PERMUTATIONS, GROUPS, ISOMORPHISMS AND SUBGROUPS, POWER SERIES, GENERATING FUNCTIONS, PARTITIONS, SHORTEST PATHS, LATIN SQUARES; STRANG: GRAPHS – NODES AND EDGES, DFS AND BFS, SPANNING TREES, COMPLETE MATCHING, THE GREEDY ALGORITHM)

IB MATHS HL: Discrete mathematics (Option) – Strong induction. Pigeon-hole principle. For example, proofs of the fundamental theorem of arithmetic and the fact that a tree with n vertices has $n - 1$ edges. $a|b \Rightarrow b = na$ for some $n \in \mathbb{Z}$. The theorem $a|b$ and $a|c \Rightarrow a|(bx \pm cy)$ where $x, y \in \mathbb{Z}$.

The division algorithm $a = bq + r$, $0 \leq r < b$. Division and Euclidean algorithms. The greatest common divisor, $\gcd(,)$, and the least common multiple, $\text{lcm}(,)$, of integers a and b . Prime numbers; relatively prime numbers and the fundamental theorem of arithmetic. The Euclidean algorithm for determining the greatest common divisor of two integers. Linear Diophantine equations $ax + by = c$. General solutions required and solutions subject to constraints. For example, all solutions must be positive. Modular arithmetic. The solution of linear congruences. Solution of simultaneous linear congruences (Chinese remainder theorem). Representation of integers in different bases. Fermat's little theorem. $a^p = a \pmod{p}$, where p is prime.

Graphs, vertices, edges, faces. Adjacent vertices, adjacent edges. Degree of a vertex, degree sequence. Handshaking lemma. Two vertices are adjacent if they are joined by an edge. Two edges are adjacent if they have a common vertex. Simple graphs; connected graphs; complete graphs; bipartite graphs; planar graphs; trees; weighted graphs, including tabular representation. Subgraphs; complements of graphs. It should be stressed that a graph should not be assumed to be simple unless specifically stated. The term adjacency table may be used. Euler's relation: $v - e + f = 2$; theorems for planar graphs including $e \leq 3v - 6$, $e \leq 2v - 4$ leading to the results that K_5 and $K_{3,3}$ are not planar. If the graph is simple and planar and $v \geq 3$, then $e \leq 3v - 6$. If the graph is simple, planar, has no cycles of length 3 and $v \geq 3$, then $e \leq 2v - 4$.

Walks, trails, paths, circuits, cycles. Eulerian trails and circuits. A connected graph contains an Eulerian circuit if and only if every vertex of the graph is of even degree. Hamiltonian paths and cycles. Simple treatment only. Graph algorithms: Kruskal's; Dijkstra's. Chinese postman problem. To determine the shortest route around a weighted graph going along each edge at least once. *Not required:* Graphs with more than four vertices of odd degree. Travelling salesman problem. Nearest-neighbour algorithm for determining an upper bound. Deleted vertex algorithm for determining a lower bound. To determine the Hamiltonian cycle of least weight in a weighted complete graph.

Recurrence relations. Initial conditions, recursive definition of a sequence. Solution of first- and second-degree linear homogeneous recurrence relations with constant coefficients. Includes the cases where auxiliary equation has equal roots or complex roots. The first-degree linear recurrence relation $u_n = au_{n-1} + b$. Modelling with recurrence relations. Solving problems such as compound interest, debt repayment and counting problems.

Assessment in Upper Secondary Mathematics: A Comparison between the International Baccalaureate Diploma Programme and International Qualifications

Submitted to the International Baccalaureate by UK NARIC

The National Recognition Information Centre for the United Kingdom

The national agency responsible for providing information and expert opinion on
qualifications and skills worldwide

July 2015

Commercial in confidence



recognising international expertise

Contents

Executive Summary	4
1. Introduction and Scope	7
2. Methodology	9
2.1 Methodological Process	9
2.2 Caveats and Limitations	19
3. Overview of the International Qualifications	20
3.1 International Baccalaureate	20
3.2 Advanced Placement	21
3.3 GCE A levels	23
3.4 Gāokǎo	25
3.5 Singapore-Cambridge General Certificate of Education Advanced level (GCE A level)	26
3.6 Alberta Diploma	28
4. Qualification Level Comparative Analysis	30
4.1 Qualification Purpose and Structure	30
4.2 Mathematics in Context	33
5. Programme Level Analysis	37
5.1 Programme Purpose	37
5.2 Comparison of Aims	39
5.3 Assessment Objectives	43
5.4 Assessment Methods	48
5.5 Marking	70
5.6 Grading, Performance and Progression	73
6. Key Findings: Summary and Analysis	88
6.1 Summary of Qualification Level Findings	88
6.2 Programme-Level Analysis of Assessment	89
6.3 Observations on Demand	93
7. Reference list and bibliography	95
Appendix 1: Glossary of Terms	114
Appendix 2: Programme Cohort Size for 2014	116
Appendix 3: Grade Distribution by Programme	117

Acronyms

Term	Meaning	
A level	GCE Advanced level	[UK]
A2	Second year units of the GCE A Level	[UK]
AP	Advanced Placement	[USA]
AS	Advanced Subsidiary	[UK]
AS level	GCE Advanced Subsidiary level	[UK]
CIE	University of Cambridge International Examinations	[Singapore]
DM	Discrete Mathematics	[IB]
DPCR	Diploma Programme course results	[IB]
FRQ	Free Response Question	
GCE	General Certificate of Education	[UK]
GCSE	General Certificate of Secondary Education	[UK]
H1	Higher 1	[Singapore]
H2	Higher 2	[Singapore]
H3	Higher 3	[Singapore]
HL	Higher Level	[IB]
IB DP	International Baccalaureate Diploma Programme	[IB]
IBO	International Baccalaureate Organisation	[IB]
IP	Integrated Programme	[Singapore]
MCQ	Multiple Choice Question	
MYP	Middle Years Programme	[IB]
Ofqual	The Office of Qualifications and Examinations Regulation	[UK]
QCF	Qualifications and Credit Framework	[UK]
SEAB	Singapore Examinations and Assessment Board	[Singapore]
SIPCAL	The Singapore-Cambridge A Level	[Singapore]
SE	Calculus	[IB]
SL	Standard Level	[IB]
SG	Sets, Relations, and Groups	[IB]
SP	Statistics and Probability	[IB]

Executive Summary

Recognising the value of international comparisons to inform qualification development, the International Baccalaureate (IB) commissioned UK NARIC to undertake a study on the assessment of mathematics in its Diploma Programme (DP) in the context of five international qualifications.

As such UK NARIC has conducted an objective comparative analysis of the four mathematics programmes currently offered within the IB DP (Mathematical Studies Standard Level (SL); Mathematics SL; Mathematics Higher Level (HL); and Further Mathematics HL) with the Advanced Placement in the USA, the GCE A Level in the UK, the Singapore-Cambridge A Level in Singapore (hereafter referred to as SIPCAL), the Alberta Diploma in Canada, and the Gāokǎo in China¹, based wholly on documentation available in the public domain.

The study focused on the assessment of the mathematics programmes offered within each international qualification, with particular attention to the assessment objectives, assessment methods, and marking guidelines. A detailed review of past exam papers, where available, or released past questions in the absence of full papers, enabled an in-depth comparison both in terms of factual information such as duration, number and types of question, weighting, as well as a comparison of individual questions on five selected topics: differentiation; sequences and series; trigonometry; vectors and probability. Whilst the qualifications reviewed varied in structure – encompassing three single-subject qualifications, two diploma- or baccalaureate- style qualifications, and one university entrance exam – all offered more than one mathematics programme. As such, for the purpose of this study, UK NARIC has reviewed those considered to be most relevant to the IB DP in terms of broad content and focus so as to facilitate the comparative analysis of assessment.

It is important to note firstly that, as a study designed to inform the IB's own review and development, this study seeks to provide an outward referencing of the DP assessment to determine how the DP compares to international qualifications in its assessment of upper secondary mathematics. No judgement of quality is made or intended on the international qualifications in isolation.

Secondly, it is important to acknowledge that the concept of comparability for the purposes of recognition for further study or employment encompasses considerations beyond assessment. As such, the report does not seek to comment on the overall comparability of the qualifications under review.

The study found many key similarities and differences between the qualifications reviewed and their corresponding mathematical programmes. The qualifications reviewed for this study are all taken at upper secondary school level and include a similar amount of prior study; 12 or 13 years of education. Whilst many of the qualifications examined are designed

¹ Given that the Gāokǎo is organised and administered at a provincial level, this study focussed on the assessment of mathematics within the Gāokǎo set in Shanghai province.

primarily to provide and assess upper secondary school education, all are widely accepted for admission to higher education in their respective countries, at institutional discretion.

It is clear that the constituent mathematics programmes seek to provide a solid preparation for further study in higher education, whether for:

- Degrees involving a substantial mathematics component (IB DP Mathematics HL, GCE A Level Mathematics, SIPCAL Higher 2 (H2) Mathematics programme and the Alberta Diploma Mathematics 30-1)
- Degrees in areas such as business, economics and social sciences where students would benefit from a solid background in mathematics but would not necessarily need advanced calculus / pure mathematics (IB DP Mathematics SL, SIPCAL H1 Mathematics, and again the GCE A Level Mathematics)
- Degrees in arts, humanities and other areas, where an understanding of mathematics application in everyday life would nevertheless be of benefit to the student (IB DP Mathematical Studies SL and the Alberta Diploma Mathematics 30-2).

Three programmes (the IB DP Further Mathematics HL, GCE A Level Further Mathematics and the SIPCAL H3) are intended specifically for those who have a strong aptitude and enthusiasm for mathematics, and whilst these would all be useful for those looking to study mathematics or mathematics-related degrees, are not typically a pre-requisite for admission.

Most of the programmes reviewed also assessed across a range of mathematics topics, where much or all of the content was core: the exception to this is the AP which offers three specialised programmes (two in calculus, one in statistics).

All programmes assess students through one or more external written examinations, conducted under exam conditions. The Alberta Diploma and IB DP programmes, with the exception of Further Mathematics HL, also include internal assessment however in the absence of documentation on internal assessment, the analysis focusses exclusively on external assessment.

The most notable differences in assessment observed between the mathematics programmes pertain to:

- Question types: both Alberta Diploma programmes (Mathematics 30-1, and 30-2), the AP and to a lesser extent the Gāokǎo, employ multiple choice questions which are not used at all within the IB DP mathematics exams. The IB DP and GCE A Level programmes, by contrast, primarily use multi-part, free response questions. The Gāokǎo and the AP also use free-response questions, whilst the Alberta Diploma has short numerical response questions.
- Marking approaches. Directly relevant in some cases to the question types, was the varying emphasis on accuracy versus method marks. Throughout the IB DP mathematics exams, along with those set for the GCE A Levels, students demonstrating their workings can obtain method marks for a correct or appropriate approach, independent of whether they reach the correct answer. This is similarly true for the AP free-response questions which account for 50% of the overall grade.

In programmes where only accuracy marks are awarded; a mathematical error in a student's calculations would have a greater impact on their marks than is the case in the IB DP exams.

- Grading systems. There was considerable variation in the number and range of distinct grades awarded for each programme, and not all included the concept of a pass or fail. Overall the IB DP mathematics programmes allow for the finest level of differentiation in student achievement by grade, which can be considered beneficial for facilitating identification of the highest levels of student achievement for competitive higher education admissions purposes.

The study also sought to consider the extent to which the IB DP programmes could be considered to assess mathematics with a comparable level of demand. In doing so, it identified that across the programmes examined, different factors have contributed to the demand of assessment. Whilst some exams may have a higher proportion of questions judged to be of medium or high demand, they may also enable students of varying abilities to obtain some marks for demonstrating appropriate workings, whilst in others, the demand of individual questions may be lower but students must reach the correct answer in order to obtain marks. Similarly some programmes – most notably the IB DP SL programmes and the GCE A Levels – had questions pitched at a range of levels to enable the typical threshold student (for example, those at UK grade E) to obtain some marks across the majority of questions, whilst having a proportion that would also test students expected to achieve at the top grade(s).

Overall, it is evident that the IB DP mathematics programmes demonstrate clear similarities to each of the international programmes examined, whether in terms of aims, assessment objectives, methods and marking; whilst not mirroring any one programme. Combining both a detailed programme and assessment-level analysis with wider contextual considerations outlined above, it is clear that the IB DP assesses mathematics at a level and breadth consistent with the individual purpose for which each of its four mathematics programmes were designed; assessing with a broadly comparable level of demand to one or more of the international programmes.

Copyright © 2015 UK NARIC

All rights reserved. Short sections of text may be quoted without express permission provided that full credit and reference are given to the source. For any permissions enquiries, please contact UK NARIC using the address below.

Published by:
UK NARIC
Oriol House, Oriol Road
Cheltenham, Gloucestershire
United Kingdom
GL50 1XP

Email: alyssa.templer@naric.org.uk / projects@naric.org.uk
Telephone: +44 (0)1242 694723 Fax: +44(0)1242 258611

1. Introduction and Scope

International comparisons and benchmarking studies have long been used to inform curricula and qualification review and development. With experience in evaluating, benchmarking and mapping international qualifications, UK NARIC was commissioned by the International Baccalaureate (IB) to conduct a comparative study to help inform the review and development of its Diploma Programme (DP) courses in mathematics. The IB is a non-profit educational foundation that offers four programmes delivered internationally in authorised schools, known as IB World Schools. The four programmes include the Primary Years Programme (PYP), Middle Years Programme (MYP), Diploma Programme (DP), and Career-related Programme (CP).

This report examines the assessment of mathematics in upper secondary level qualifications, providing an independent and objective comparative analysis of the DP mathematics courses in the context of five international qualifications, selected by the IB:

Table 1: Qualifications reviewed in this study

IB DP Mathematics Courses	International Qualifications
<ul style="list-style-type: none">• Mathematical Studies Standard Level (SL)• Mathematics SL• Mathematics Higher Level (HL)• Further Mathematics HL.	<ul style="list-style-type: none">• The Advanced Placement, USA• The GCE A Level, UK• The Singapore-Cambridge A Level, Singapore²• The Alberta Diploma, Canada• The Gāokǎo, China.

In doing so, this report seeks to establish how the assessment for each of the IB DP mathematics courses compares to that in the five selected international qualifications in terms of:

- Assessment objectives
- Assessment methods used and, where more than one method is used, the weighting of each assessment in relation to the overall grade
- Exam papers (including factual information such as duration; structure; format, question types and weighting; relevant rules, where published; and objective comparison of assessment items)
- Marking schemes and guidelines.

Acknowledging that the principal purpose of this study is to help inform the IB's review of its DP mathematics programmes, the report will present an "outward" referencing, examining how the IB compares to international qualifications in its assessment of secondary mathematics, highlighting similarities and differences in the assessment methods and objectives, and crucially, the relative level of demand of the DP assessments. No judgement of quality is made or intended on the international qualifications in isolation.

² Hereafter referred to as the SIPCAL.

Structure of the Report

Section 2 describes the methodology developed and used for carrying out this comparative study.

Section 3 provides an overview of the IB DP and five international qualifications and identifies the mathematics programmes available within them at the time of writing.

Section 4 provides a comparative analysis at qualification level, examining the purpose, structure, duration and other orientating factors of each award to contextualise the mathematics programmes reviewed in Section 5.

Section 5 compares the mathematics assessment in the programmes including assessment objectives, assessment methods, marking and grading.

Section 6 examines the key findings from the comparative analysis and provides conclusions on the demand of the assessment in the IB DP mathematics courses in the context of the international programmes.

Section 7 includes a bibliography of the materials and references used to inform the study.

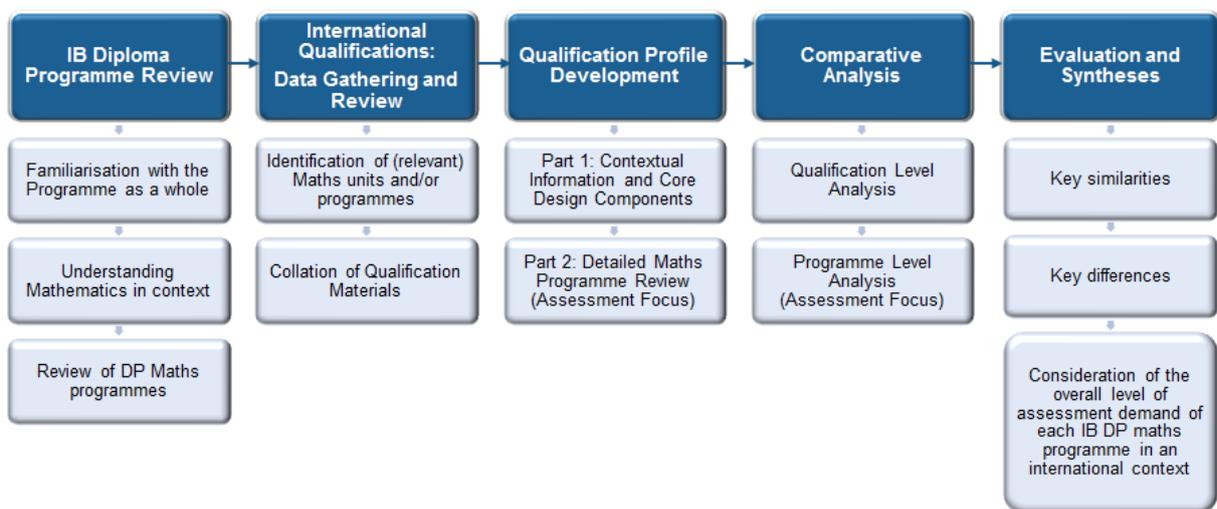
2. Methodology

2.1 Methodological Process

An inherent challenge in international comparison studies is in trying to compare, what can be, very different qualifications. To arrive at a point at which reliable comparisons could be drawn between the international awards; it was important to thoroughly explore each qualification and its mathematics programme(s) in isolation, building up comprehensive and objective profiles.

The process undertaken by UK NARIC in conducting this study can be illustrated as follows:

Figure 1: Methodological process



At this point, it is important to highlight a terminology point. For the purposes of this study, “qualification” has been used to refer to the overall qualification awarded (e.g. IB Diploma Programme, Alberta Diploma, GCE A Level), whilst “programme” has been used to refer to the distinct mathematics course/assessment reviewed (GCE A Level Mathematics, [IB DP] Mathematical Studies SL, AP Calculus AB).

2.1.1 IB Diploma Programme Review

The first step of the study was to review the DP programme documentation to ensure a thorough understanding of the programmes and inform the identification of the most suitable mathematics programme(s) in each system during the next stage. The documentation gathered from the IB included, but was not limited to:

- Examination papers and mark schemes from the May 2012-2014 exam sessions
- Teacher and student guides
- Formula booklets
- Subject guides, outlines, and reports
- Grade distribution data
- Documents related to regulations and policies
- DP guides and informational documents.

A full list can be found in the [Bibliography](#).

2.1.2 International Qualifications Review

In order to review the international qualifications, documentation similar to that gathered from the IB needed to be collated for each international qualification and its mathematical programmes. To begin, awarding bodies or government websites were reviewed and any publically available documentation or information was collected. Using this information, or sources provided through the awarding bodies or governments, additional websites providing information pertaining to the international qualifications were reviewed with any relevant information collated. Where documentation was lacking, which was the case for most of the awards, an overall web search was completed for any remaining websites containing information related to the international qualifications. The main documents searched for included:

- Examination papers and mark schemes
- Student or teacher handbooks/guides
- Grade distribution data.

Please note that the GCE A Level is separately awarded by different awarding bodies in the UK, and the Gāokǎo is awarded at a provincial level in China. For the purposes of this study, UK NARIC agreed, in consultation with the IB, to focus on the assessment of the GCE A Level and Gāokǎo from a single examining body and province: Edexcel (Pearson Education Ltd) and Shanghai respectively.

2.1.3 Qualification Profile Development

To inform the comparative analysis, a qualification profile was completed for the IB DP Mathematics programmes as well as the five international qualifications selected.

The profile comprised two parts, as follows:

Table 2: Qualification profile data requirements

Part 1: Contextual Information and Core Design Components					
Qualification level				Mathematics in Context	
Entry requirements	Duration	Structure and Content	Associated Outcomes	Compulsory / optional as part of the qualification	Mathematics programme(s) available
Part 2: Detailed Mathematics Programme Review (Assessment Focus)					
Purpose and Aims	Structure and Scope of Content (General vs specialised)	Assessment Objectives	Assessment Methods	Marking Methodology	Grading, Performance and Progression

The intention of Part 1 was to provide an overview of the international qualification, to contextualise the mathematics programme(s) reviewed; whilst Part 2 provided detailed information to enable comparisons.

2.1.4 Comparative Analysis: Factors and Considerations

The comparative analysis involved comparing each of the IB DP Mathematics programmes with the international counterparts considering factual information gathered, with the aim of highlighting similarities and differences. Some aspects of the comparative analysis were, by their very nature, more subjective. It is important however to highlight that UK NARIC, both in its capacity as a national agency and as a credential evaluation organisation rather than awarding body, undertook a fully independent and objective analysis.

2.1.4.1 Qualification Level Analysis

Purpose and Other Orientating Factors

Qualification purpose, entry level and associated outcomes are useful considerations in evaluating and comparing qualifications. In this study, focusing on secondary education qualifications, the review sought to establish the following contextual information:

- **Whether the qualifications are the first exit qualifications in their respective systems.** Qualifications representing the first exit qualification for school students may encompass a wider range of academic abilities than one with selective entry or other entry requirements. This is important information to understand, for example, when later considering student performance (grade distribution).

Similarly, it is useful to understand the duration of the qualification. Although neither the sole nor primary indicator of academic level, considered in conjunction with the entry requirements it can nevertheless provide an indication of the volume, depth and breadth of study that can be completed within the specified timeframe of the programme.

- **The purpose of the qualification**, for example a general upper secondary level leaving award or one designed specifically to prepare for or provide access to higher education.
- **The style of award.** It was important to identify whether a given qualification was a single-subject award or a baccalaureate / diploma style qualification. This, considered in tandem with the number of subjects a student would expect to study and be assessed in, provides an indication of the likely subject breadth and depth that might be expected from each qualification.

Mathematics Programmes in Context

This summarised the mathematics programme(s) available within each international qualification. An initial scoping for this study had shown that the awarding bodies either offered:

- A general mathematics programme in which all mathematics topics were core;
- A general mathematics programme encompassing a common core of mathematics topics and a range of electives;
- One or more specialised programmes focussing on a specific area of mathematics.

It was anticipated that these aspects would be discussed at length within the parallel curriculum analysis being conducted by the IB; however in the context of this study it was agreed that where multiple programmes, or multiple options within programmes were available; the assessment analysis would focus on those programmes most relevant in content to the IB.

2.1.4.2 Programme Level Analysis

Having gained a solid understanding of each qualification, the next step was to centre in on the mathematics programmes offered in each qualification.

Purpose and Aims of Each Mathematics Programme

Any reviews of assessment methods and items need first to be contextualised by an understanding of what the awarding body is looking to achieve as part of the programme. Since it can be reasonably expected that a programme's aims should be reflected throughout the programme specifications and assessment (from content, learning outcomes, assessment methods and marking practices), it follows that identifying the aims of each mathematics programme was an important step in this study.

The international qualification aims were initially isolated and reviewed to identify the key themes, whilst bearing in mind variations in the wording and expression of similar aims between qualifications. Each theme was intended to highlight a distinct skill area or behavioural attribute which features in the aims of one or more of the international qualification specifications. Where similar or shared aims were identified between the IB DP and one or more international programmes, the wording used for the themes reflects the IB DP wording, both for ease of reference and due to permissions restrictions (as detailed in Section 2.2); for other themes, not covered by the IB DP aims, the international programme aims have been paraphrased. Aims specified in each individual qualification specification were then mapped against the identified international themes to gauge the extent of their coverage, the results of which are presented in a table. A tick represents where a specific qualification's aims cover an identified theme.

Assessment Objectives

In general terms, assessment objectives set out the key skills and competencies to be evaluated in the assessment of a programme, which may include coursework and

examinations. They are typically a specification-level factor as they intend to set out to teachers, students and examiners, the expected skills and / or content to be covered in the assessment.

From experience, UK NARIC was aware that the format, scope and even the existence of prescribed assessment objectives can vary from one international qualification to the next. This was certainly the case with the international qualifications reviewed as part of this study. In particular, it was apparent that outcomes – specifically the skills and aptitudes expected to be developed throughout a course of study – and assessment objectives were sometimes used interchangeably. In theory, learning outcomes are used mainly to describe skills acquired through the learning process rather than skills to be assessed, however in practice outcomes and assessment objectives can cover broadly similar themes, which include the mathematical and broader cognitive skills required to be successful in a particular programme or assessment.

Equally, there was also sometimes an overlap between the use of assessment objectives and learning aims. In most programmes, learning aims are distinct from assessment objectives and outcomes, as they intend to set out the teaching objectives and include broader intentions separate from the assessment, such as behavioural attributes or developing appreciation and awareness of mathematical ideas. However, in the AP Calculus AB and BC specifications, aims fulfil a similar role to assessment objectives used in other upper secondary programmes in that they are competency based and closely linked to assessment. In this instance, therefore AP aims have been selected as the basis for comparison.

As such this section of the analysis sought to identify key themes evident within some or all of the programmes that would support or contextualise the wider analysis of assessment.

A comparative review of the international qualification assessment objectives was initially conducted to identify themes covered by the different specifications, at the same time acknowledging differences in wording in the expression of similar skills and objectives across qualifications. Each theme was intended to highlight a specific cognitive skill or ability mentioned in one or more of the selected qualification specifications. Having developed the overarching set of key themes, the assessment objectives of each individual international qualification were mapped against the identified themes. The results of the mapping exercise are presented in a table, with a tick indicating where an international qualification's objectives explicitly references a specific theme.

Assessment Methods and Demand

This aspect of the analysis formed a substantial part of the study, comparing assessment methods at three levels:

- Overall assessment methods for mathematics in each programme
- The specific assessments conducted for each programme (reviewing whole papers, where available)
- Assessment items.

Overall Assessment of Mathematics

This section examined and compared how each mathematics programme would be assessed, identifying the following information:

- Whether assessment was internal, external or a combination of the two (and if so, the relative weighting of each needed to be considered).
- The methods of summative assessment used. For example, written examinations under test conditions; open-book examinations; coursework; other.

Comparative Review of Assessment Materials

This involved a review of (whole) external assessments conducted for each programme, where available³. Exam papers from the last three years were sought to identify any considerable differences from one year to the next in terms of assessment format. The 2014 papers from each programme, where identified, were reviewed and compared in the following aspects:

- The type(s) of questions used and how/whether questions were weighted (mark allocation).
- Assessment duration in conjunction with the number and type of questions to be answered within that timeframe: specifically identifying whether the time constraints are considered to be considerably stricter or less demanding from one programme to the next.
- The element of choice: whether all questions were compulsory or whether students were offered a choice from a range of questions.
- The total number of marks available.
- Any resources available to the student (e.g. formula sheet, calculator).
- Whether students were expected to show their workings out (also discussed in greater detail during the Marking section).
- Content breadth and depth covered within the assessment. Reviewing the programme specifications, it was possible to broadly gauge the scope, breadth and depth of subject content taught within each qualification⁴, which could then be further examined within the subsequent examination paper analysis; including whether papers assess one particular area of mathematics or assess a range of mathematical topics. Any comparisons of assessment item demand need to be contextualised by the content breadth and depth of the respective programme. For example, it might be expected that an international programme focusing in-depth on a particular topic of mathematics may (though not necessarily) pose more questions, and of a more complex nature, on a given topic than those programmes designed to test a broad range of mathematics topics.

³ As outlined in the Assessment Methods section, few programmes reviewed used internal assessment and where used, the information on these was very limited and as such not considered within the study.

⁴ Topics covered, and the complexity of these, were also considerations though to some extent this aspect overlaps with the IB's parallel study in curriculum comparisons. Ofqual highlighted for example that the exclusion of topics such as trigonometry decrease the level of demand.

Comparative Review of Assessment Items

The focus of this review was on comparing questions from the different qualifications to understand the demands placed on the student by each.

The following five topic areas were identified for analysis:

- Differentiation. Identified as a common pure maths / calculus sub-topic across the international qualifications.
- Sequences and series. A common principal or sub-topic across the international qualifications.
- Probability. Often covered as a statistics sub-topic or as part of a major statistics and probability topic.
- Vectors. Covered as a pure maths topic, and additionally within mechanics units for the UK GCE A Level.
- Trigonometry.

As can be seen from the table below, the first four topics are covered in many of the mathematics programmes and in all of the qualifications. Trigonometry, though less widely covered, has been identified in prior studies as a topic which can increase demand in mathematics assessment⁵.

⁵ Ofqual, 2012. International Comparisons in Senior Secondary Assessment Full Report. [pdf] Published by: Office of Qualifications and Examinations Regulation in 2012. Available at: <https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/372211/2012-06-12-international-comparisons-in-senior-secondary-assessment.pdf> [Accessed 1st April 2015]..

Table 3: Sample topics for assessment comparison

Qualification	Programme	Topic				
		Differentiation	Sequences and/or Series	Trigonometry	Vectors	Probability
IB DP	Mathematical Studies SL	✓	✓	✓	-	✓
	Mathematics SL	✓	✓	✓	✓	✓
	Mathematics HL	✓	✓	✓	✓	✓
	Further Mathematics HL	✓	✓	-	✓	✓
Advanced Placement	Calculus AB	✓	-	-	-	-
	Calculus BC	✓	✓	-	-	-
	Statistics	-	-	-	-	✓
GCE A level	A Level Mathematics	✓	✓	✓	✓	✓
	A Level Further Mathematics	✓	✓	-	✓	✓
SIPCAL	H1	✓	-	-	-	✓
	H2	✓	✓	-	✓	✓
	H3	✓	✓	-	-	-
Gāokǎo	Arts Stream	-	✓	✓	✓	✓
	Science Stream	-	✓	✓	✓	✓
Alberta Diploma	Mathematics 30-1	-	-	✓	-	✓
	Mathematics 30-2	-	-	-	-	✓

As such, questions related to these topic areas were selected for the IB DP and international qualifications, drawing on past examination papers from 2014, where available. Where full papers were not available in the public domain, relevant published sample questions were considered. No exam papers or sample questions were found for the SIPCAL.

For each programme, an exam question sheet was created for each of the five topics, where covered, listing all the exam questions relevant to that topic from the year in question. A two-step review process was then undertaken: the first to grade each question according to demand.

The demand of each question was identified as Low, Medium or High, or on occasions as borderline between these two (Low-Medium; Medium-High), taking into consideration the following aspects:

- **The use of single- and/or multiple-part questions** and the extent to which these provide students with a strategy for response or require students to create their own.
- **The complexity / level of depth at which the topic was tested.**
- **The level of resources (or direction) given:** to what extent is the information needed to answer the question given in the question / exam paper itself.
- **In the case of multiple choice questions, the number and plausibility of distractors:** to what extent do these either help students get to the right answer or provide highly plausible, incorrect answers that would trip up many students.

The second step was to calculate the distribution of marks to questions of low / medium / high for the topic within each paper and each programme. This approach was taken due to the significant differences in the number of questions and marks per programme. This enabled comparisons to be drawn about the proportion of high demand questions per programme as well as the range of low / medium / high demand questions.

Marking

To support the review of assessment items and exam papers, it was important to consider how students were marked in the assessments; identifying any similarities and/or differences in the way in which marks are awarded and, where appropriate, considering how any differences might be reflected in the relative level of demand of the programme. As such three initial questions were posed:

- **Whether papers were machine- or human- marked.** Where human-marked, marking guidelines were reviewed, where available, to understand the level of response, knowledge and skills expected of students.
- **Whether students were expected to show their workings out** and if so, whether failure to do so would impact on the marks awarded.
- **Similarly, whether marks awarded for correct answers (accuracy) were conditional on or independent of method marks.** Specifically, whether students would:
 - be awarded marks for getting an incorrect answer if they had demonstrated understanding of the correct process.
 - be awarded marks for using an alternative method.

Grading, Overall Performance and Progression

Whilst reviewing assessment design and materials provides a significant insight into assessment methods and demand, it is important to also consider how the assessment and marking work in practice to test and differentiate student performance. For this aspect of analysis, there were three key points of reference:

- **Grading systems used.**
This identified the grades used in each system to reflect different levels of student achievement.

- **Quantitative data on student performance.**
Where available, this included grade distribution data, showing the numbers of students achieving each grade over a period of, ideally, five years.
- **Qualitative data on student progression.**
This involved identifying the DP and international qualifications grades typically specified for higher education admission by taking a sample of admission policies from universities in each country. General admission requirements as well as admission requirements for STEM subjects were reviewed to determine (i) whether the qualification(s) in question would be accepted and (ii) if so, what grade would be needed for admission.

2.1.5 Evaluation and Synthesis

The purpose of this stage was to draw on the findings of the comparative analysis to document:

- **A summary of key qualification-level (contextual) findings.**
- **The key similarities in assessment** between the IB DP and international programmes, outlining areas where the IB DP programmes compare closely to one or more of the selected international qualifications.
- **The key differences in assessment** between the IB DP and international programmes, from which the IB DP could consider whether there was a need or opportunity for further development; or indeed to identify areas of strength or alternative provision.
- **Whether, in terms of assessment, each IB DP programme could be considered to offer a comparable level of demand to one or more of the international programmes.**
This considered which demand factors were observed in the programmes.

2.1.5.1 Defining Demand and its Contributing Factors

It is important to first define what is meant by ‘demand’ in the context of assessment, and specifically in the context of this study. In view of the scope and aims of this project, it is necessary to consider both what is assessed and what is marked. In essence, in order to understand what is expected of students, it is necessary not only to investigate what is *asked* of them (through critical review of examination and other summative assessments, where available, including breadth, depth, complexity and cognitive demand of assessment items, duration of exam) but what is *expected* of them in order to achieve a passing grade (through critical review of marking guidelines / practices).

2.2 Caveats and Limitations

A few limitations were encountered at various stages of the project, mostly within the data collection process for the international qualifications. In line with the IB's project specifications, this report examines, for each international qualification, only documentation which is publically available. For some of the international qualifications, many documents were available in the public domain; however this was not the case for the SIPCAL for which no examination papers, mark schemes, or sample questions were available. As such the analysis for the SIPCAL addresses the aims, purpose, assessment objectives, and associated outcomes and progression for SIPCAL holders.

Copyright laws were another limitation for some of the documents found in the public domain. Although the information could be included in our review and analysis, they could not be quoted as permissions could not be obtained without contacting the bodies.

When collating grade distribution data for the international qualifications, we were also limited to what was available in the public domain. This meant that some of the data did not cover the full five years, or, in the case of the Alberta Diploma, only applied to the performance standards and not the final mark.

For the international qualifications that did not provide full examination papers in the public domain, sample questions or released exam items were used. Although these questions did not allow for an analysis of a full examination paper, they did contribute to the analysis of the assessment items by demonstrating the types of questions posed in the international programmes exam and the cognitive demands placed on the students. However, the questions did not demonstrate the full range and number of questions in the exam. This limitation was accounted for in our findings. Full examination papers were only available for the IB DP, GCE A level, and the Gāokǎo.

Although the limitations and caveats must be acknowledged and discussed, it is important to note that the analysis worked around these and provided valuable key findings using the available information.

3. Overview of the International Qualifications

3.1 International Baccalaureate

3.1.1 Overview

The DP is a two year baccalaureate-style programme, offered to students internationally between the ages of 16-19. No specific entry requirements are set for the DP, due to the different countries, and therefore education systems, it is offered within. For some students the DP may be their first exit qualification for secondary school, but this varies between countries⁶. Students are able to transition from the MYP to the DP, however not all IB World Schools offer both the MYP and DP, so some students enter the DP programme with no previous experience in IB courses⁷.

The associated outcomes of the qualification vary depending on the country in which a student is looking to progress their education, though the DP is widely accepted for undergraduate admissions⁸.

The curriculum is made up of six subject groups and the DP core. The DP core includes three compulsory elements: Theory of Knowledge; the extended essay; and creativity, action, service⁹. Students then choose at least one course from each of the following subject groups:

- Studies in language and literature
- Language acquisition
- Individuals and societies
- Sciences
- Mathematics
- The arts¹⁰.

Courses are offered at either higher level (HL) or standard level (SL), with each providing a different scope. For instance, HL students need to demonstrate greater knowledge, understanding and skills than those in SL. Students must select at least three courses (four maximum) at HL, while the rest are taken at SL¹¹.

⁶ Students may also sit exams in different languages which may or may not be their native language. Different languages are available depending on the subject group, course, and examination series. Group 5 Mathematics courses are available in English, French, and Spanish for the May 2015 exam series.

⁷ International Baccalaureate Organization, 2014. Handbook of procedures for the Diploma Programme 2014. Internal International Baccalaureate Organization document.

⁸ International Baccalaureate Organization, 2015. FAQs. [online] Available at: <<http://www.ibo.org/en/university-admission/recognition-of-the-ib-diploma-by-countries-and-universities/faqs/>> [Accessed 13th April 2015].

⁹ International Baccalaureate Organization, 2015. Curriculum. [online] Available at: <<http://www.ibo.org/en/programmes/diploma-programme/curriculum/>> [Accessed 7th April 2015].

¹⁰ Students have the option to study an additional sciences, individuals and societies, or languages course, instead of a course in the arts.

¹¹ International Baccalaureate Organization, 2015. Curriculum. [online] Available at: <<http://www.ibo.org/en/programmes/diploma-programme/curriculum/>> [Accessed 7th April 2015].

It is also possible for students not enrolled in the full DP to take individual DP subject courses of their choosing (when offered and allowed at their school), undertaking the same assessment in that subject as those taking the full DP. These students do not receive the award of Diploma, but rather individual scores referred to as Diploma Programme course results (DPCR). Students enrolled in the full DP but that do not meet the minimum score requirements, or that fail to meet the other minimum requirements for the Diploma, will also receive the DPCR¹².

3.1.2 Mathematics in Context

As seen above, at least one programme must be selected from the mathematics subject group. Students have the option to study:

- Mathematics SL
- Mathematics HL
- Further Mathematics HL
- Mathematical Studies SL.

In line with the scope and objectives of the project, each of the above four programmes will be reviewed within the comparative analysis.

3.2 Advanced Placement

3.2.1 Overview

The Advanced Placement (AP) programme is developed and awarded by the CollegeBoard, but delivered in many schools in over 100 countries and territories around the world¹³. Students enrol in the course taught by AP approved teachers, and take a final exam developed and marked by the CollegeBoard.

There are no formal entry requirements for an AP programme, however some subjects may include a pre-requisite in that subject¹⁴. Students in the USA would take this course alongside other high school classes leading to the first exit qualification for secondary school students, the high school diploma. Students often take AP classes in their 11th or 12th year of education, with some taking them in the 9th or 10th year.

Most AP programmes are run over the course of one academic school year, but this can vary by course and school. Students are also able to take different courses over different years within their education.

¹² International Baccalaureate Organization, 2014. International Baccalaureate Diploma Programme: A guide to assessment. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/assessmentbrief_web.pdf> [Accessed 1st May 2015].

¹³ The College Board, 2015. AP Around the World. [online] Available at: <<https://apstudent.collegeboard.org/creditandplacement/ap-and-college/ap-around-the-world>> [Accessed 20th April 2015].

¹⁴ The College Board, 2012. Calculus: Calculus AB Calculus, BC Course Description Effective Fall 2012. [pdf]. Available at: <<http://media.collegeboard.com/digitalServices/pdf/ap/ap-calculus-course-description.pdf>> [Accessed 20th April 2015].

There is no set structure for the AP programme overall and therefore no mandatory or required courses, students are able to take as many or as few courses as they would like. The number they enrol in is determined in discussion with their school counsellor. Students can undertake the course without sitting the examination however no award would be granted. Equally, students can take AP examinations without being enrolled in AP courses¹⁵.

Currently, there are 37 AP subjects, falling under the following subject groups:

- AP Capstone
- Arts
- English
- History and Social Science
- Math and Computer Science
- Sciences
- World Languages and Cultures¹⁶.

A course description is developed for each AP programme with information included on the content, goals, and sample examination questions. The programme is then taught by AP teachers who are responsible for developing the curriculum¹⁷.

In some schools, students have the option to combine specific subjects to receive an award. The AP Capstone subjects, AP Seminar and AP Research, can result in an AP Seminar and Research Certificate. When the two subjects are also taken in conjunction with four other AP courses and exams, the student may receive an AP Capstone Diploma¹⁸.

In terms of purpose, each AP course is designed to be representative of the content covered in introductory level university courses: the AP courses and exams do not provide direct entry to undergraduate admission in the USA. AP can be used for course credit allowing a student to skip introductory courses in their first year¹⁹.

¹⁵ The College Board, 2014. AP® Bulletin for AP Students and Parents 2014-2015. [pdf]. Available at: <https://secure-media.collegeboard.org/digitalServices/pdf/professionals/2014-15_ap_student_bulletin.pdf> [Accessed 28th April 2015].

¹⁶ The College Board, 2015. AP Courses. [online] Available at: <<https://apstudent.collegeboard.org/apcourse>> [Accessed 22nd April 2015].

¹⁷ The College Board, 2015. AP Central. [online] Available at: <<http://apcentral.collegeboard.com/home>> [Accessed 20th April 2015].

¹⁸ The College Board, 2015. AP Capstone. [online] Available at: <<https://lp.collegeboard.org/ap-capstone/how-ap-capstone-works>> [Accessed 1st May 2015].

¹⁹ The College Board, 2015. How to Earn Credit for Your Scores. [online]. Available at: <<https://apstudent.collegeboard.org/creditandplacement/how-to-earn-credit-for-your-scores>> [Accessed 28th April 2015].

3.2.2 Mathematics in Context

APs are not compulsory for high school students, and as single subject qualifications, mathematics is not compulsory. For students wishing to take a mathematics-related AP course, the following three courses are available:

- Calculus AB
- Calculus BC
- Statistics.

Each programme focusses on a particular area of mathematics, calculus or statistics. All three courses (hereby referred to as 'programmes') will be considered as part of the overall comparative analysis of the AP and the IB DP.

3.3 GCE A levels

3.3.1 Overview

GCE A levels are single-subject upper secondary qualifications offered at schools and colleges in England, Wales and Northern Ireland, and many international schools worldwide. They are developed by awarding organisations²⁰ and regulated by the Office of Qualifications and Examinations Regulation (Ofqual).

GCE A levels can be studied at any age, although they are normally studied at age 16-18, following compulsory secondary education and completion of General Certificate of Secondary Education qualifications (GCSEs). There are no formal entrance requirements, although some schools and colleges may request a minimum of five GCSE qualifications at grades A*-C. When studied at secondary school, the normal duration is two academic years (Years 12 and 13).

A full GCE A level currently consists of Advanced Subsidiary (AS) units followed by Advanced (A2) units²¹. Students typically study three to five subjects as AS level, and then take some or all of these forward to A2 level, where they study the subjects in greater depth.

Though not specifically designed as a higher education access qualification, GCE A levels are one of the principal qualifications giving access to higher education in the UK, and are placed at level 3 on the Qualifications and Credit Framework (QCF) in England, Wales and Northern Ireland²².

²⁰ AQA (Assessment and Qualifications Alliance); CCEA (Council for the Curriculum Examinations and Assessment); Edexcel (Pearson Education Limited); OCR (Oxford Cambridge and RSA); and WJEC / CBAC (Welsh Joint Education Committee).

²¹ Changes to the existing structure, content and assessment of GCE A Levels are planned to start first teaching from September 2015 in certain subjects following a reform of secondary qualifications conducted by Ofqual and the Department for Education (DfE). In other subjects (e.g. Mathematics and Further Mathematics) the changes will come into effect in first teaching in September 2017. As a study focussed on assessment, this study considers the assessment of the current GCE A Levels.

²² QAA, SCQF, CCEA Accreditation, Ofqual, CQFW & QQI, 2014. Qualifications Can Cross Boundaries- a guide to comparing qualifications in the UK and Ireland. [pdf]. Available at: <<http://www.qaa.ac.uk/en/publications/documents/qualifications-can-cross-boundaries.pdf>> [Accessed 11th August 2015].

Students can choose the subjects they wish to study, often basing their selections on personal interest or subjects relevant to their intended programme of study at undergraduate level. Completion of three full GCE A levels is the usual requirement for admission to undergraduate study in the UK²³.

3.3.2 Mathematics in Context

As indicated in the methodology, schools and colleges are able to choose which awarding organisation's GCE A level programmes they teach, which can vary on a subject-by-subject basis. For the purpose of this study, the specifications and assessment materials of Pearson Edexcel (UK-regulated GCE A Level) have been considered.

Mathematics is not compulsory. The Edexcel GCE A Level Mathematics programme offers numerous pathways leading to various awards:

- AS / A Level Mathematics
- AS / A Level Pure Mathematics
- AS / A Level Further Mathematics (can be taken alongside or after Advanced GCE Mathematics)
AS / A Level Further Mathematics (Additional) (awarded in addition to Advanced GCE in Mathematics and Advanced GCE Further Mathematics)²⁴.

Within these programmes, there are also a range of units (modules) that make up the GCE A Level:

Table 4: GCE A Level units offered²⁵

Area of Mathematics	Units offered by Edexcel
Core Mathematics	C1, C2, C3, C4
Further Pure Mathematics	FP1, FP2, FP3
Mechanics	M1, M2, M3, M4, M5
Statistics	S1, S2, S3, S4
Decision Mathematics	D1, D2

Table 5: Edexcel Advanced GCE Mathematics awards²⁶

Edexcel Qualification	Number of Units Required	Compulsory	Elective
GCE A Level Mathematics	6	C1, C2, C3 and C4	One combination from the following: <ul style="list-style-type: none"> • M1 and S1, or • M1 and D1, or • S1 and D1, or

²³ UK NARIC (2015), *International Comparisons: UK file*. Similarly observed through a sample of UK university admissions policies including: University of Birmingham, Cardiff University, Kings College London, University of Cambridge, and University of York.

²⁴ Pearson Education Ltd, 2013. Specification GCE Mathematics. [pdf]. Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/UA035243_GCE_Lin_Maths_Issue_3.pdf> [Accessed 23rd April 2015].

²⁵ Ibid.

²⁶ Ibid.

			<ul style="list-style-type: none"> • S1 and S2, or • M1 and M2, or • D1 and D2.
GCE A Level Further Mathematics ^{27*}	6	FP1 and FP2 or FP3	Any units (excluding C1-C4).
GCE A Level Pure Mathematics	6	C1, C2, C3, C4 and FP1	FP2 or 3
GCE A Level Further Mathematics (Additional) ²⁸	6 (18 total encompassing A Level Mathematics and A Level Further Mathematics)	Any units not undertaken as part of the student's A Level Mathematics or A Level Further Mathematics	

For the purposes of this study, the GCE A Level Mathematics and A Level Further Mathematics units (FP1-FP3) will be considered. A Level Pure Mathematics requires a combination of units sat in both the A Level Mathematics and A Level Further Mathematics and as such, it is not necessary to consider it separately. From the review of the IB DP programmes, it is clear that mechanics is not covered within any of the IB DP programmes. It is anticipated that this will be discussed within the IB's parallel curriculum review and since the purpose of this study is to consider assessment, the units considered most relevant for this study were:

- All Core Mathematics units (C1, C2, C3, C4)
- Statistics units which may be covered within GCE A Level Mathematics (S1 and S2)
- Further Pure units which may be covered within the GCE A Level Further Mathematics (FP1, FP2, FP3).

3.4 Gāokǎo

3.4.1 Overview

The National University College Entrance Examination, known as the 高考 (Gāokǎo), is an examination taken by high school students who wish to enter higher education in China. The Gāokǎo is administered by provincial examinations authorities in each of the 31 provinces, autonomous regions and municipalities. Some provincial authorities are permitted to set their own examination papers and some use papers that are set centrally by the Ministry of Education²⁹. A number of provinces use a combination of their papers and those set by the Ministry of Education.

As outlined in the methodology, this report will focus on the assessment of the Gāokǎo in Shanghai province, overseen by the 上海市教育考试院 (Shanghai Municipal Educational Examinations Authority)³⁰.

²⁷ Can be taken alongside or after GCE A Level Mathematics (contains Core Mathematics units C1-C4).

²⁸ Awarded in addition to Advanced GCE Mathematics and Advanced GCE Further Mathematics.

²⁹ 陈鹏, 2015. 明年 25 个省将统一命题 高考命题重回“全国卷”时代. [online] Available at: <http://news.xinhuanet.com/politics/2015-04/03/c_127653654.htm> [Accessed 16th May 2015].

³⁰ 上海市教育考试院, 2015. 上海招考热线. [online] Available at: <<http://www.shmeea.com.cn/node2/node118/node119/index.html>> [Accessed 16th May 2015].

The Gāokǎo – specifically designed as an undergraduate admissions test – is required for entry to higher education, and is taken in the final year of high school, after a total of 12 years of education. Prior to this, there are two formal secondary (exit) qualifications: the first is the Lower Secondary School Graduation Certificate, 初中毕业证书, (*Chūzhōng bìyè zhèngshū*), examined after nine years of schooling. Students who wish to continue to upper secondary level sit an entrance examination. After completion of their upper secondary education all students sit examinations for the Academic Level Examination, 会考/ 学业水平测试 (*Huikao*)³¹ in order to obtain the Senior Secondary School Graduation Certificate, 高中毕业证书 (*Gāozhōng bìyè zhèngshū*). Students sit examinations in all 13 subjects studied at upper secondary level and are required to pass all subjects examined in order to graduate.³²

In Shanghai, the Gāokǎo consists of four subjects, however students can choose to sit the examination in one of two streams: arts or science with subjects as follows³³:

Table 6: Gāokǎo subjects for the Arts and Science streams

Arts Stream	Science Stream
Chinese*	Chinese*
Foreign Language*	Foreign Language*
Mathematics*	Mathematics*
Geography or History or Politics	Biology or Chemistry or Physics

*denotes 'compulsory'

3.4.2 Mathematics in Context

As outlined above, all students who take the Gāokǎo must sit an examination in mathematics from either the arts stream or the science stream. Assessment from both streams will be considered in the context of this study.

3.5 Singapore-Cambridge General Certificate of Education Advanced level (GCE A level)

3.5.1 Overview

The Singapore-Cambridge General Certificate of Education Advanced Levels (GCE A Levels, hereafter referred to as SIPCAL for clear distinction with the UK GCE A Level examined in this study) are single-subject, post-secondary qualifications, awarded jointly by University of Cambridge International Examinations (CIE); Ministry of Education, Singapore; and Singapore Examinations and Assessment Board (SEAB).

³¹ The exact title varies between provinces.

³² 新加坡教育, 2015. 上海公布普通高中学业水平考试实施办法. [online] Available at: <<http://edu.sina.com.cn/gaokao/2015-04-27/1556466862.shtml>> [Accessed 15th May 2015].

³³ 中华人民共和国教育部, 2014. 改变用一个分数评价录取学生. [online] Available at: <<http://www.moe.gov.cn/publicfiles/business/htmlfiles/moe/s8298/201409/175293.html>> [Accessed 16th May 2015].

SIPCALS are typically studied by 17-19 year olds and awarded after two or three years of study³⁴. Entry to SIPCALS is typically based on achievement at GCE O level although it is possible to undertake an Integrated Programme (IP)³⁵.

The SIPCAL curriculum comprises:

- Life Skills
- Knowledge Skills
- Content-Based Subjects in:
 - Languages
 - Humanities and the arts
 - Mathematics and sciences³⁶.

Subjects are offered at different levels of study: Higher 1 (H1), Higher 2 (H2), and Higher 3 (H3). Whilst the academic depth of H1 and H2 is intended to be the same, H1 covers half the breadth of curriculum than that covered at H2³⁷. H3 subjects are aimed at particularly able students who wish to study a subject at greater depth, or in a more specialised area³⁸. Students studying H3 subjects are assumed to be studying a corresponding subject at H2.

Students take a combination of subjects from the curriculum, but the range of subjects available varies by institution. An example of a typical combination is as follows:

- Three H2 Content-based subjects and one H1 Content-based subject (which must be different to those subjects studied at H2 level). At least one content-based subject must be from a contrasting discipline (i.e. outside a student's main area of specialisation).
- H1 Mother Tongue Language (MTL) or H2 Mother Tongue Language and Literature
- H1 General Paper (GP) or H1 Knowledge and Inquiry (KI)
- H1 Project Work (PW)³⁹.

Private students can study a maximum of five H1 subjects and three H2 subjects, or three H1 subjects and four H2 subjects⁴⁰.

The SIPCAL is defined as a post-secondary award preparing for work, continuing education and training, or university⁴¹. The SIPCAL is accepted for admission to undergraduate study by higher education institutions in Singapore.

³⁴ Two years of study at Junior Colleges or three years of study at Centralised Institutes (CI)

³⁵ Ministry of Education, Singapore, 2015. Integrated Programmes (IP). [online] Available at: <<http://www.moe.gov.sg/education/secondary/other/integrated-programme/>> [Accessed 18th May 2015].

³⁶ Ministry of Education Singapore, 2015. Bringing out the best in every child. Education in Singapore. [pdf] Ministry of Education Singapore. Available at: <<http://www.moe.gov.sg/about/files/moe-corporate-brochure.pdf>> [Accessed 11th May 2015].

³⁷ Ministry of Education, 2008. Frequently Asked Questions. [online] Available at: <<http://www.moe.gov.sg/cpdd/alevel2006/faqs.htm>> [Accessed 7th May 2015].

³⁸ Ministry of Education Singapore, 2015. Bringing out the best in every child. Education in Singapore. [pdf] Ministry of Education Singapore. Available at: <<http://www.moe.gov.sg/about/files/moe-corporate-brochure.pdf>> [Accessed 11th May 2015].

³⁹ Ministry of Education, 2008. Frequently Asked Questions. [online] Available at: <<http://www.moe.gov.sg/cpdd/alevel2006/faqs.htm>> [Accessed 7th May 2015].

⁴⁰ Singaporean Examinations and Assessment Board, (n. d.). Singaporean-Cambridge GCE N(T)-, N(A)-, O- & A-Level Examinations. *Instructions for Private Candidates*. [pdf] Singaporean Examinations and Assessment Board. [Accessed 1st May 2015].

⁴¹ Ministry of Education, Singapore (2015) *The Singapore Education Landscape*.

3.5.2 Mathematics in Context

Mathematics is not compulsory at this level, though does appear in the norm SIPCAL combinations published by the Ministry of Education. The following mathematics programmes are currently available:

- H1 mathematics
- H2 mathematics
- H3 mathematics⁴².

All three programmes were reviewed as part of this study.

3.6 Alberta Diploma

3.6.1 Overview

The High School Diploma is the school leaving qualification for pupils studying at upper secondary education in Alberta⁴³. It is generally awarded on successful completion of a three year upper secondary level course in grades 10 and 11, culminating in the Diploma examinations at the end of grade 12. The Diploma is administered by the Government of Alberta and is based on the curriculum set by Alberta Education, the provincial department of education.

The Diploma programme comprise a number of courses, each of which are credit based and assigned one of three levels, which are 10 Level, 20 Level and 30 Level. The 30 Level is designed to be the most demanding, with the 20 Level programmes typically a pre-requisite⁴⁴. The Diploma Examinations Program includes diploma examinations in the following subject areas:

- Biology
- Chemistry
- English language arts
- French
- French Language Arts
- Mathematics
- Physics
- Science
- Social studies⁴⁵.

The purpose of the Alberta Diploma is to provide and assess upper secondary level education. It is accepted for admission to undergraduate study in Canada. In order to obtain

⁴² As an alternative to the Cambridge-SEAB H3 mathematics, students could undertake a H3 programme in Linear Algebra, offered by one of the Ministry of Education's partners, the National University of Singapore (NUS).

⁴³ The territorial governments of the Northwest Territories and Nunavut also follow and complete the curriculum set by Alberta Education, with students in these territories receiving the Alberta Diploma.

⁴⁴ Alberta Government, 2014. Guide to Education: ECS to Grade 12 2014-2015. [pdf]. Crown in Right of the Province of Alberta. Available at: <http://education.alberta.ca/media/8765464/guide_to_education_2014.pdf> [Accessed 22nd April 2015].

⁴⁵ Alberta Government, 2014. Guide to Education: ECS to Grade 12 2014-2015. [pdf]. Crown in Right of the Province of Alberta. Available at: <http://education.alberta.ca/media/8765464/guide_to_education_2014.pdf> [Accessed 22nd April 2015].

the Diploma, students must obtain 100 credits (courses are usually worth 5 credits each), and score at least 50% in each course.⁴⁶ The minimum requirements at an English speaking school include the following⁴⁷:

- 10 level courses:
 - Physical education (minimum 3 credits)
- 20 level courses
 - A mathematics course
 - A science course or a specific combination of 10 level courses
- 30 level courses:
 - English language arts
 - Social studies course
 - Any other 30-level course, chosen by the student (10 credits)
- Career and life management (minimum 3 credits)
- 10 credits earned from any combination of a prescribed set of optional courses^{48 49}.

In addition to the course requirements for the Diploma in Alberta, students are required to write at least two diploma examinations out of the following: English 30-1 and 30-2, social studies 30-1 and 30-2⁵⁰. Examinations are also provided in a select group of 30 level courses. Courses without examinations are assessed solely through school-based assessment.

3.6.2 Mathematics in Context

The following mathematics programmes are offered within the Diploma:

- 10-Level: Mathematics 10C, 10-3, and 10-4
- 20-Level: Mathematics 20-1, 20-2, 20-3, 20-4
- 30-Level: Mathematics 30-1, 30-2, 30-3
- Mathematics 31⁵¹.

As outlined in section 3.6.1, a 20-Level programme in mathematics is required to pass the Alberta Diploma. The 20-Level mathematics programmes are assessed solely via school-based assessment and do not require an external examination. Both 30-1 and 30-2 Mathematics programmes require an external examination and as such were considered to provide the most relevant points of reference for comparison against the IB DP.

The Mathematics 30-1 programme is focused on pure mathematics whilst Mathematics 30-2 concentrates on the application of mathematics in real world situations.

⁴⁶ Ibid.

⁴⁷ The requirements differ slightly for Francophone schools, with 30 level French also being required.

⁴⁸ Including Advanced level Career and Technology Studies, Green Certificate specialisation courses, Registered Apprenticeship Programme, special projects, work experience courses, and other appropriate locally developed / acquired / authorised courses.

⁴⁹ Alberta Government, 2014. Guide to Education: ECS to Grade 12 2014-2015. [pdf]. Crown in Right of the Province of Alberta. Available at: <http://education.alberta.ca/media/8765464/guide_to_education_2014.pdf> [Accessed 22nd April 2015].

⁵⁰ Alberta Government, 2015. General Information Bulletin: Introduction to the Diploma Examinations Program. [pdf]. Available at: <http://education.alberta.ca/media/6902694/02-dip-gib-2014-15_introrevisions.pdf> [Accessed 22nd April 2015].

⁵¹ Alberta Government, 2014. Guide to Education: ECS to Grade 12 2014-2015. [pdf]. Crown in Right of the Province of Alberta. Available at: <http://education.alberta.ca/media/8765464/guide_to_education_2014.pdf> [Accessed 22nd April 2015].

4. Qualification Level Comparative Analysis

4.1 Qualification Purpose and Structure

As can be seen from the qualification profiles above, all qualifications are taken at upper secondary school level, and lead to higher education; however the structure and style of the awards are very different.

The table below summarises the purpose, structure and general associated outcomes for each qualification and places the mathematics programmes offered in the context of the overall qualification:

Table 7: Comparison of qualifications

Qualification Features	IB Diploma Programme	Advanced Placement	GCE A levels	Gāokǎo	SIPCAL	Alberta Diploma
Formal entry requirement	No	No	No (may vary by school)	High school graduation	No (may vary by school)	Completion of first 9 years of school
Years of study	2 years	No set number, but often 1 year (10-12 years education)	2 years (13 years total education)	Exam only (12 years education)	2-3 years	3 years (12 years education)
Number of subjects studied	7	No set number	No set number (3-5 normally)	N/A	6-7 (6-8 for private students)	No set number.
Number of subjects assessed	7	No set number	No set number	4	6-7 (6-8 for private students)	2 minimum (1 exam for every other 30 level course)
Certification type	Baccalaureate / Diploma ⁵²	Single subject qualification (Can study for the Diploma ⁵³)	Single subject qualification	Entrance examination	Single subject qualification	High School Diploma
Purpose	Assess Upper Secondary Level	Prepare for Semester 1, Higher Education	Assess Upper Secondary Level	Assess for Higher Education Admission	Assess Upper Secondary Level	Assess Upper Secondary Level

⁵² Students can also take individual IB DP subject courses and receive a Diploma Programme course results (DPCR). Students enrolled in the full DP but that do not meet the minimum score requirements, or that fail to meet the other minimum requirements for the Diploma, will also receive the DPCR.

⁵³ The AP Capstone subjects, AP Seminar and AP Research, can result in an AP Seminar and Research Certificate. When the two subjects are also taken in conjunction with four other AP courses and exams, the student may receive an AP Capstone Diploma.

Qualification Features	IB Diploma Programme	Advanced Placement	GCE A levels	Gāokǎo	SIPCAL	Alberta Diploma
Associated outcomes:						
Accepted for undergraduate admission, at institutional discretion	✓	✓	✓	✓	✓	✓
Employment and/or continuing education and training	✓	✓	✓	✓	✓	✓

4.2 Mathematics in Context

Table 8: Mathematics in context summary

	IB Diploma Programme	Advanced Placement	GCE A levels	Gāokǎo	SIPCAL	Alberta Diploma
Is mathematics compulsory?	✓	–	–	✓	–	✓ ⁵⁴
Number of mathematics programmes available	4	3	4	N/A (2 exam streams)	4	11

As can be seen above, mathematics is compulsory in half of the qualifications reviewed in this study. All of the qualifications offer multiple programmes (with the exception of the Gāokǎo, although mathematics can be taken in two separate examination streams).

In all but the Gāokǎo, students may study one or more programmes. Figure 2 below illustrates the differing qualification structures and the various mathematics programmes available at the time of writing.

As can be seen in the table below, the programmes selected as most relevant for the study, (as identified in section 3) are primarily general in nature with all students studying core content across a range of topics. This is also discussed in section 5.3. The AP programmes are an exception, with each focusing on either statistics or calculus. The GCE A Levels and IB Mathematics HL do offer students some options to choose from in terms of units in the case of A Levels and Paper 3 in the Mathematics HL.

Table 9: Mathematics programmes reviewed in this study

Qualification	Programme(s) for review	General (all content core)	General with specialist electives	Specialised
IB DP	Mathematical Studies SL	✓		
	Mathematics SL	✓		
	Mathematics HL		✓	
	Further Mathematics HL	✓		
GCE A Level	A Level Mathematics ⁵⁵		✓	
	A Level Further Mathematics ⁵⁶		✓	

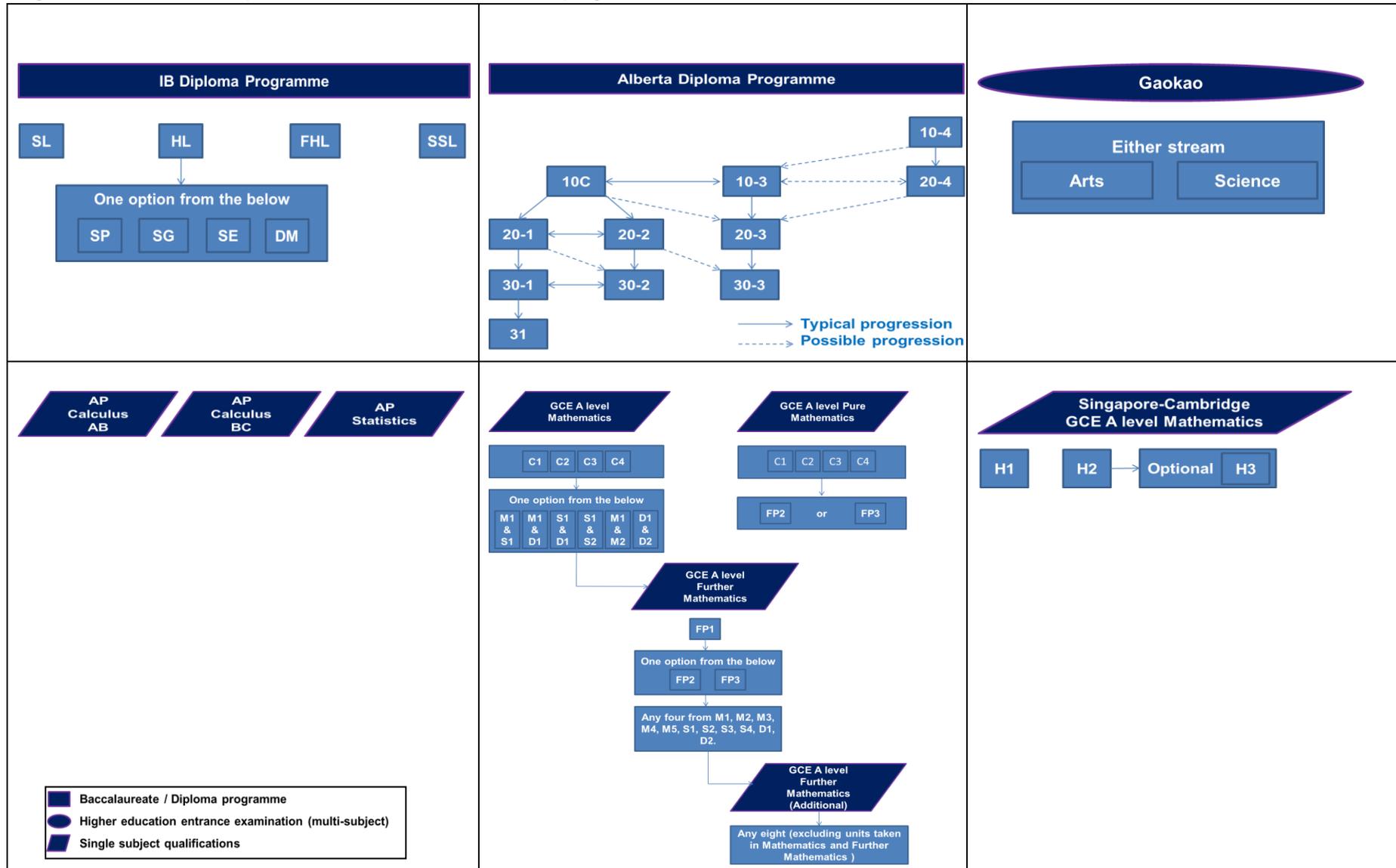
⁵⁴ Note, whilst mathematics is a compulsory subject within the Alberta Diploma, the two programmes reviewed within this study are not compulsory.

⁵⁵ For the A Level Mathematics, the analysis focusses on the compulsory units [C1, C2, C3, C4] and S1 and S2, the combination of units considered to be most relevant to the IB DP content areas.

⁵⁶ For the A Level Further Mathematics, the analysis focusses primarily on FP1, FP2, FP3, with all A Level Further Mathematics students required to take FP1 and FP2/3.

Qualification	Programme(s) for review	General (all content core)	General with specialist electives	Specialised
Advanced Placement	Calculus AB			✓
	Calculus BC			✓
	Statistics			✓
SIPCAL	Higher 1 (H1)	✓		
	Higher 2 (H2)	✓		
	Higher 3 (H3)	✓		
Alberta Diploma	Mathematics 30-1	✓		
	Mathematics 30-2	✓		
Gāokāo	Mathematics (Art Stream)	✓		
	Mathematics (Science Stream)	✓		

Figure 2: Structure of the qualifications and their mathematical programmes



Key:

H1	Higher 1	[Singapore]
H2	Higher 2	[Singapore]
H3	Higher 3	[Singapore]
DM	Discrete Mathematics	[IB]
SE	Calculus	[IB]
SL	Standard Level	[IB]
SG	Sets, Relations, and Groups	[IB]
SP	Statistics and Probability	[IB]
SL	Mathematics SL	[IB]
HL	Mathematics HL	[IB]
FHL	Further Mathematics HL	[IB]
SSL	Mathematical Studies SL	[IB]
C1, C2, C3, C4	Core	[UK]
FP1, FP2, FP3	Further Pure Mathematics	[UK]
M1, M2, M3, M4, M5	Mechanics	[UK]
S1, S2, S3, S4	Statistics	[UK]
D1, D2	Decision Mathematics	[UK]

5. Programme Level Analysis

5.1 Programme Purpose

The table below summarises the purpose for each individual mathematics programme reviewed as part of this study.

Table 10: Programme Purpose Summary

Qualification	Programme	Purpose
IB DP	Mathematical Studies SL	Mathematical Studies SL is for students who are likely to study or work in areas not directly linked to mathematics, (such as arts, languages, humanities) but who would nonetheless benefit from learning about a broad range of mathematical topics and their application.
	Mathematics SL	Mathematics SL is designed for students with basic mathematics knowledge who are looking to improve their background in important mathematical concepts to apply in their future studies in areas requiring some mathematical background (such as business, economics and psychology).
	Mathematics HL	Mathematics HL includes the same concepts at Mathematics SL but in more depth, and is intended more for students who wish to pursue mathematics at higher education level or subjects that will directly involve mathematics (such as engineering and physics); or for those with a strong interest in the subject.
	Further Mathematics HL	Further Mathematics HL requires a strong mathematics background and interest in the subject and is intended to cover different branches of mathematics and equip students for studying mathematics at university level. This course is designed to be at the same level of difficulty as the Mathematics HL, but covers all of the four Maths HL optional topics in greater depth.
Advanced Placement	Calculus AB	Calculus AB is designed to mimic the first semester of a college calculus course that is devoted to the topics in differential and integral calculus.
	Calculus BC	Calculus BC is designed to mimic the first and second semester of a college calculus course, and expands on Calculus AB topics in differential and integral calculus, and also includes the different types of equations and introduces the topic of sequences and series.
	Statistics	The AP Statistics course is designed to mimic a one semester introductory, non-calculus based, university level course in statistics.
GCE A level	A Level Mathematics	Students will develop an understanding of mathematics and mathematical processes, develop the ability to reason logically and construct mathematical proofs, and understand coherence and progression in mathematics and how different areas of mathematics can be connected. These programmes intend to allow progression into related courses in higher education.
	A Level Further Mathematics	
SIPCAL	H1	H1 is intended for students who intend to study social sciences, business, or economics at tertiary level.

Qualification	Programme	Purpose
	H2	H2 is intended more for students wishing to pursue courses in maths, physics and engineering.
	H3	H3 is designed for students with a passion and ability in mathematics and who wish to study it at a higher level and more in depth. The programme extends on Functions and Graphs, Sequences and Series, and Calculus included in H2.
Gāokǎo	Arts Stream	The Arts Stream exam is taken by students who intend to study arts or humanities subjects at undergraduate level.
	Science Stream	The Science Stream exam is taken by students who intend to pursue a Bachelor degree in a science or mathematics related field of study.
Alberta Diploma	Mathematics 30-1	Mathematics 30-1 is intended for students wishing to progress onto programmes that may require knowledge of pure maths at a higher level.
	Mathematics 30-2	Mathematics 30-2 is for students who intend to use mathematics in applied, everyday life settings and is also designed to prepare students for further study in courses which may involve some mathematics but not calculus.

All the reviewed international programmes share a similar goal in providing an assessment of mathematical skills and knowledge required at upper secondary level in their respective systems, at the same time it is important to note differences in purpose that are also reflected in assessment format and design.

In terms of purpose, IB DP Mathematical Studies is most similar to the Alberta Diploma Mathematics 30-2 programme in the sense that they are designed to provide a broad, general course in mathematics, focussed on application.

The IB DP Mathematics SL is closest in purpose to the SIPCAL H1 Mathematics, with both designed to provide a sound background in mathematics to support work or higher education study in areas such as business, economics and social sciences.

IB DP Mathematics HL is intended by the IB to be the primary programme undertaken by those wishing to work in areas or study for degrees involving a substantial use of mathematics. A number of the international qualifications have a programme which share this purpose: the Mathematics 30-1 [Alberta Diploma]; the GCE A Level Mathematics; the SIPCAL H2 Mathematics.

The IB DP, SIPCAL and GCE A Level all have programmes designed specifically for those with a particular passion for mathematics, which will further support study of programmes that are substantially or wholly mathematics-focussed without necessarily being pre-requisites for such degrees. These programmes are the IB DP Further Mathematics HL, the GCE A Level Further Mathematics and the SIPCAL H3 Mathematics.

The IB DP mathematics programmes, along with many of the other international programmes, are quite different in overall purpose to the Gāokāo. Of the international programmes reviewed, the purpose of the Gāokāo, being a university entrance examination, is most similar to the AP mathematics programmes as it intends to assess skills and knowledge which are relevant for the first year of undergraduate study.

Table 11: Summary of programme purpose in relation to higher education study

Qualification	Programme	For further study in non-STEM subjects	For further study in STEM
IB DP	Mathematical Studies SL	✓	
	Mathematics SL	✓	
	Mathematics HL		✓
	Further Mathematics HL		✓
Advanced Placement	Calculus AB		✓
	Calculus BC		✓
	Statistics	✓	✓ ⁵⁷
GCE A level	A Level Mathematics	✓	✓
	A Level Further Mathematics	✓	✓
SIPCAL	H1	✓	
	H2		✓
	H3		✓
Gāokāo	Arts Stream	✓	
	Science Stream		✓
Alberta Diploma	Mathematics 30-1		✓
	Mathematics 30-2	✓	

5.2 Comparison of Aims

Aims can be defined as broad statements of what students are expected to learn and are generally written at qualification level. Aims may include abstract concepts such as personal or professional qualities, behavioural attributes or include appreciation and enjoyment of a subject. Aims may describe wider learning that may be difficult to measure but which is nonetheless important. Aims are typically written to apply across all elements of the programmes; this is the case for the IB DP mathematics and is similarly the case for most of the international qualifications included in this review, with the exception of SIPCAL H3.

⁵⁷ The AP Statistics course prepares students for upper-level calculus based courses in statistics at university; a usual requirement for science, engineering and mathematics degree programmes.

Table 12: Summary of programme aims

Key Themes	IB Diploma Programme	Advanced Placement	GCE A level	SIPCAL		Alberta Diploma
	All mathematics programmes	Calculus AB and Calculus BC	Mathematics and Further Mathematics	H1 and H2	H3	Mathematics 30-1 and 30-2
Developing enjoyment of mathematics	✓	✓	✓	✓	✓	-
Understanding the key principles of mathematics	✓	✓	✓	✓	✓	✓
Mathematical communication	✓	✓	✓	✓	-	✓
Logic and problem solving skills	✓	Partial	Partial	✓	✓	✓
Abstract reasoning and generalisation	✓	-	✓	-	✓	-
Application of mathematics to alternative situations	✓	✓	✓	Partial	✓	Partial
Awareness of the impact of mathematics on technology	✓	Partial	Partial	Partial	-	Partial
Understanding the moral, social and ethical implications of mathematics	✓	-	-	-	-	-
Appreciation of the international dimension of mathematics	✓	-	-	-	-	-
Awareness of the contribution of mathematics to other disciplines	✓	-	✓	✓	✓	-
Experiment, interpret results, draw conclusions	-	✓	-	-	-	-
Determining the reasonableness of solutions	-	✓	-	-	-	-

Key Themes	IB Diploma Programme	Advanced Placement	GCE A level	SIPCAL		Alberta Diploma
	All mathematics programmes	Calculus AB and Calculus BC	Mathematics and Further Mathematics	H1 and H2	H3	Mathematics 30-1 and 30-2
Solving unstructured problems	-	-	✓	-	-	-
Construction of mathematical proofs	-	-	✓	-	✓	-
Taking responsibility for learning and own development	-	-	✓	-	-	-
Awareness of the limitations of technology	-	-	✓	-	-	-
Learning independently and cooperatively	-	-	-	✓	-	-
Production of imaginative and creative work	✓	-	-	✓	-	-
Developing skills for everyday life and continuous learning	-	-	-	✓	-	✓

As illustrated in the table above, there are some clear similarities in the aims of the IB DP and international programmes.

There are key similarities between the aims of the IB DP and GCE A Level mathematics programmes, such as enabling students to enjoy mathematics, to communicate effectively using mathematics, and to recognise the relevance of mathematics in wider contexts. Notably, the IB DP makes reference to the development of creative thinking skills through the study of mathematics which is not referenced in the GCE A Level aims.

In comparison with IB DP aims, the AP aims, referred to as 'goals' for the Calculus AB and BC⁵⁸ place greater emphasis on developing knowledge and understanding of specific topics and on being able to demonstrate general mathematical skills in the assessment. Given the skills-based orientation and specificity of the majority of the AP aims, they could be viewed as conceptually closer to objectives than learning aims; as such they form the basis of comparison against IB DP assessment objectives in the following section. Despite differences in focus, the IB DP and the AP share some common themes, including reference to use of technology; solving a variety of problems; logical thinking; and promoting enjoyment of mathematics.

The Singapore SIPCAL H1 and H2 specification aims demonstrate broad similarities with those specified for the IB DP. As with the IB DP, there is also mention in the SIPCAL aims of developing creative thinking skills in mathematics and the aim to promote awareness of the application of maths to other disciplines, themes which are not included in the other international qualification specifications reviewed. In terms of differences, the SIPCAL aims also mention acquiring the ability to use mathematics in everyday life and in continuous learning, which is not explicitly stated as an aim in the IB specification.

In contrast to the IB DP specifications in which the same aims are written for the four Mathematics programmes reviewed in this study, the Singapore SIPCAL provides a separate set of aims for candidates taking H3 (the H1 and H2 share the same aims). The H3 aims contain many of the same themes as those covered in the H1 and H2 syllabus, although there is more emphasis on broadening knowledge of mathematics, understanding connections, and creative thinking in mathematics and in particular appreciation of the beauty of mathematics, an aim also included in the IB DP.

As opposed to aims which relate specifically to the learning process, the aims of the Alberta Diploma programmes, Mathematics 30-1 and 30-2, focus on the development of cognitive skills⁵⁹ assessed in the respective examinations: no direct reference is made to the behavioural attributes and attitudes towards mathematics that are demonstrated in the IB DP. Both the Alberta Diploma and IB DP cover some common themes such as developing knowledge and understanding of mathematical principles and using this to solve routine and non-routine problems using different strategies.

⁵⁸ No goals are included with the AP Statistics specifications.

⁵⁹ Defined by three cognitive levels: Conceptual understanding, procedural understanding, and problem solving.

In line with its purpose as a higher education entrance examination, with no set programme of study, the Gāokǎo, understandably, does not define learning aims; instead there are three prescribed examination objectives, considered in section 5.3. The absence of learning aims is a reflection of the purpose of the Gāokǎo mathematics which is to provide a standardised examination of mathematical knowledge.

Developing awareness of the international dimension and historical perspectives on mathematics, found in the IB DP aims, is not explicitly included in the other international qualification aims. Its inclusion reflects the distinct international orientation of the IB DP and the interdisciplinary aspect which is emphasised in the portfolio work. A further aim of the IB DP, which is not evidenced across the other qualification specifications, is the development of moral, social and ethical implications arising from the work of mathematicians. The broader, interdisciplinary focus also relates to the aim and requirements of the IB DP in general, the Theory of Knowledge element of the IB DP allows for the further exploration of links between disciplines including mathematics, and of its wider societal impact. It is also noted that a larger proportion of the IB DP aims focus on attitudes than in the other specifications, in particular developing appreciation and enjoyment of mathematics.

5.3 Assessment Objectives

The aim of this section will be to compare the assessment objectives for each programme, in terms of the following:

- Scope
- Skills coverage
- Weighting.

5.3.1 Scope

One of the key differences between the assessment objectives observed in the upper secondary qualification specifications is that between the use of skills-based and content-based objectives. Skills-based objectives describe the key skills and in the case of mathematics, processes assessed in the examinations whereas content-based objectives are predominantly focused on the specific mathematical knowledge and procedures required for assessment. Generally speaking, assessment objectives that are skills-based are not designed to be too prescriptive, allowing for flexibility in the way in which they are linked to assessment.

The IB DP, GCE A Level, SIPCAL and Gāokǎo specifications, for instance, all include skills-based assessment objectives, making reference to the broader competencies, such as demonstrating knowledge of mathematics interpreting mathematical models and constructing mathematical arguments. These skills are expressed at a level that is applicable to most or all topic areas covered within the syllabus.

The Alberta Diploma assessment outcomes are primarily content-based, although the assessment standards do list seven underpinning cognitive processes. These processes are linked to the content based outcomes and highlight cognitive skills expected of candidates in the examinations; as such they can also provide a basis for comparison with other

international qualification assessment objectives. The cognitive processes include mathematical communication, problem solving, reasoning, technology use, visualization mental mathematics and connections.

The AP goals, used in the absence of defined assessment objectives, present a combination of skills-based and content-based approaches. Four of the ten goals reference developing knowledge, understanding and problem solving relating to subject-specific content, whilst the majority of the other goals relate to general mathematical skills covering themes also included in the A Level, IB DP and SIPCAL objectives. One of the ten AP goals is presented as a traditional learning aim rather than an assessment objective in that it refers to developing appreciation of calculus as a coherent body of knowledge⁶⁰.

Whilst examining variations in the focus of assessment objectives in qualification design, it has been observed that the use of content-based objectives does not necessarily reflect the nature of the learning or the assessment. For instance, although the Alberta Diploma emphasises theoretical and procedural knowledge in its assessment outcomes, the assessment tasks focus heavily on process- and skills-based mathematics, drawing upon knowledge of mathematical procedures and knowledge mentioned in the assessment objectives.

5.3.2 Skills Coverage

The table below demonstrates the key similarities and differences in the skills covered by the different mathematics programmes.

⁶⁰ The College Board, 2012. Calculus: Calculus AB Calculus, BC Course Description Effective Fall 2012. [pdf]. Available at: <<http://media.collegeboard.com/digitalServices/pdf/ap/ap-calculus-course-description.pdf>> [Accessed 20th April 2015].

Table 13: Skills coverage in mathematics programmes

Mathematical Skills and Knowledge	IB DP	Advanced Placement	GCE A level	Gāokǎo	SIPCAL		Alberta Diploma
	All mathematics programmes	Calculus AB and Calculus BC	Mathematics and Further Mathematics	Arts and Science stream	H1 and H2	H3	Mathematics 30-1 and 30-2
Demonstrate knowledge and understanding of mathematical concepts and principles	✓	✓	✓	✓	✓	✓	✓
Problem solving in real world and abstract contexts	✓	✓	✓	✓	✓	✓	✓
Solving substantial problems in unstructured form	-	-	✓	-	-	-	-
General Mathematical Communication and interpretation	✓	✓	✓	Partial	✓	✓	✓
Mathematical communication orally and in written sentences	-	✓	-	-	-	-	-
Technology use	✓	✓	✓	-	✓	✓	✓
Understanding the limitations of technology use	-	-	✓	-	-	-	-
Reasoning and development of mathematical arguments	✓	Partial	✓	Partial	✓	✓	✓
Reading critically, and understanding longer mathematical arguments	-	-	✓	-	-	-	-
Inquiry Approaches	✓	Partial	✓	Partial	Partial	Partial	Partial
Determining the reasonableness of solutions and degrees of accuracy	-	✓	✓	-	-	-	-
Personal qualities and test taking skills.	-	-	-	✓	-	-	-

The assessment objectives of the GCE A Levels reference many of the same general mathematical and transferrable skills as those specified for the IB DP. Both sets of assessment objectives, for example, cover mathematical communication; the use of technology; construction of mathematical arguments using deduction and logical thinking; manipulation of mathematical expressions. However, the A Level assessment objectives appear to go further than the IB DP objectives in intending to assess the candidate's ability at constructing extended arguments for handling substantial problems in unstructured form. Nonetheless, it is noted that A Level objectives do not specifically mention inquiry approaches, experimentation and investigation, although one of the assessment objectives does make reference to making predictions and translating findings into realistic contexts.

The Gāokǎo has fewer and broader assessment objectives than the IB DP mathematics programmes though these similarly make reference to problem solving and the development of knowledge and understanding of mathematical principles and processes. There is also reference to solving mathematical problems and mathematical proof although there is no explicit reference to logical deduction, manipulation of mathematical expressions or notably calculator use. In addition to the mathematics related skills, one of the three Gāokǎo objectives focuses on standardised test taking skills in a general sense, for instance, the ability to remain calm and work quickly and accurately under timed conditions.

Singapore SIPCAL objectives also cover problem solving in real and abstract contexts, mathematical communication and the use of graphical calculators. Manipulation of mathematical expressions, logic and deduction are similarly expressly referenced in the SIPCAL objectives as is the ability to solve problems in real and abstract contexts. There is no explicit mention of inquiry approaches or the ability to investigate areas within mathematics, although the skills of interpreting and evaluating mathematical results, making predictions and commenting on context, as specified in the IB DP objectives, are all covered by the SIPCAL objectives.

The Alberta Diploma Examination assessment outcomes, being content-specific, place emphasis on developing knowledge and understanding and problem solving skills in particular topic areas. There is however reference made to some of the more generic skills referenced in the IB DP, such as logical reasoning, deduction and inference and manipulation in the seven cognitive processes included within the assessment standards. Equally, while there is no direct reference to calculator use in the assessment outcomes, the ability to use technology as a tool for learning and solving problems is covered as one of the seven key cognitive processes.

As previously discussed, the AP goals cover knowledge and understanding in specific subject areas relevant to calculus. A number of aims also reference more generic skills, including mathematical communication, use of technology and problem solving in abstract and actual physical situations. Unlike in the IB DP objectives, however, there is no specific reference to using logical deduction or inference as general skills, although these skills may be implied through some of the subject specific aims. Notably, one of the AP goals, not explicitly covered in the IB DP objectives, highlights the ability to determine the reasonableness of solutions.

5.3.3 Weighting

Both the IB DP and the GCE A Level specifications assign varying weighting values to their objectives, reflecting the extent to which the overall assessment focuses on each specific skill area. The IB DP and GCE A Level place similar emphasis on knowledge and understanding, problem solving and reasoning. The IB DP notably places greater emphasis on calculator use, with 10-20% of the marks allocated overall in each programme as opposed to 5% in the GCE A Level. Weightings are also given to the assessment of specific objectives in individual papers in the IB DP and GCE A Level. By linking the weighting of assessment objectives to individual papers, these specifications show how the objectives apply in the context of the practical assessment of individual elements of the programme.

The IB DP objective weightings are similar for all papers, apart from the use of technology objective which is naturally given a weighting of 0% for the non-calculator papers. In addition to the individual papers, the IB DP also includes a weighting for the exploration component, which places emphasis on inquiry skills. In contrast to the IB DP, the weighting of assessment objectives in the A Level can vary depending on the units undertaken and in this respect the A Level specification provides more specific guidance on the range of marks which should be awarded in each skill area. Variations in the weighting of assessment objectives in this case demonstrates how the assessment of different subject areas within mathematics draw more heavily on particular skill areas than others.

In contrast to the IB DP and the GCE A Level, the Singapore SIPCAL Mathematics assessment objectives, as outlined in the specifications, are at each level weighted equally overall whilst the Gāokāo objectives do not indicate a weighting. Given the subject specific focus of the Alberta Diploma Examination assessment objectives, their weighting is implied by the emphasis given to the assessment of each topic area in the Mathematics 30-1 and 30-2 examinations.

5.3.4 Summary of Key Observations

In summary, the IB DP assessment objectives compare well to other skills-based objectives prescribed by international qualifications reviewed as part of this study. All qualification objectives cover knowledge and understanding of mathematical concepts and processes, problem solving in real world and abstract contexts. All except for the Gāokāo cover mathematical communication, whilst the Gāokāo and Alberta Diploma objectives do not reference the use of graphical calculators. Both the IB DP and the international programmes' objectives place similar emphasis as the IB DP on mathematical reasoning, including logic, deduction and inference, with the Gāokāo, GCE A Level and SIPCAL objectives also making reference to mathematical proof. However, whilst the international qualification objectives provided partial reference to skills underpinning inquiry approaches and investigative mathematics, the IB DP objectives are more explicit in their intention to assess experimentation, interpretation and validity testing in investigative contexts, reflective of the exploration component in the IB DP.

5.4 Assessment Methods

This section presents the assessment methods for each mathematics programme.

5.4.1 Overall Assessment of Mathematics

Table 14: Assessment methods in mathematics programmes

		IB Diploma Programme				Advanced Placement			Alberta High School Diploma		GCE A level		SIPCAL			Gāokǎo
		Further Mathematics HL	Mathematics HL	Mathematics SL	Mathematical Studies SL	Calculus AB	Calculus BC	Statistics	Mathematics 30-1	Mathematics 30-2	A Level Mathematics	A Level Further Mathematics	H1	H2	H3	
Internal Assessment	Is it used?	No	Yes	Yes	Yes	No			Yes	Yes	No		No			No
	Weight for final grade	-	20%	20%	20%	-			50%	50%	-		-			-
	Format	-	Project				-			School-based assessment varies between schools		-		-		
External Assessment	Is it used?	Yes	Yes	Yes	Yes	Yes			Yes	Yes	Yes		Yes			Yes
	Weight for final grade	100%	80%	80%	80%	100%			50%	50%	100%		100%			100%
	Format	Unseen test / exam setting				Unseen test / exam setting			Unseen test / exam setting		Unseen test / exam setting		Unseen test / exam setting			Unseen test / exam setting

The assessment methods table presented above demonstrates how all of the mathematical programmes have external assessment in the form of an unseen written exam taken in an exam setting. Internal assessment is used in two of the qualifications, the Alberta Diploma and the IB DP (with the exception of the Further Mathematics HL programme which is assessed wholly through external assessments).

The table also highlights how all programmes employ unseen written examinations, completed under test conditions. At the time of writing, the weighting of the external examination(s) ranged from 50-100% although the weighting of external assessment in the Alberta Diploma will decrease from September 1st 2015.

5.4.2 Comparative Review of External Assessment Materials

5.4.2.1 Assessment Duration in Conjunction with Number and Type of Questions

The following table summarises the number and type of assessment materials reviewed, the duration and total number of marks available for each paper. "NA" indicates where no information was available at the time of writing.

Table 15: Exam duration, No. of questions and total marks

Qualification	Programme	Exam Papers Reviewed in this Study	Exam Duration	No. of Questions (for 2014/15)	Total number of marks awarded per paper
IB DP	Mathematical Studies SL	Paper 1	1 hr 30 min	15	90
		Paper 2	1 hr 30 min	6	90
	Mathematics SL	Paper 1	1 hr 30 min	10	90
		Paper 2	1 hr 30 min	10	90
	Mathematics HL	Paper 1	2 hr	13-14 ⁶¹	120
		Paper 2	2 hr	12-14 ⁶²	120
		Paper 3 Students select one: <ul style="list-style-type: none"> • Discrete Mathematics • Calculus • Sets, Relations and Groups • Statistics and Probability 	1 hr	4 ⁶³	60
	Further Mathematics HL	Paper 1	2 hr 30 min	16 ⁶⁴	150
		Paper 2	2 hr 30 min	9 ⁶⁵	150

Notes:

⁶¹ In 2014, timezone 1 had 13 questions, timezone 2 had 14 questions. In 2013, both had 13 questions. In 2012, timezone 1 had 12 questions, and timezone 2 had 13.

⁶² In 2014, timezone 1 had 12 questions and timezone 2 had 14 questions. In 2013, both had 13. In 2012, timezone 1 had 13 and timezone 2 had 12.

⁶³ In 2013 and 2012, all four had 5 questions.

⁶⁴ In 2013, this programme was previously Further Mathematics SL and had 5 questions, in 2012 it had 6.

⁶⁵ In 2013, this programme was previously Further Mathematics SL and had 6 questions, in 2012 it had 5.

Qualification	Programme	Exam Papers Reviewed in this Study	Exam Duration	No. of Questions (for 2014/15)	Total number of marks awarded per paper
Advanced Placement	Calculus AB	Single paper	3 hr 15 min	51	NA
	Calculus BC	Single paper	3 hr 15 min	51	NA
	Statistics	Single paper	3 hr	46	NA
GCE A level	A Level Mathematics	C1	1 hr 30 mins	11	75
		C2	1 hr 30 min	10	75
		C3	1 hr 30 min	9	75
		C4	1 hr 30 min	8	75
		S1	1 hr 30 min	8	75
		S2	1 hr 30 min	6	75
	A Level Further Mathematics	FP1	1 hr 30 min	9	75
		FP2	1 hr 30 min	8	75
		FP3	1 hr 30 min	9	75
SIPCAL	H1	Single paper	3 hr	11-13 ⁶⁶	95
	H2	Paper 1	3 hr	10-12 ⁶⁷	100
		Paper 2	3 hr	9-12 ⁶⁸	100
	H3	Single paper	3 hr	8 (approx.) ⁶⁹	100
Gāokǎo	Arts Stream	Single paper	2 hr	23	150
	Science Stream	Single paper	2 hr	23	150
Alberta Diploma	Mathematics 30-1	Single paper	2hr 30 min - 3 hr ⁷⁰	40	NA
	Mathematics 30-2	Single paper	2hr 30 min - 3 hr ⁷¹	40	NA

Across all the reviewed examination papers for upper secondary qualifications in mathematics, all questions are compulsory, ensuring that all candidates are required to revise and develop knowledge of the entire mathematics curriculum and are assessed on the full range of techniques and methods required across the syllabus.

The GCE A Level Mathematics, employing a modular assessment format, requires candidates to sit a number of separate examination papers, three in the first year of study and a further three in the second year. Six papers are required in order to gain the full A

⁶⁶ The H1 specification states that the examination will have 5 questions in Section A, and between 6-8 questions in Section B.

⁶⁷ The H2 specification states that the Paper 1 examination will have between 10-12 questions.

⁶⁸ The H2 specification states that the Paper 2 examination will have 3-4 questions in Section A and 6-8 questions in Section B.

⁶⁹ As stated in the specification.

⁷⁰ The exam is designed for students to be able to complete in 2hr 30 minutes however students are permitted an additional 30 minutes if needed.

⁷¹ As above.

Level, each requiring one hour and 30 minutes and a total of nine hours examination time for the overall single subject qualification. This allows for a large volume of material to be assessed across the two year programme and across a broad range of topic areas including pure mathematics and optional topics as discussed. GCE A Level Further Mathematics comprises a further six modules, all of which have the same exam duration as the A Level papers.

Papers for the IB DP Mathematical Studies SL and Mathematics SL papers are each one hour and 30 minutes in duration and the Mathematics HL Papers 1 and 2 are two hours in duration. Mathematics HL Paper 3 is the shortest exam, requiring one hour whilst the Further Mathematics HL papers are two hours 30 minutes each. As such the total time for external assessment ranges from three hours for SL programmes, to five hours for HL programmes, though it should be noted that the actual time spent on assessment will be longer for the Mathematical Studies SL, Mathematics SL and Mathematics HL given the internal assessment component.

This is similarly true for the Alberta Diploma, which allows three hours for the written exam in each programme and additional time for the internal assessment tasks. SIPCAL exams are each three hours and comprise 100 marks per paper

The AP Calculus AB and BC and AP Statistics examinations are each three hours and 15 minutes, which allows for a thorough assessment of calculus and statistics related topics. With a narrower focus than the other papers reviewed as part of this exercise, the duration and specific focus of the AP provides opportunity to assess in-depth knowledge and understanding of calculus or statistics.

Reviewing the assessment papers, the number of marks and questions demonstrates that programmes with longer duration may either assess a broader range of topics or assess a narrow range in more depth and a longer duration does not necessarily mean increased demand. In particular the Gāokǎo, with the shortest *overall* assessment of mathematics, places considerable demands on candidates in terms of timing, with 150 marks spread across the 23 questions, some of which present extended problems, to be completed in a timeframe of two hours.

5.4.2.2 Question Types and Weighting

A number of different types of question are used in the mathematics programmes reviewed, although their presentation and the ways in which they are structured vary by examination paper. The following broad question types have been identified:

- Multiple choice
- Short answer/numerical response
- Multi-part structured
- Extended problem
- Combination of multi-part structured and extended problem.

It is important to note that the awarding bodies each define short-answer questions differently. Accordingly definitions for these question types, as used in the context of this study, can be found in the Glossary.

The following table summarises the types of questions which are set across the various international qualification examination papers reviewed as part of this study:

Table 16: Comparative review of question types

Qualification	Programme	Multiple choice	Short answer	Multi-part structured	Extended problem	Combination of multi-part structured and extended problem
IB DP	Mathematical Studies SL	-	✓ ⁷²	✓	-	-
	Mathematics SL	-	✓ ⁷³	✓	✓	✓
	Mathematics HL	-	✓	✓	✓	✓
	Further Mathematics HL	-	✓	✓	✓	✓
Advanced Placement	Calculus AB	✓	-	✓	-	✓
	Calculus BC	✓	-	✓	-	-
	Statistics	✓	-	✓	-	-
GCE A level	A Level Mathematics	-	✓	✓	✓	✓
	A Level Further Mathematics	-	✓	✓	✓	✓
Gāokǎo	Arts Stream	✓	✓	-	✓	✓
	Science Stream	✓	✓	-	✓	✓
Alberta Diploma	Mathematics 30-1	✓	✓	-	-	-
	Mathematics 30-2	✓	✓	-	-	-

⁷² The IB DP Mathematical Studies SL papers include a number of short answer sub-questions as part of a larger multi-part question.

⁷³ The IB DP Mathematics SL papers include a number of short answer sub-questions as part of a larger multi-part question.

Multiple choice

Multiple choice questions are not used within the IB DP mathematics programmes or the GCE A Levels, but form a significant part of the assessment in the AP, Alberta Diploma and Gāokǎo examinations; however these questions exhibit some similarities and differences in format and presentation across the papers. Although multiple choice questions included in the papers typically present well-defined problems, for instance, there can be differences in the number of steps required to reach solutions and the technical complexity of the relevant strategy, impacting on the demand of individual questions.

The number of possible answers (distractors) varies between the papers, for example the Alberta Diploma multiple choice questions have four alternative answers whilst the Gāokǎo and the AP multiple choice questions have five. In both examinations, distractor answers are designed to be plausible so that students would find it difficult to guess the correct answer. The level of plausibility of distractor items in addition to the number of alternative answers can influence question demand. Multiple choice questions set in the Alberta Diploma and Gāokǎo examinations can also vary in the amount of written information presented and would need to be read and understood by the candidate in order to solve the related problems. Reading time may therefore be a factor that can influence examination demand, particularly in shorter examinations such as the AP in Calculus and Statistics and the Alberta Diploma, in which candidates are expected to complete a large number of such questions.

Short answer/numerical response

Short answer questions are used across some of the examination papers reviewed. This type of question is designed to assess knowledge and correct application of a limited range of mathematical techniques, the demand of such questions often relies on the technique or method being assessed. The IB DP Mathematical Studies SL and Mathematics SL papers include a number of short answer sub-questions to assess candidate's ability to solve problems typically involving a single- or two-step calculation or transformation as part of a larger multi-part question. The GCE AS Mathematics Core Papers 1 and 2 also include a similar proportion of short answer sub-questions.

Both the Gāokǎo and the Alberta Diploma include a significant number of gap fill, numerical response questions which, similar to the short answer questions on the IB DP and the A Level papers, and typically assess candidate's response to well-defined problems. Nevertheless, it is noted that some of the numerical response questions in the Gāokǎo can require more complex multi-step problems.

Multi-part structured

Multi-part structured questions are generally used throughout the IB DP Mathematical Studies and Mathematics SL and GCE A Level examination papers, with complex problems divided into sub-questions, each with separate mark allocations, that typically require candidates to identify a particular value or quantity in the first instance, and use this to answer subsequent questions.

Extended problem

The IB DP Mathematics HL, Further Mathematics HL, the GCE A Level Mathematics and Further Mathematics papers and the Gāokǎo examination free-response section contain a small number of extended problem questions which are not broken down into sub-parts. Extended problems are more open-ended in that the candidate must select and apply the correct procedures and work through a series of steps in providing a solution. Given the lack of guidance in selecting the appropriate procedures and adopting specific steps, and depending on the subject matter being assessed, these questions are typically of higher demand in comparison with the shorter answer questions and multi-part structured questions. In the absence of a set structure or a given method, these questions often have a number of acceptable strategies or alternative methods, some of which are specified in the accompanying mark schemes.

Combination of multi-part structured and extended problem

Questions which are broken into multiple parts but with one or more parts presenting more substantial, unstructured problems, typically assigned a higher allocation of marks, are also prominent across the GCE A Level, IB DP Mathematics HL and Further Mathematics HL, AP Calculus and Statistics and the Gāokǎo examination papers. Given the extended problem element, these questions can sometimes be of comparable demand or of higher demand as the standalone extended problems which are not broken down into parts.

Weighting of Questions

The examination papers reviewed vary in terms of the weighting and allocation of marks per question and question type. The IB DP examination papers all indicate the number of marks allocated to each question, and each sub-question. The IB DP Mathematical Studies SL papers contain the highest number of short answer sub-questions worth between 1 and 3 marks. The IB DP Mathematical Studies and Mathematics SL papers contain predominantly multi-part, structured questions in which guidance is given in completing all or most parts, most sub-questions are worth from 1 to 4 marks. Less structured multi-part questions, incorporating extended problems in sub-parts worth over 4 marks, are more prominent across the IB DP Mathematics HL papers. The majority of questions in the IB DP Further Mathematics HL papers are also multi-part with extended problems which are typically allocated a high number of marks. The IB DP Further Mathematics HL papers typically have a greater number of extended problems compared with the SL and HL papers, which carry a higher proportion of the overall marks.

As observed in the IB DP exam papers, the weighting of each multi-part question in the A Level papers varies depending on the precise requirements of each question, with marks ranging from 3 marks to over 20 marks per question. The AS and A2 Level examinations similarly comprise multi-part questions, in which a greater number of extended answer elements and stand-alone extended questions are included in the A2 and in the Further Mathematics papers. The GCE A Level Further Mathematics papers predominantly contain multi-part questions in which candidates are expected to develop their own strategy with little or no guidance from the examiner, with the majority of sub-parts worth at least 4 marks. A

small number of extended stand-alone problems with no sub-parts are also set, which typically carry a higher mark allocation.

In the Gāokǎo, free response questions are worth 50% of the overall marks in the paper and comprise a combination of structured multi-part and extended problem questions, which vary in length but are generally allocated from 12 to 20 marks each. The gap fill, numerical response questions in Part I, are each allocated 4 marks are worth 36% of the overall marks for the paper, whilst the multiple choice in Part II carry 5 marks per question and account for 13% of the overall assessment. Part marks are not allocated in the numerical response and multiple choice sections (Parts I and II respectively) due to the objective testing method, so candidates either gain the full marks for the questions in these sections or no marks if they are unable to provide the correct answers. The Gāokǎo Part III free response questions incorporate extended questions carrying up to 15 marks. Multi-part questions comprise the majority and firstly involve solving a number of structured questions, which carry 2 to 4 marks and are followed by two or more substantial problems, typically assigned 5 to 8 marks.

As with the Gāokǎo, the AP free response questions account for half of the marks on the Calculus AB and BC and Statistics papers overall, whilst the remaining marks are allocated to the multiple choice sections. In contrast to the other international examination papers reviewed, the AP free response section does not include any information for the student about the allocation of marks per sub-question, although candidates are informed that questions are given equal weighting, revealed in the mark scheme as 9 marks per question. Questions typically start with short answer problems carrying 1 to 3 marks, which lead onto at least one extended problem which is typically assigned a higher mark allocation of between 3 and 6.

The Alberta Diploma 30-1 and 30-2 mathematics examinations contain the highest proportion of multiple choice questions, comprising 70% of the available marks for external assessment. All multiple choice questions are weighted equally in the examination. The remaining 30% of the marks are allocated to 12 numerical response questions. Not all numerical response questions in the Diploma examination are worth the same number of marks as some have sub-questions.

5.4.2.3 Exam Paper Resources and Requirements

All examinations reviewed as part of this study provide formulae to assist in answering the set questions, with the exception of the Gāokǎo examinations which require students to commit to memory the relevant formulae in their preparation.

The majority of examinations reviewed permit or require the use of a graphical calculator. The A Level and the SIPCAL require graphical calculators for all papers, although it is understood that not all questions may involve the use of one. This allows for demanding questions to be set which can only be solved using a graphical calculator, it also ensures that candidates can check their answers quickly and accurately in the timeframe given for each exam. The AP Calculus AB and BC examination Sections A and B allow calculators whilst only Part B of the free response section does not permit use of a calculator. Whilst the

Gāokǎo paper does not specify whether a calculator is permitted, the exam regulations as specified on the Ministry of Education website indicates that it is.

The IB DP Mathematics SL Paper 1 and HL Paper 1 do not permit use of a calculator in any of the sections. The availability of a calculator may have an impact on the relative demand of some of the IB DP questions in the relevant papers in comparison to other examinations. For instance non-calculator papers cannot, by design, assess topics which require complex graphical interpretation which could potentially lower the demand. At the same time it is acknowledged that non-calculator papers can assess candidates' ability to solve problems manually and require candidates to check their own workings thoroughly and independently, lending more emphasis to precision and accuracy and ability to work quickly and efficiently under timed conditions.

Although non-calculator papers are compulsory in the IB DP, it is also noted that the IB DP Mathematics HL Papers 2 and 3 include a greater number of questions than any of the other reviewed papers that are specifically designed to assess the candidate's ability at using and interpreting outputs from graphical calculators. The inclusion of such questions serves to assess candidate's thorough knowledge and application of graphical calculators.

5.4.2.4 Breadth and Depth of Content Assessed

When considering content, it is important to consider the breadth and depth of both what is taught and assessed, although it is envisaged that the curriculum content taught will be considered in more depth within the parallel curriculum analysis being conducted.

As shown in the table below, desk-based research revealed no reliable means of comparing the volume of study across the different qualifications and programmes:

- The IB DP and the Alberta Diploma propose a number of teaching hours (at topic level in the IB DP, and at programme level within the Alberta Diploma)
- The GCE A Levels have guided learning hours attached to each overall programme
- SIPCAL programmes are expressed only in terms of years of study
- There is no assigned programme of study for the Gāokǎo.

Table 17: Study time/volume for mathematics programmes

Qualification	Programme	Study Time/Volume
IB DP	Mathematical Studies SL	Number and algebra: 20 teaching hours (TH) Descriptive statistics: 12 TH Logic, sets and probability: 20 TH Statistical applications: 17 TH Geometry and trigonometry: 18 TH Mathematical models: 20 TH Introduction to differential calculus: 18 TH Project: 25 TH Total teaching hours 150
	Mathematics SL	Algebra: 9 TH Functions and equations: 24 TH Circular functions and trigonometry: 16 TH Vectors: 16 TH Statistics and probability: 35 TH Calculus: 40 TH Mathematical exploration (internal assessment): 10 TH Total teaching hours 150
	Mathematics HL	Algebra: 30 TH Functions and equations: 22 TH Circular functions and trigonometry: 22 TH Vectors: 24 TH Statistics and probability: 36 TH Calculus: 48 TH Statistics and probability / Sets, relations and groups / Calculus / Discrete mathematics: 48 TH Mathematical exploration (internal assessment): 10 TH Total teaching hours 240
	Further Mathematics HL	Linear algebra: 48 teaching hours (TH) Geometry: 48 TH Statistics and probability: 48 TH Sets, relations and groups: 48 TH Calculus: 48 TH Discrete mathematics: 48 TH Note: One of topics 3–6 will be assumed to have been taught as part of the Mathematics HL course and therefore the total teaching hours will be 240 not 288. Total teaching hours 240
Advanced Placement	Calculus AB	The curriculum is set by the school and there are no guided learning hours (GLH). If each programme is taught over one school year, for 180 days, five days a week, in 47 minute class periods, the total classroom hours would be 141. If the programme is taught for one half of the school year, it would be roughly 70 hours.
	Calculus BC	
	Statistics	

Qualification	Programme	Study Time/Volume
GCE A level	A Level Mathematics	360 guided learning hours (GLH) (180 GLH per year). Students have two total years of study.
	A Level Further Mathematics	360 GLH (180 GLH per year). Students have two total years of study.
SIPCAL	H1	Students have 2-3 years of study.
	H2	Students have 2-3 years of study.
	H3	Students have 2-3 years of study.
Gāokǎo	Arts Stream	-
	Science Stream	-
Alberta Diploma	Mathematics 30-1	125 hours of classroom-based teaching.
	Mathematics 30-2	125 hours of classroom-based teaching.

There are similar limitations in estimating the proportion of time associated with mathematics study given that the AP programmes would typically be studied alongside the US High School Diploma.

In addition to the volume of study, it is important to examine the mathematics content assessed. Therefore, in terms of assessment, this section presents the type, breadth and depth of content assessed in each of the programmes. Please note that where percentages are quoted next to topics (within the table), this reflects the proportion of marks that are formally prescribed by the awarding body for that topic.

Alberta Diploma

The Alberta Diploma programmes specify the proportion of assessment to be assigned to each topic areas as follows:

Alberta Mathematics 30-1

		Core		
External exams (50%)	Multiple choice questions (35%)	Relations and Functions (55%)	Trigonometry (29%)	Permutations, Combinations and Binomial Theorem (16%)
	Numerical response questions (15%)	Relations and Functions (55%)	Trigonometry (29%)	Permutations, Combinations and Binomial Theorem (16%)
Internal assessment (50%)				

Alberta Mathematics 30-2

		Core		
External exams (50%)	Multiple choice questions (35%)	Relations and Functions (50%)	Probability (33%)	Logic and Reasoning (17%)
	Numerical response questions (15%)	Relations and Functions (50%)	Probability (33%)	Logic and Reasoning (17%)
Internal assessment (50%)	50%			

Detailed information on internal assessment in the Alberta Diploma was not publicly available at the time of writing and as such it is not possible to provide a definitive comment on the breadth and depth of content assessed across the programme. Nevertheless, it is clear that there are three main areas of content tested within external assessment of the Mathematics 30-1 and 30-2, with each programme assigning at least half of the weighting for both multiple choice and numerical response questions to the assessment of relations and functions.

Gāokǎo

The Gāokǎo covers a comparatively broad range of topics given the duration of the exam. There are around a dozen topics identified as compulsory, and covered within the exam. Each year, one of three further topics will be selected by the examining body for inclusion in the paper. The topics covered in both the Arts and Science streams are very similar, reviewing the 2014 papers revealed that some mathematics questions were included within both papers, whilst some may pose questions on the same topics, with the same structure and mark allocation but with different values or quantities specified in the questions.

Gāokǎo mathematics exam (both streams)

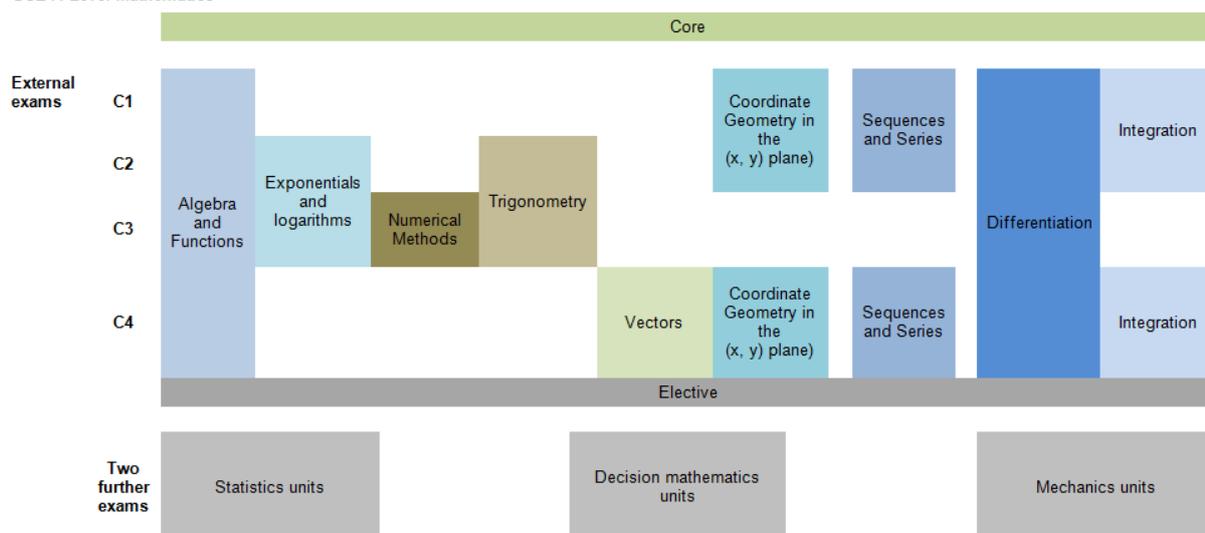
External exam (100%)	Core
	Algorithms Basic functions I (Exponentials, logarithms) Basic functions II (trigonometric functions) Complex Numbers Derivatives Geometry Inequalities Logic Mathematical Reasoning and Proof Sequences Sets Statistics and Probability Trigonometry
	Elective
	Geometric proof Inequalities (further) Parametric equations and polar coordinates

GCE A Levels

The GCE A Level Mathematics comprises six units, four of which are compulsory. The compulsory units (C1-C4) are designed to provide an in-depth focus on pure mathematics and overall the assessment of pure mathematics topics accounts for two thirds of the overall assessment. The exam papers assess similar areas of content, but with increasing depth.

The remaining two units can be selected from a choice of combinations so students could study statistics (S1 and S2) which would provide the most in-depth assessment; or choose a combination of units from different areas of mathematics – such as statistics (S1) and mechanics (M1).

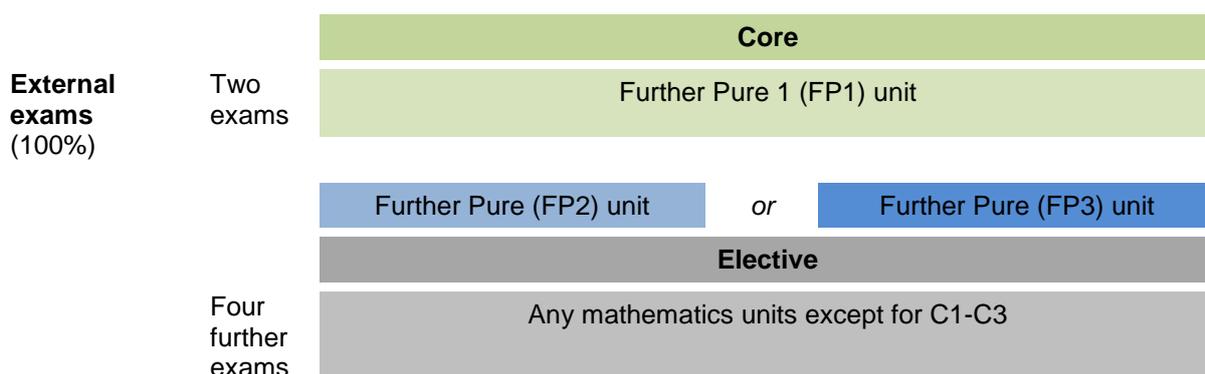
GCE A Level Mathematics



As can be seen from the chart above, many topics are reflected in the assessment of two or three units in GCE A Level mathematics. As such, the C4 examination assesses these topics in considerable depth; this was a key factor in the rating of questions by demand in the subsequent item level analysis. It is also important to highlight that the A Level actively looks to link these topics to one another in both the curriculum, teaching (as evidenced through Pearson teacher guides) and in the assessment. For example, algebra and functions, a topic taught and assessed in each of the four units, is also linked to coordinate geometry in the (x, y) plane and sequences and series; whilst it was observed within the exam papers that differentiation and integration are often tested together, as part of a multi-part structured question.

By comparison to the GCE A Level Mathematics, the GCE A Level Further Mathematics can vary considerably in breadth and depth due to the larger range of unit combinations available.

GCE A Level Further Mathematics



IB DP

All of the IB DP programmes aim to assess across the whole curriculum. The tables below present the key topics covered.

IB DP Mathematical Studies

External assessment (80%)	Papers 1 and 2	Core			
		Number and Algebra	Descriptive Statistics	Statistical applications	Introduction to differential calculus
		Geometry and trigonometry	Logic, sets and probability	Mathematical models	
Internal assessment (20%)		Project			

The proportion of teaching time allocated to these areas is relatively even with less time spent on descriptive statistics.

IB DP Mathematics SL

External assessment (80%)	Papers 1 and 2 (40% each)	Core		
		Algebra	Functions and Equations	Circular Functions and Trigonometry
		Vectors	Statistics and probability	Calculus
Internal assessment (20%)		Individual exploration of an area of mathematics		

The proportion of teaching time allocated to these areas varies, with the lowest proportion (6%) assigned to *Algebra*, 11% each to *Vectors* and *Circular Functions and Trigonometry*. The highest proportion of teaching time is allocated to *Calculus* (27%), followed by *Statistics and probability* (23%).

IB DP Mathematics HL

External Assessment (80%)	(60% core)	Core		
		Algebra	Functions and Equations	Circular Functions and Trigonometry
		Statistics and Probability	Vectors	Calculus
	(20% elective)	Elective		
Statistics and Probability		Sets, Relations and Groups	Calculus	Discrete Mathematics
Internal Assessment (20%)	Individual exploration of an area of mathematics			

As can be seen from the table above the IB DP Mathematics HL offers the same core topics as the Mathematics SL, but with the addition of optional topics, from which students select one. The total teaching time is much higher, at 240 compared to 150 for the Mathematics SL. Both the allocation of teaching time and the assessment materials reviewed highlight a stronger emphasis on algebra in the Mathematics HL programme (more than triple the number of teaching hours, and 12.5% of the total hours for Mathematics HL).

IB DP Further Mathematics HL

External Assessment (100%)	Core		
	Linear Algebra	Geometry	Sets, Relations and Groups
	Statistics and Probability	Discrete Mathematics	Calculus

AP programmes

AP Calculus AB

External Assessment
(100%)

Core
Calculus

AP Calculus BC

External Assessment
(100%)

Core
Calculus

AP Statistics

External Assessment
(100%)

Core
Statistics

SIPCAL programmes

H1 Mathematics

External Assessment
(100%)

Core	
Pure Mathematics (37%)	Statistics (63%)

H2 Mathematics

External Assessment
(100%)

Core	
Pure Mathematics (70%)	Statistics (30%)

As can be seen from the H1 and H2 tables above, the two main topic areas are pure mathematics and statistics although the number of sub-topics associated with each is higher within the H2 Mathematics. The syllabi for both programmes specify the number of marks which must be assigned to each and it is on this basis that the above percentages have been calculated.

H3 Mathematics

External Assessment
(100%)

Core		
Functions and Graphs, Sequences and Series and Calculus (40%)	Combinatorics (25%)	Differential Equations as Mathematical Models (35%)

5.4.3 Comparative Analysis of Assessment Items

This section considers the relative demand of questions posed within the external assessments for each mathematics programme, where materials were available through comparison of assessment in selected topics and general observations and themes drawn from this. It is important to highlight however that the differences in question demand would not be considered in themselves to indicate a higher overall level of demand. As indicated in the methodology, it is important to consider also how marks are awarded to these questions.

5.4.3.1 Probability

In terms of the type and complexity of questions posed, there were both some clear similarities and some marked differences in the assessment of probability across the mathematics programmes reviewed. A number of questions considered to be of a low demand were identified across several programmes. For example, in the Mathematics 30-2, students were given three ratios, though introduced differently (odds in favour of; odds against; and probability). From this students were asked to select the answer, from a choice of four, which correctly ordered the ratios from lowest to highest probability, requiring a conversion of ratios to probability fractions. In a numerical response question, students were asked to calculate the probability of selecting a sweet of one of two particular colours from a range of seven. Both of the two questions described from the Mathematics 30-2 programme are pitched at an “Acceptable” standard, a term used within the Alberta Diploma to reflect questions which could be answered by students achieving an “Acceptable standard” (a mark of 50-79%). Two probability questions in the sample were identified as “SE” or “Excellence” indicating that a high achieving (“Standard of Excellence”, 80% and above) student should be able to answer these. Within the context of this study, these questions were considered to be of medium demand.

There were some clear similarities in the probability questions posed within the IB DP Mathematics SL and IB DP Mathematical Studies SL. Both papers for example, included multi-part questions, which started by asking students to complete a tree diagram where three of the values were given, either in the question or on the diagram itself, and the student needed to identify the remaining three values. The completed tree diagram could then be used to support the student in answering the next sub-questions. The questions were typically considered to be of low, low-medium, and medium demand though it was observed that given the use of multi-part questions and the mark allocation, it was in some cases possible to obtain over half the marks from a question through answering the low demand sub-questions.

The Gāokǎo included two questions on probability from a total of 23 questions across the paper. These were both considered to be of a medium demand, presenting problems – such as calculating minimal probability from a random variable – which are not broken down into sub-questions to provide students with a strategy for responding.

Probability is assessed in the AP Statistics programme through both multiple choice and free response questions. The multiple choice questions present a scenario and ask students to select the correct probability from a choice of five possible answers. These were considered to be of medium and medium-high demand, taking into account the plausibility of the distractors; that some questions incorporate mean, distribution and standard deviation in to the question; and that the questions are not broken down into sub-parts. A free response question asks students to make a recommendation on which photocopier would have the lowest overall cost based on the sale price and the distribution and probability of the number of repairs. The question, though extended and not broken down into sub-parts, does provide some direction to the student in the question and accordingly was also considered to be of medium demand.

The key similarities evident within the above five programmes (Mathematics 30-2, Gāokǎo, AP Statistics, IB DP Mathematical Studies SL and Mathematics SL) is the testing of probability through real life, non-abstract examples. This is similarly true in the S1 paper from the GCE A Level Mathematics although the A Level questions were typically multi-part and considered to be of medium demand. The IB DP Mathematics HL by contrast started with some real-life questions, becoming increasingly more abstract throughout the paper. The IB DP Further Mathematics HL probability questions were similarly abstract, and assessed probability distribution of discrete random variables.

The assessment of probability was considered to be particularly demanding in the IB DP Mathematics HL where the Statistics and Probability paper is taken; and within the Further Mathematics HL. Half of the marks were assigned to questions judged to be of high demand with all questions considered to be of at least medium demand and though questions were broken down into sub-questions, a substantial number of marks were often awarded to the final sub-questions which were considered to be of high demand. The Gāokǎo and the GCE A Level assigned a similar proportion of marks to questions determined to be of medium demand or above. Both the IB DP Mathematics SL and the IB DP Mathematical Studies SL included a range of questions, though overall of a lower demand than the other IB DP and some of the international programmes. As with other topics examined, direct comparisons with the Alberta Diploma and AP programmes are challenging due to the use of sample questions without guidance on the proportion of marks assigned to these questions. Nevertheless, considering the level of question demand, on balance, indicates that the IB DP Mathematical Studies SL and IB DP Mathematics SL and Alberta Diploma Mathematics 30-2 questions assess probability at a broadly comparable level.

5.4.3.2 Differentiation

Of the five topics reviewed, this topic had the highest number of questions across the international qualifications. Of all qualifications, the GCE A Level Mathematics had the highest proportion of marks related to the topic with differentiation featuring in all four of the compulsory units (C1, C2, C3, and C4). Differentiation is also covered in some of the A Level Further Mathematics units.

Overall the GCE A Level Further Mathematics papers had the largest proportion of marks assigned to high demand questions on differentiation. This is followed closely by the IB DP Mathematics HL programme. GCE A Level Mathematics, whilst having around a fifth of marks assigned to high demand questions, also encompasses questions of varying levels of demand, with clear progression between the units: for example C1, the first unit taken in the AS Level, had the highest proportion of low demand questions but by C4, no low level questions were included in the paper with almost two thirds of questions considered to be of high demand. By contrast, the IB DP Mathematics HL did not have any differentiation questions considered to be of low demand.

In terms of differentiation, the demand of questions within the IB DP Mathematics HL lies between GCE A Level Mathematics and GCE A Level Further Mathematics. Questions on differentiation in IB DP Mathematical Studies SL and IB DP Mathematics SL vary from low to medium, posing a lower level of demand than the GCE A Level programmes.

Comparison to AP assessments is more challenging since, in the absence of published full papers, the questions reviewed were from sample / past questions meaning that the marks for the individual questions were not known. To compare based on the number of questions would distort the analysis since the type and structure of questions differ substantially between the awards. Whilst the AP assessments do contain some free-response questions, many of the ones relating to differentiation were single-part, multiple choice questions. The IB DP programmes by contrast include several multi-part questions with the level of demand and associated marks increasing from one part to the next. Differentiation was covered within the AP Calculus AB and BC specifications, with the questions typically falling at a medium-high level, accounting for the depth of knowledge required in the topic and the plausibility of the distractors. The Alberta Diploma also covers differentiation, though it had substantially fewer questions relevant for analysis.

5.4.3.3 Sequences and/or Series

Similar to differentiation, the proportion of marks assigned to high-demand questions was high within the IB DP Mathematics HL. The proportion of marks assigned to questions rated medium and above was broadly comparable between the IB DP Mathematics HL and Gāokāo. However, both sit slightly below the GCE A Level Further Mathematics but above the GCE A Level Mathematics which, though including questions at all levels of demand, assigned a similar proportion of marks to sub-questions considered of low- and low-medium demand. IB DP Mathematical Studies SL and Mathematics SL questions were primarily judged to be of low demand; being broken down into short, clear sub-questions each worth a similar or equal number of marks.

Sample questions from the AP Calculus BC included both multiple choice and free response questions, ranging in demand from medium to high. Where questions were considered to be of high demand, it was typically due to the complexity of the topic, with some questions covering content sometimes found in first semester/year university study in the USA (reflecting the purpose of the AP and the consideration given to sequences and series in the specialised calculus programme).

5.4.3.4 Vectors

In the assessment of vectors, the GCE A Level Further Mathematics (paper FP3), Gāokāo and the IB DP Mathematics HL questions were considered overall to be the most demanding, with the questions largely considered to be of a high demand. In particular the IB DP Mathematics HL questions are similar both in structure, range of demand and mark allocation to those found in the GCE A Level Further Mathematics. IB DP Mathematics SL included some high mark questions on vectors which were broken down into sub-questions. In places these sub-questions were broken down into further sub-questions which took the student through a problem step by step, albeit with the demand and mark allocation increasing in the later sub-questions. Overall these were considered less demanding than those in the GCE A Level Mathematics.

5.4.3.5 Trigonometry

As outlined in the methodology, trigonometry was selected for analysis both due to its inclusion in a number of the programmes reviewed in this study, and its identification in previous studies as a complex topic, with potential to raise the demand of assessment where included.

The overall proportion of marks assigned to trigonometry questions rated medium and above, was consistent with the findings of the other topics. The IB DP Mathematics HL included both single- and multi-part questions. Some single-part questions were considered to be of medium demand, while others were high, with both assigned a similar number of marks. The GCE A Level Mathematics questions all had two or three sub-questions, which like the IB DP Mathematics HL, ranged from medium to high demand with a similar allocation of marks to each. Both the IB DP Mathematical Studies SL and Mathematics SL included some high demand sub-questions but in the IB DP Mathematical Studies SL, a higher proportion of marks were assigned to sub-questions of a low or low-medium demand. For example, in a 15-mark question divided into four sub-questions, the first two sub-questions, asking students to find the size of an angle and the area of a triangular field (each judged to be simple operations), were allocated seven of the 15 marks. A third low-medium sub-question, calculating the length between two points, was worth a further three marks. The high demand sub-question was therefore allocated the remaining five marks. The IB DP Mathematics SL had a relatively even distribution of marks between low - medium - high questions.

Released trigonometry questions from the Alberta Diploma Mathematics 30-1 exams ranged in demand, echoing the Alberta Education's rating of questions as Acceptable or Excellence. For example, a question identified as Acceptable, and rated as low-medium demand in this

study, asked students to calculate the coordinates of an intersection point of the unit circle and the terminal arm at a particular angle; selecting the correct answer from a choice of four possible answers which were similar (involving the same numbers but representing x or y; and with the inclusion of some positive and negative numbers). Other questions, identified as Excellence, were rated as medium, medium-high and high.

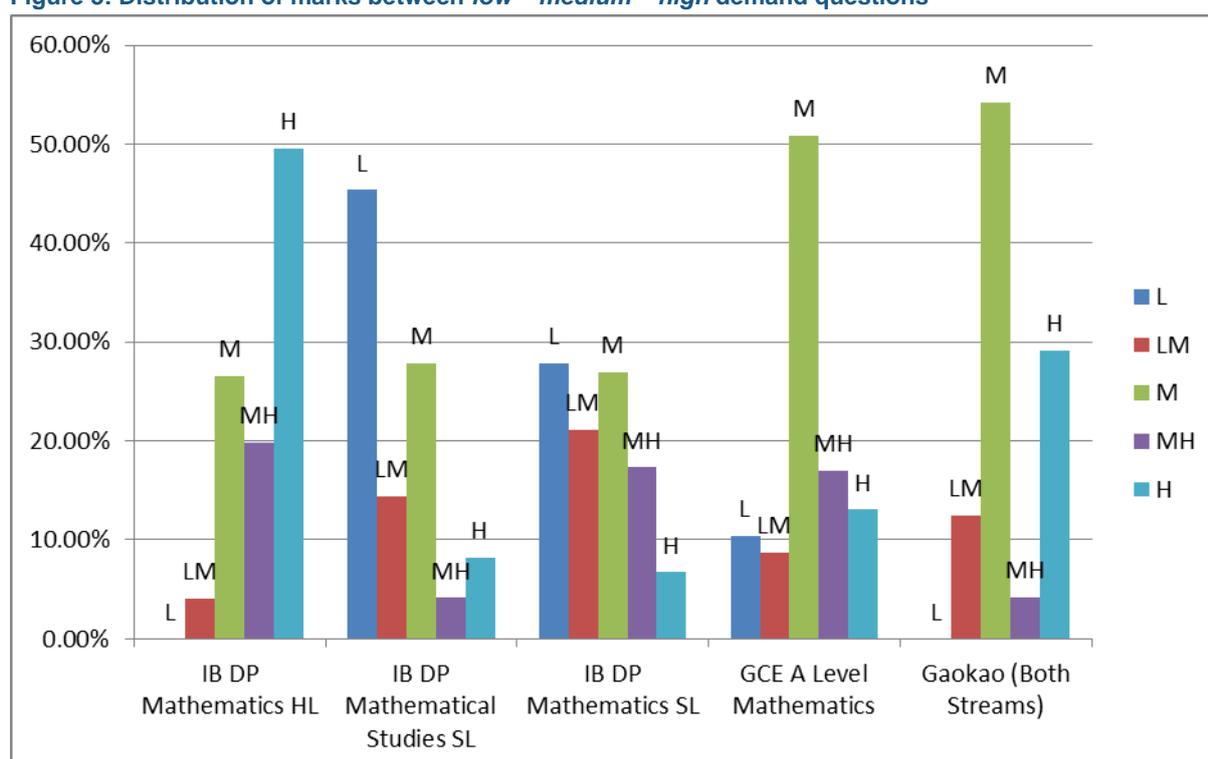
The Gāokāo also had a number of questions on trigonometry. In contrast to many of the other programmes examined, which had questions at a range of demand levels, all of the Gāokāo trigonometry questions were considered to be of medium demand, in particular due to the absence of sub-questions or alternative direction; the student needs to devise a strategy to answer these questions correctly.

Since all three AP programmes examined are specialised in either Calculus or Statistics, trigonometry was not featured within the assessment.

5.4.3.6 General observations

The chart below indicates the distribution of marks between low - medium - high demand questions in each programme, drawing on the findings of the comparative analysis by topic.

Figure 3: Distribution of marks between *low – medium – high* demand questions



Notes

1. Due to the various combinations that could be encompassed within the GCE A Level Further Mathematics (encompassing AS or A2 units), it was found that the distribution of marks across questions of different demand could vary depending on the elective modules taken by the student. This is discussed below.
2. Since the IB DP Further Mathematics included few questions related to the sample topics, this has been excluded from the chart above.

As can be seen from the chart above, the sample indicates that the IB DP Mathematics HL assigns the highest proportion of marks to questions judged to be of high demand. Of the questions reviewed, the IB DP Mathematics HL also had the highest number of marks considered to be of medium demand or higher (96%).

Both the IB DP Mathematical Studies SL and IB DP Mathematics SL had higher proportions of low demand questions than the GCE A Level Mathematics and the Gāokǎo, although marks were most evenly distributed across low / medium / high demand questions in the IB DP Mathematics SL.

Although the IB DP Mathematics and Further Mathematics HL papers typically included a mixture of real-life and abstract problems, the majority of the programmes examined focused exclusively or near-exclusively on real life scenarios, reflecting a shared purpose among a number of the programmes (identified in section 5.1, in particular Alberta Mathematics 30-2 and the IB DP Mathematical Studies SL which place particular emphasis on the application of mathematics in every-day life) to focus on the application of mathematics, thereby supporting them in work or higher education study areas where mathematical background would support study but where a high and/or in-depth level of mathematical knowledge is not required.

5.5 Marking

As outlined above, the mathematics programmes reviewed use a variety of question types which then reflects how the programme assessments are marked. Multiple choice examination questions in the Alberta Diploma, the first section of the AP and Part I Gāokǎo examinations are generally marked by a machine. Given that there is no requirement or marks awarded to method or reasoning in the respective papers, machine scoring is a fast and effective marking practice for this type of questions. In the case of the Alberta Diploma, numerical response questions are also marked by a machine, lending objectivity to the marking process.

Table 18: Marking approach

Qualification [All mathematics programmes unless specified]	Machine marked [Answer key]	Marking guidelines used
IB DP	–	✓
Advanced Placement	✓ for multiple choice questions	✓ for free response questions
GCE A Level	–	✓
Gāokāo	✓ for multiple choice questions	✓ for free response questions
Alberta Diploma	✓	–

The marking approach and question types have a direct impact on the way in which marks are awarded for assessment of each programme.

Table 19: Comparative Review of Marking Methodologies for Written Examinations

Qualification [All mathematics programmes unless specified]		Method Marks Awarded	Credit Given for Alternative Methods ⁷⁴	Accuracy Marks	Breakdown of marks given in the mark scheme	
					Per sub-question	Per step
IB DP		✓	✓	✓	✓	✓
AP	MCQ	–	–	✓	N/A	N/A
	FRQ	✓	NE*	✓	✓	✓
GCE A Level		✓	✓	✓	✓	✓
Gāokāo	Part I	–	–	✓	N/A	N/A
	Part II	–	–	✓	N/A	N/A
	Part III	✓	✓	✓	✓	NE
Alberta Diploma		–	–	✓	N/A	N/A

* Not evident.

The clearest similarities can be seen between the IB DP marking and the GCE A Level. Mark schemes for all IB DP and GCE A Level programmes clearly show the breakdown of marks for each step in multi-step calculation questions, and the allocation of credit per question. IB DP students receive marks for showing the correct method, accuracy and reasoning, whilst in the GCE A Level, method marks cover both method and reasoning skills. Both the IB DP and A Level mark schemes allow credit to be given in some cases for wrong

⁷⁴ In comparison to the best method, as identified in the mark scheme.

final answers, allowing for errors in the working to be carried through from previous sub-parts in which mistakes were made, although absurd answers are never accepted.

The GCE A Level mark schemes also make a distinction between accuracy marks, which can only be awarded if the candidate has demonstrated the correct method, and unconditional accuracy marks where no method needs to be provided. In the IB DP, marks for accuracy are often, but not always, dependent on the preceding reasoning and method. Both the IB DP and A Level mark schemes clearly indicate cases in which alternative methods can be used and awarded credit. The flexibility of the IB DP and GCE A Level mark schemes in allowing credit to be awarded for different methods and levels of response to questions can be seen as a key strength of the assessment process. At the same time, clear notation and guidance with respect to how marks should be allocated lends consistency and objectivity to the marking process to ensure that all candidates receive the grade their performance merits in accordance with the mark scheme.

As with the GCE A Level and IB DP mark schemes, the AP free response marking guidelines similarly demonstrate the breakdown of marks per question and for each correct step in the relevant method. AP scoring guidelines do not distinguish between method marks and accuracy marks in the same way as the IB DP and GCE A Level mark schemes. The AP guidelines nonetheless indicate that both are taken into consideration when marking candidate responses, with credit awarded for correctly stated the relevant steps required in solving a given problem. However, they do not list alternative methods or responses to questions, there is no additional guidance given to examiners regarding the use of their own judgement in accepting alternative methods. It is also important to highlight that since 50% of the marks are weighted to multiple choice questions, where no method marks are available, a notably lower proportion of marks can be attained for demonstrating understanding and application of the method in comparison to the IB DP.

The Gāokāo sample answers document, like the A Level, AP (free-response sections) and IB DP mark schemes, provides comprehensive sample responses to the free response questions in its mark scheme with full method and reasoning indicated. However, in contrast to the A Level, AP and IB DP mark schemes which provide breakdown of marks for method and answer, only the total number of marks per question and per sub-question is given in the Gāokāo mark scheme. There is no subdivision of marks allocated for method in the solutions to problems and accordingly UK NARIC has been unable to discern whether examiners allocate marks to partially correct answers which may demonstrate a number of correct steps but fail to arrive at the final solution.

As such it is clear that making a minor error in the calculation could have different implications within the international programmes: whilst in the IB DP programme assessments, a candidate may still obtain some marks for demonstrating the method without reaching the correct answer, candidates of the AP, Gāokāo and Alberta Diploma would not obtain any for the multiple choice questions which typically make up a significant proportion of the student's marks.

5.6 Grading, Performance and Progression

The purpose of this section is to identify how different levels of student achievement/performance are represented within each programme, and what, in practice, these levels mean in terms of progression to higher education.

5.6.1 Grading Systems

A key challenge in comparing student performance across the different qualifications reviewed within this study is the variation between the grading systems used; both in terms of the format, the number of grades available and the notion of a “pass” / “fail”.

The table below summarises the grading systems used for each programme:

Table 20: Grading systems

	IB Diploma Programme Mathematics	Edexcel GCE A level Mathematics	Advanced Placement Mathematics	Alberta High School Diploma Mathematics 30-1 / 30-2	SIPCAL		Gāokǎo Mathematics (Shanghai municipality)
					H1 and H2	H3	
Grade system used	Numeric	Letter	Numeric	Pass / Fail	Letter	Pass / Fail	Numeric
Total number of grades	7	7	5	3	7	4	Students are graded out of 150 for mathematics, 600 overall
Total number of pass grades	7	6	5	2	5	3	
Highest pass grade	7	A*	5	Standard of Excellence ⁷⁵	A	Distinction	150, 600 overall
Lowest pass grade	1	E	1	Acceptable standard	E	Pass	N/A
Fail grades used?	-	✓	-	✓	✓	✓	-
Summary of all grades (in descending order, highest to lowest)	7 6 5 4	A* A B C	5 4 3 2	Standard of Excellence (80-100%) Acceptable standard (50-79%)	A B C D	Distinction Merit Pass	
Fail grades, where used, are denoted in red.	3 2 1	D E U	1	<i>Fail (below 49%)</i>	E S (sub-pass) ⁷⁶ <i>Ungraded</i>	<i>Ungraded</i>	

⁷⁵ Students are also given an overall percentage. The percentages corresponding to "Standard of Excellence" and "Acceptable Standard" are defined in the summary row.

⁷⁶ Students who fail to pass a subject at the lowest pass grade (E) will be awarded an S (sub-pass) or Ungraded. These grades will not appear on the GCE A level certificate, but will however be included on the student's result slip.

As can be seen above, the IB DP, GCE A Level and SIPCAL use the highest number of distinct grades, enabling for the greatest differentiation of student performance.

5.6.2 Grade Descriptors

Overarching and/or subject grade descriptors are published for three of the qualifications reviewed, the IB DP, the GCE A Level and the Alberta Diploma. Differences in the level of detail, skills coverage and use of terminology are evident across the different descriptors. In all descriptors, however, performance is indicative of the skills and aptitude of the typical candidate achieving each grade, whilst there is little differentiation between varying levels of performance across distinct areas of the syllabi.

The following table lists out key sections of IB DP and GCE A Level grade descriptors which highlight the level of knowledge understanding and problem solving expected to achieve each grade.

Table 21: IB DP Grade descriptors in comparison with the GCE A Level descriptors

IB DP Grade	Descriptor Key Features	GCE A Level Grade	Descriptor Key Features ⁷⁷
7	<ul style="list-style-type: none"> • Demonstrates a thorough knowledge • Applies mathematical principles at a sophisticated level • Problem-solving techniques in challenging situations 	Grade A	<ul style="list-style-type: none"> • recall or recognise almost all the mathematical facts, concepts and techniques that are needed • When confronted with unstructured problems, can often devise and implement an effective solution strategy.
6	<ul style="list-style-type: none"> • Demonstrates a broad knowledge • Successfully applies mathematical principles • Uses problem-solving techniques in challenging situations 		
5	<ul style="list-style-type: none"> • Demonstrates a good knowledge of the syllabus • Successfully applies mathematical principles in performing routine tasks; • Successfully carries out mathematical processes in a variety of contexts 	Grade C	<ul style="list-style-type: none"> • recall or recognise most of the mathematical facts, concepts and techniques that are needed • When confronted with unstructured problems, can sometimes devise and implement an effective and efficient solution strategy.
4	<ul style="list-style-type: none"> • Demonstrates a satisfactory knowledge of the syllabus • Applies mathematical principles in performing some routine tasks; • Successfully carries out mathematical processes in straightforward contexts 		

⁷⁷ Pearson Education Ltd, 2013. Specification GCE Mathematics. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/UA035243_GCE_Lin_Maths_Issue_3.pdf> [Accessed 23rd April 2015].

IB DP Grade	Descriptor Key Features	GCE A Level Grade	Descriptor Key Features ⁷⁷
3	<ul style="list-style-type: none"> • Demonstrates partial knowledge of the syllabus and limited understanding of mathematical principles • Limited understanding of mathematical principles in performing some routine tasks; • Attempts to carry out mathematical processes in straightforward contexts 	Grade E	<ul style="list-style-type: none"> • recall or recognise some mathematical facts, concepts and techniques that are needed • sometimes select appropriate standard models to use in some real world contexts
2	<ul style="list-style-type: none"> • Limited knowledge of the syllabus • Attempts to carry out mathematical processes at a basic level 		
1	<ul style="list-style-type: none"> • Minimal knowledge of the syllabus • Demonstrates little or no ability to use mathematical processes. 		

IB DP grade descriptors provide a holistic summary of generic skills expected at the various levels of achievement. By contrast, the GCE A Level grade descriptors provide more detail regarding the subject specific skills indicative of performance at particular grades. For instance whilst the IB DP descriptors make reference to problem solving on a general level, the GCE A Level provides additional detail on the skills involved in using, interpreting and refining mathematical models in solving problems. IB DP grade descriptors are provided for all grades ranging from 1 to 7, whilst the GCE A Level provides descriptors for performance at three grades, grade A, C and E, omitting grades A*, B and D.

Both A Level and IB DP descriptors demonstrate progression in the level of skills and knowledge expected from lower level to higher levels of performance required to gain the high grades, i.e. a 7 in the IB DP (highest) or an A in the GCE A Level (second highest, after A*). Candidates achieving IB DP grades 6 and 7 typically demonstrate a broad to thorough knowledge of the syllabus and use problem solving techniques in challenging situations, which correlates broadly to the level of knowledge described by the Grade A descriptor in the GCE A Level. For instance, A Level candidates achieving grade A in the A Level, typically recall or recognise almost all mathematical facts, concepts or techniques and can solve unstructured problems in a variety of contexts.

The ability to recall and recognise most mathematical facts, concepts and techniques indicative of performance at grade C in the GCE A Level can be interpreted as broadly comparable to the level of performance described by the IB DP grade 5 and grade 4 descriptors. IB DP candidates achieving this level of performance are similarly expected to demonstrate a satisfactory to good knowledge of the syllabus, and an ability to solve problems in a variety of contexts. Moreover, the GCE A Level grade E descriptor, which makes reference to the ability to recall some mathematical facts, corresponds to the IB DP grade 3 descriptor, which expects candidates to demonstrate a partial knowledge of the syllabus, yet limited conceptual understanding of mathematical principles.

The Alberta Diploma grade descriptors are broader in comparison with those used for the IB DP mathematics programmes, and also include reference the percentage required to gain either the Acceptable or Standard of Excellence. Some similarities are nevertheless

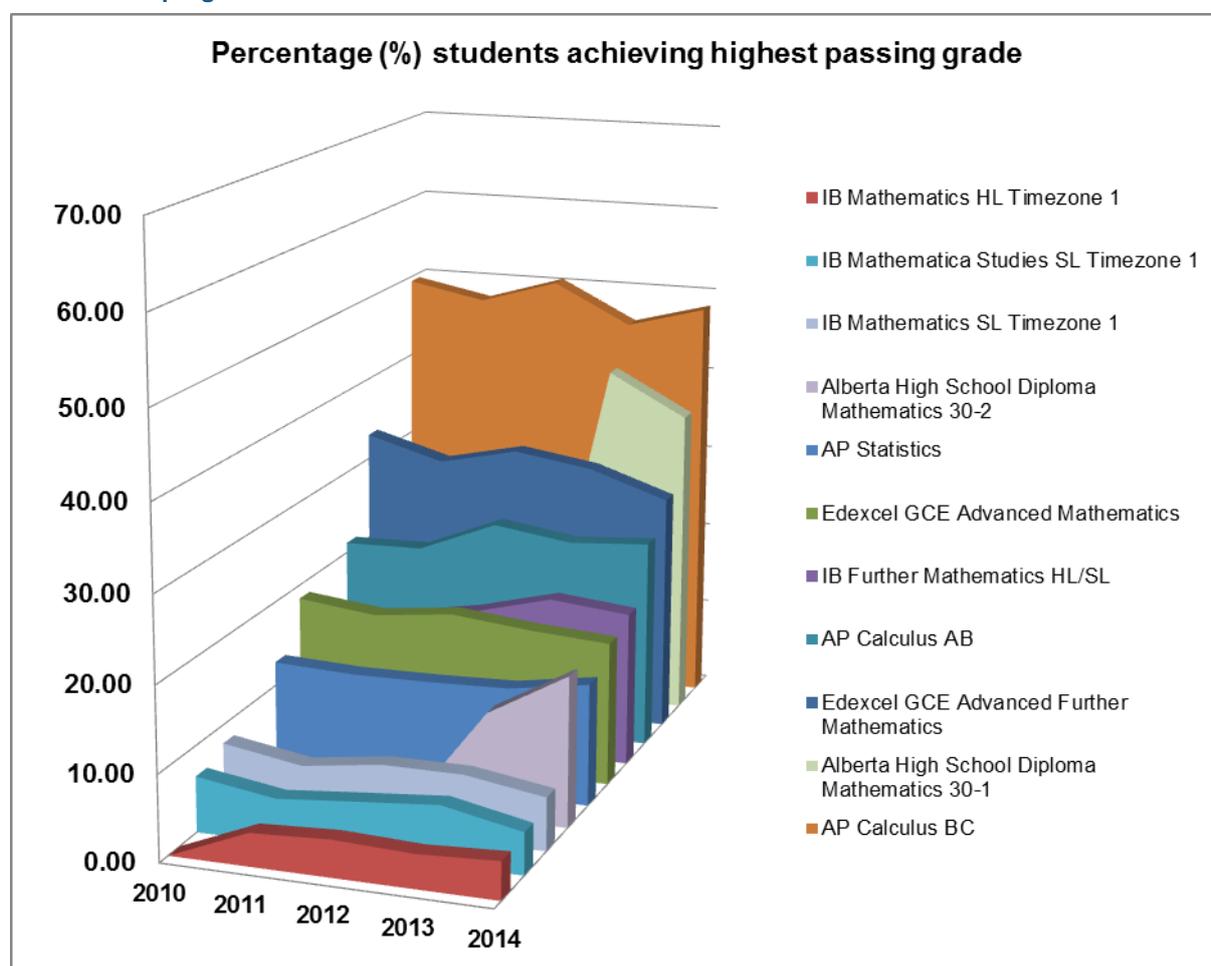
observable between the IB DP higher level descriptors (grades 5-7) the Alberta Diploma Standard of Excellence, in reference to depth and breadth of expected mathematical knowledge. As is also evident in the IB DP grade 6 and grade 7 descriptors, reference is made to being able to apply conceptual understanding to solve problems in unfamiliar contexts at the Standard of Excellence level in the Mathematics 30-1 and 30-1 courses. Nonetheless, many of the skills and attributes mentioned in the IB DP descriptors, particularly grades 6 and 7, are not explicitly mentioned in the Alberta Diploma descriptors.

In referencing a basic knowledge of mathematical principles and procedures, the descriptor for the Acceptable standard in the Alberta Diploma Mathematics programmes corresponds in part to the level of performance expected at IB DP grade 2. However, the expectation that candidates show a conceptual understanding of mathematics may indicate comparability in some part to the level of performance described at higher IB DP grades, 3 and 4.

5.6.3 Grade Distribution

Within this study, UK NARIC sought to collate grade distribution information for each programme. Since grade distribution can vary year on year and over time, distribution over a five year period was considered wherever possible. The figure below illustrates the percentage of students obtaining the highest grade in each programme in each year, where available.

Figure 4: Percentage of students achieving the highest passing grade for IB DP Timezone 1 and international programmes



Notes

- 1 For ease of reading, data for the IB DP programmes is based on performance in timezone 1 (one of two timezones in which the IB DP examinations are held).
- 2 Performance data for the Alberta Diploma Mathematics 30-1 is only reported for 2013 and 2014.
- 3 Please note that the IB DP Further Mathematics HL / SL grade data contains Further Mathematics SL grades until 2013 and Further Mathematics HL grades from 2014. The 2014 grade distribution is therefore not completely comparable with previous years.
- 4 IB DP Mathematics HL data combines all four Mathematics HL options: Calculus; Discrete; Statistics and Probability; and Sets, Relations and Groups. Grade data for 2010 was presented to UK NARIC individually for the four Mathematics HL options and is thus not reported overall for this grade distribution analysis.

As can be seen from above, with the exception of Further Mathematics HL, the IB DP mathematics programmes (timezone 1) show the least variation year on year in the percentage of students obtaining the top grade.

The chart also demonstrates that overall, and across all the programmes for which data was available, the IB DP Mathematics HL (timezone 1) consistently awards the top grade (7) to the lowest percentage of students. In the context of the four IB DP mathematics programmes, this would suggest that the overall demand encompassing assessment and marking, is the highest within the IB DP Mathematics HL (timezone 1).

For the AP Calculus BC and AB the top grade is awarded to a notably higher proportion of students (mean: 48.30% and 23.26% respectively) than is the case for the four IB DP programmes examined in timezone 1⁷⁸. To some extent that must be caveated by the fact that the AP uses fewer grades (5, in comparison to the IB DP's 7), although the AP Statistics, also using a 5-point system, awards the top grade to around 12%. For the GCE A Level Mathematics, which uses a 6-point pass grade system, the proportion of students achieving the top grade is also higher than the IB DP mathematics programmes. It is difficult to draw reliable comparisons between the Alberta Diploma and IB DP distributions because of the differences in the number of distinct pass grades/performance bands awarded (2 and 7 respectively).

It is important to highlight however, that there are some small but evident differences in grade distribution between the two timezones used for the IB DP mathematics programmes as shown within both the chart below (blue denotes distribution data for timezone 1; green denotes data for timezone 2); and the table, which compares the mean percentage of students obtaining the top grade over a five year period.

⁷⁸ Mean percentage achieving the top grade for each IB DP mathematics programme (timezone 1) is as follows: Mathematics HL TZ1 4.14%; Mathematics SL TZ1 7.12%; Mathematical Studies SL TZ1 5.84%

Figure 5: Percentage of students achieving the highest passing grade (IB DP TZ1 and TZ2)

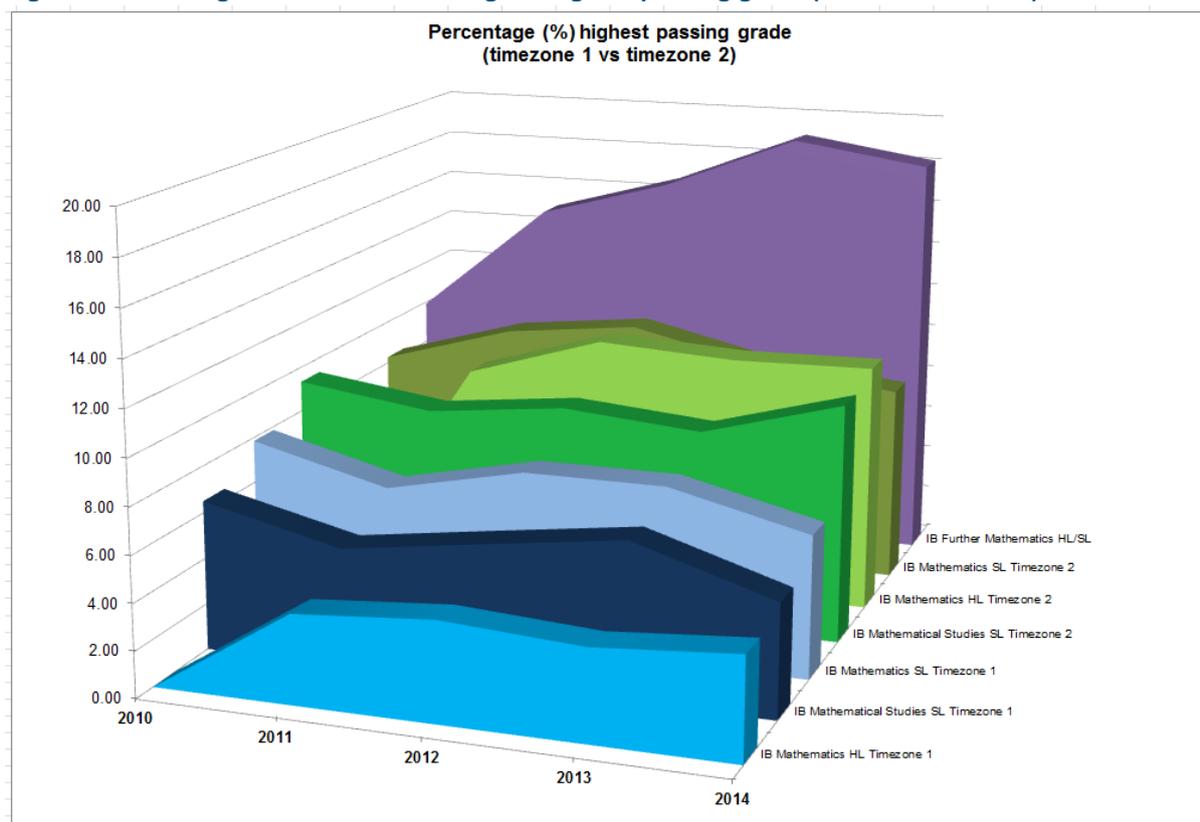
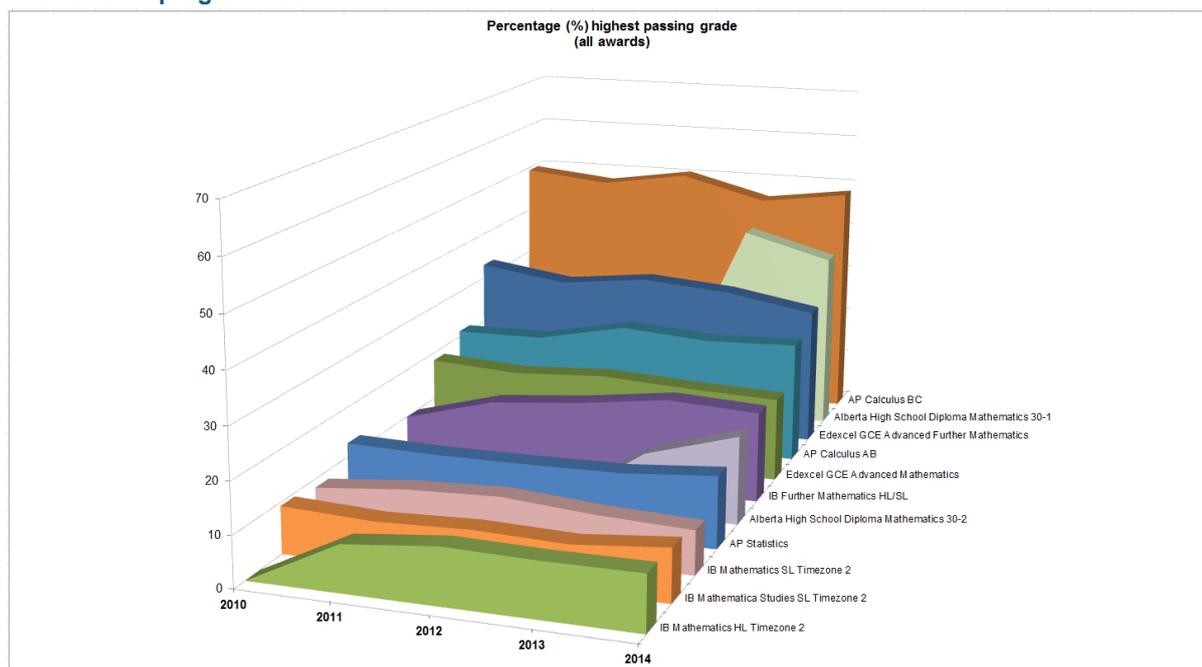


Table 22: Summary of differences between IB DP mathematics for Timezones 1 and 2

Programme	Mean percentage 2010-14 ⁷⁹ (standard deviation is noted in brackets)	
	Timezone 1	Timezone 2
Mathematical Studies SL	5.84 (0.74)	9.23 (0.65)
Mathematics SL	7.12 (0.71)	9.40 (0.89)
Mathematics HL	4.14 (0.24)	10.46 (0.74)
Further Mathematics	not available	not available

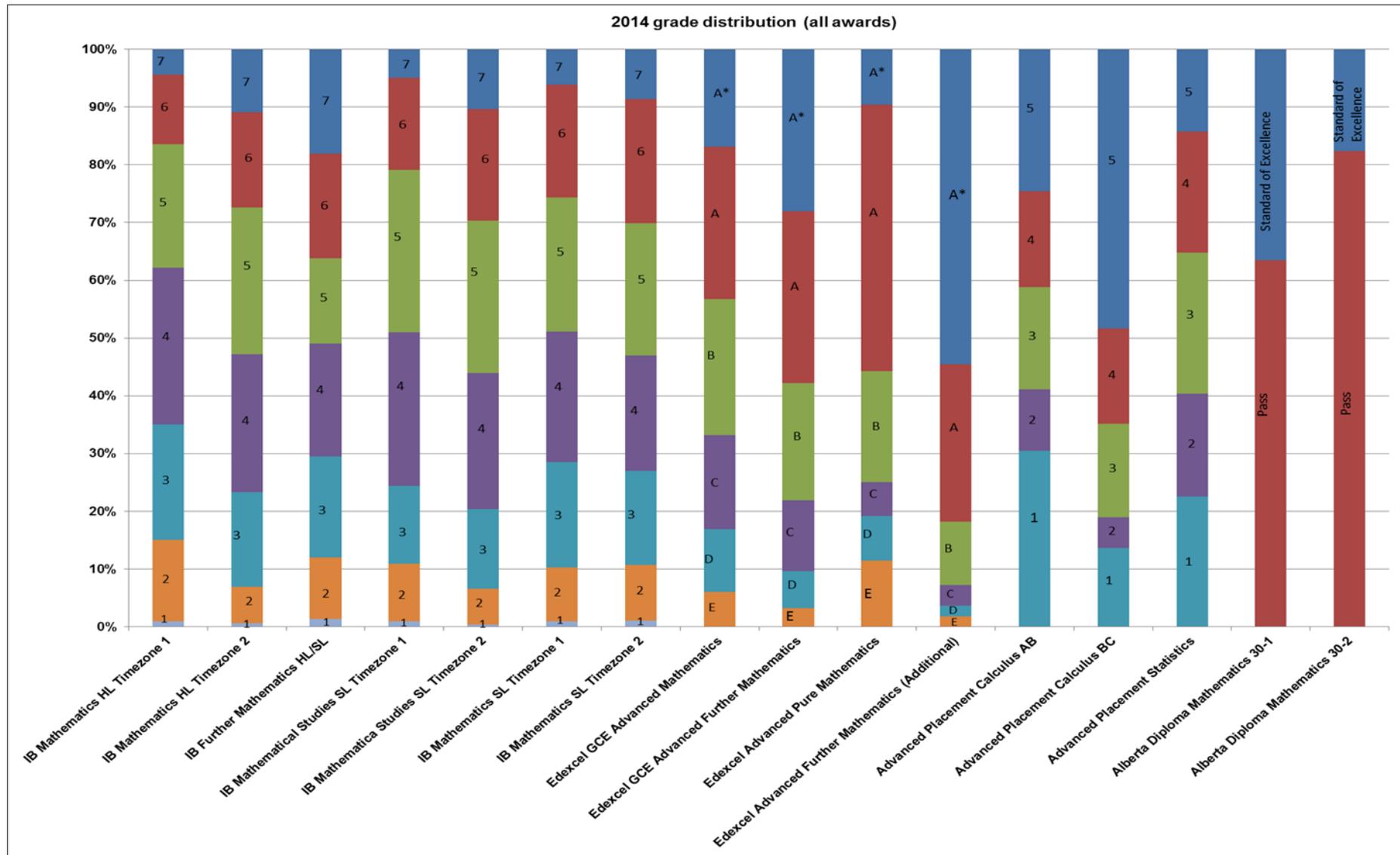
⁷⁹ With the exception of Mathematics HL which is calculated over a four year period.

Figure 6: Percentage of students achieving the highest passing grade for IB DP Timezone 2 and international programmes



The chart below illustrates the proportion of students obtaining each grade in 2014. It is important to highlight that, in line with UK NARIC's international grade comparison methodology, data on fail grades (where fail grades are used) has been omitted. The chart therefore reflects, for the UK GCE A Level and Alberta Diploma, the students obtaining each pass grade as a percentage of the overall number of students passing.

Figure 7: Grade distribution for 2014 by programme



Analysis of Figure 7 reveals that the percentage of IB DP Further Mathematics HL / SL⁸⁰ students in 2014 achieving the highest grade 7 (18.12%) is similar to the highest grades awarded in the Edexcel GCE Advanced Mathematics (Grade A*) and Alberta High School Diploma optional Mathematics 30-2 (Standard of Excellence) at 16.53% and 17.59% respectively. However, the IB DP Further Mathematics HL and Edexcel GCE Mathematics grade distribution overall further differentiates student achievement compared with the Alberta High School Diploma, which reports either a Pass or Standard of Excellence for students overall⁸¹.

In comparison, the percentage of students in 2014 achieving grade 7 in IB DP Mathematics HL timezone 2 (10.85%); IB DP Mathematical Studies SL timezone 2 (10.39%); and IB DP Mathematics SL timezone 2 (8.65%), broadly resembles the proportion achieving the highest grade (grade A*) in Edexcel GCE Pure Mathematics (9.62%). The cohort of students who took the Edexcel GCE Advanced Pure Mathematics award in 2014 is however considerably smaller than the proportion who took Edexcel GCE Advanced Mathematics or IB DP Mathematics timezone 2 qualifications (Appendix 2).

Overall, less than 7% of IB DP Mathematics timezone 1 students achieved grade 7, which is the smallest proportion achieving the highest grade of all the qualifications.

5.6.4 Opportunity for Resits

As with the IB DP, examinations in the other programmes can be re-sat in an attempt to improve overall grades.

For the IB DP, students can retake an examination twice within the three examination sessions required to obtain the diploma⁸². The highest grade achieved for a subject will contribute towards the IB DP⁸³. Similarly, the GCE A Level Mathematics students can re-sit any unit in a later examination series to try to improve on the grade for certification at AS or A level. A student's best attempt will be accepted, and contribute to the overall A Level grade.

Students can re-sit the Alberta Diploma examinations at an accredited Alberta high school if they are retaking the course⁸⁴ or resit the examinations in November and April (which can be done without re-taking the course)⁸⁵. Marks from their highest and most recent examination and/or school mark are blended to determine their final mark in the Diploma⁸⁶.

⁸⁰ IB DP Further Mathematics was a SL programme from 2010-2013.

⁸¹ The Alberta High School Diploma Mathematics 30-1 / 30-2 grade distribution data reflects the grade awarded to students who pass the Diploma overall (combining the school-awarded mark and the examination mark).

⁸² International Baccalaureate Organization, 2015. FAQs. [online] Available at: <<http://www.ibo.org/en/university-admission/recognition-of-the-ib-diploma-by-countries-and-universities/faqs/>> [Accessed 13th April 2015].

⁸³ International Baccalaureate Organization, 2011. *International Baccalaureate Diploma Programme: General regulations: Diploma Programme*. [pdf] Available at: <<http://www.ibo.org/globalassets/publications/become-an-ib-school/dp-general-regulation-2014.pdf>> [Accessed 2nd July 2015].

⁸⁴ Alberta Government, 2012. *Information for Students Planning to Write Diploma Examinations*. [pdf]. Available at: <http://www.education.alberta.ca/media/6744476/03%20info4student_2012-2013_signoff_nolabel.pdf> [Accessed 2nd July 2015].

⁸⁵ Alberta Government, 2014. *Security of Diploma Examination Materials*. [pdf]. Available at: <http://education.alberta.ca/media/6446744/04-dip-gib-2014-15_security-examination-rules.pdf> [Accessed 22nd April 2015].

⁸⁶ Ibid.

Students can re-sit the AP exams in any subject every time it is offered in May each year⁸⁷. A student's score is cumulative and the report includes the scores for all the AP examinations they have sat (including yearly re-sits in subjects). Similarly, students can re-sit the Gāokāo as many times as they like, however, they will need to re-sit the whole Gāokāo and not just the mathematics element.

Singapore Examinations and Assessment Board's SIPCAL instructions for private candidates⁸⁸ include re-sit policies in some subjects, although due to the lack of information in the public domain it is unclear whether a policy for re-sits applies to mathematics H1, H2 or H3.

5.6.5 Progression

As outlined in Section 4, all of the qualifications examined in this study would be accepted for admission to undergraduate study in the home country and in various countries internationally. In the case of the IB DP, which is offered internationally, the IB *University Policy Index* lists the universities worldwide which accept the IB DP for admission.

To better understand the grades for the respective qualifications and programmes, admission requirements for Bachelor degrees in the following countries were reviewed:

- Canada
- China
- Singapore
- UK
- USA.

5.6.5.1 Progression for holders of the AP

An overview of USA universities shows that the AP scores of 3 to 5 are often accepted for university course credit, with the higher scores often allowing a student to receive more credit or enter a higher maths course. Credits are generally awarded to AP students scoring a minimum of grade 3 or above in Calculus AB. Students achieving grade 3 in Calculus BC are likely to be awarded more credits (or entry to a higher maths course) than students achieving grade 3 in Calculus AB. This is likely because the Calculus BC syllabus covers more topics in the same amount of time as Calculus AB⁸⁹.

⁸⁷ The College Board, 2015. *AP® Students: Taking AP Exams*. [pdf]. Available at: <<https://apstudent.collegeboard.org/help#faq63>> [Accessed 2nd July 2015].

⁸⁸ Singaporean Examinations and Assessment Board, (n. d.). Singaporean-Cambridge GCE N(T)-, N(A)-, O- & A-Level Examinations. Instructions for Private Candidates. [pdf] Singaporean Examinations and Assessment Board. Available at: <<https://www.seab.gov.sg/content/privateExamInstructions/2015InstructionsForPrivateCandidates.pdf>> [Accessed 1st May 2015].

⁸⁹ Taken from a selection of universities including: Washington State University, Brown University, University of Pennsylvania, The University of Texas at Austin, and Brigham Young University.

An AP exam score may also provide additional information to a university as part of a student's application for admission. According to the College Board, the AP is also considered by 31 percent of colleges and universities for determining student scholarships⁹⁰.

5.6.5.2 Progression for holders of the SIPCAL

Selection to Singaporean universities is competitive and is based on universities' computation of a student's University Admission Score⁹¹. Norm combination subjects that count towards the University Admission Score are:

- GP or KI (counted at H1 level)
- PW (counted at H1 level)
- Three H2 and one H1 content-based subjects (at least one must be a contrasting subject)⁹².

The admission requirements for an undergraduate degree course in Mathematics / Applied Mathematics & Computer Science at the National University of Singapore is an A grade in H2 mathematics and a good H2 grade in computing, or physics; chemistry; or biology⁹³. Other Singaporean universities request a good H2 pass in mathematics for admission to undergraduate degree courses in mathematics. The admissions requirements of specialised private universities, polytechnics, colleges of arts and institutes of technical education (ITE) may also differ from those listed above.

Whilst grade requirements may not be specified, the National University of Singapore has published an Indicative Grade Profile for the 2014/15 academic year undergraduate admissions⁹⁴. This outlines the number of students admitted to each faculty and the grade combinations of the 90th and 10th percentile students. An extract of this across different subject areas is presented below:

Table 23: Indicative grade profile of admitted students at the National University of Singapore

NUS Course	Representative Grade Profile 3H2/1H1		No. of students admitted
	10 th percentile	90 th percentile	
Engineering	BBC/B	AAA/A	46
Chemical Engineering	AAA/B	AAA/A	296
Computing	AAB/B	AAA/A	373
Science	BBC/C	AAA/A	1196
Business Admin	AAA/C	AAA/A	546
Arts & Social Sciences	BBB/B	AAA/A	1591

⁹⁰ The College Board, 2015. AP and College. [online] Available at:

<<https://apstudent.collegeboard.org/creditandplacement/ap-and-college>> [Accessed 20th April 2015].

⁹¹ Ministry of Education, 2008. *Frequently Asked Questions*. [online] Available at:

<<http://www.moe.gov.sg/cpdd/alevel2006/faqs.htm>> [Accessed 7th May 2015].

⁹² Ministry of Education, 2007. *Entering University*. [online] Available at:

<<http://www.moe.gov.sg/cpdd/alevel2006/university/appl.htm>> [Accessed 11th May 2015].

⁹³ National University of Singapore (n.d.) *Course Prerequisites*. [online] Available at:

<<http://www.nus.edu.sg/oam/apply/local/prerequisites/BYA-prerequisites.html>> [Accessed 11th May 2015].

⁹⁴ National University of Singapore (n.d.) *Indicative Grade Profile* [online] Available at:

<<http://www.nus.edu.sg/oam/gradeprofile/sprogramme-igp.html>> [Accessed 11th May 2015].

5.6.5.3 Progression for holders of the Gāokǎo

University graduate numbers are regulated centrally by the Chinese Government to ensure a graduate pool that reflects the country's social and economic needs. This regulation is achieved through the establishment of quotas at provincial, institution and subject level which are determined every year between institutions, provincial and national authorities. Every year, a minimum mark for entrance into the different categories of institution is set for each province. The three categories are Group/Tier 1 institutions (prestigious universities); Group/Tier 2 (other higher education institutions for Bachelor degree study); and Group/Tier 3 (institutions for sub-degree 专科 (zhuanke) programmes). Different thresholds are set for students in the arts and science streams.

The table below shows the Shanghai thresholds for the science stream from 2012 to 2015 (out of 600):

Table 24: Threshold scores for degree admission in China

Area of Study	Arts and Humanities (Gāokǎo arts stream)		Engineering, Agriculture, Medicine etc. (Gāokǎo science stream)	
	Group 1 institutions (prestigious)	Group 2 (other)	Group 1 institutions (prestigious)	Group 2 (other)
2015 ⁹⁵	434	372	414	348
2014 ⁹⁶	444	390	423	351
2013 ⁹⁷	448	403	405	331
2012 ⁹⁸	438	379	423	342

As such it is not possible to identify a minimum mathematics score for admission. Nevertheless it is useful to consider that these thresholds are set with the aim of identifying, for admission to Group 1 institutions, the top 10% of the cohort, and for admission to Group 2 institutions, the next 20%⁹⁹.

⁹⁵ CHSI, 2014. 2014年全国各省市高考录取控制分数线. [online] Available at: <<http://gaokao.chsi.com.cn/z/gkbfslq/pcx.jsp>> [Accessed 20th April 2015].

⁹⁶ CHSI, 2014. 2014年全国各省市高考录取控制分数线. [online] Available at: <<http://gaokao.chsi.com.cn/z/gkbfslq/pcx.jsp>> [Accessed 20th April 2015].

⁹⁷ CHSI, 2013. 2013年全国各省市高考录取批次线. [online] Available at: <<http://gaokao.chsi.com.cn/z/2013gkbfslq/fsx.jsp>> [Accessed 20th April 2015].

⁹⁸ CHSI, 2012. 2012年全国各省市高考录取分数线. [online] Available at: <<http://gaokao.chsi.com.cn/z/2012gkbfslq/fsx.jsp>> [Accessed 20th April 2015].

⁹⁹ Strategy Policy and Research Education Ltd (2009), Research on China's National College Entrance Examination (the Gaokao). Commissioned and published by Australian Education International. Available at: <http://sydney.edu.au/ab/committees/admissions/2011/AEI_Gaokao_Report.pdf> [Accessed 20th April 2015].

5.6.5.4 Progression for holders of the GCE A Level

A review of UK university admissions requirements for entry into undergraduate degrees involving a substantial mathematics component indicates that the minimum grade typically accepted in GCE A Level Mathematics is grade B; however, the majority of UK universities requested a grade A in GCE A Level Mathematics. The more selective UK universities may request an A* grade in GCE A Level Mathematics and / or Further Mathematics for admission onto their undergraduate courses. GCE A Level Further Mathematics is not a compulsory requirement, although where specified, is usually required at grade A and can be considered either together or instead of GCE A Level Mathematics at grade A or A*.

5.6.5.5 Progression for holders of the Alberta Diploma

Many institutions consider exam results alongside marks obtained from internal school-based assessment. Higher education institutions within Alberta typically specify a minimum average mark of 60% in the Diploma as a requirement for admission to undergraduate courses, although a number of Canadian universities outside of Alberta asked for a minimum average mark of 70%.

5.6.5.6 Progression for holders of the IB DP

Independent of the IB's University Policy Index, UK NARIC conducted a survey of published admission requirements from universities across the five countries.

An overview of admission requirements for holders of the IB DP to UK undergraduate mathematics programs showed that 36 to 39 points overall with 16-18 at HL was needed. The IB DP Mathematics requirements, considered by the universities as an alternative to GCE A Level Mathematics grade B, generally ranged between grade 5 and 6 at HL, although some universities would also consider grade 6 at SL instead of grade 5 at HL¹⁰⁰.

Universities specifying a grade A in GCE A Level Mathematics would alternatively accept a grade 6-7 in IB DP Mathematics HL. As an alternative to an A* grade in GCE A Level Mathematics, selective universities would generally request IB DP Mathematics HL grade 7.

Most of the universities made no distinction in their requirements between the IB DP Mathematics HL and the IB DP Further Mathematics HL for admission to their undergraduate degrees. This supports their recognition at the same academic level.

Some universities in the USA offer college credits for the IB DP, with the standard entrance requirements of the SAT, ACT, and other standardised testing or the high school diploma still needed. As an alternative qualification to the minimum grade 3 AP Calculus AB / BC requirement in the US, the universities analysed requested IB DP Mathematics HL / Further Mathematics HL at a minimum grade 4. Where the highest score for Calculus BC was

¹⁰⁰ These are the subject-specific mathematics requirements and as such may only form part of the overall admissions requirements for an individual (e.g. three GCE Advanced levels at grade x with a grade B in Mathematics, or overall IB score at grade x with a grade 5 or 6 at HL).

requested (grade 5), IB DP Mathematics HL / Further Mathematics HL was generally requested at grade 7.

As an alternative qualification in Canada, IB DP Mathematics is generally accepted, with the majority of Canadian universities analysed requesting IB DP Mathematics SL or HL. Few of the Canadian universities analysed specified an overall grade in IB DP Mathematics SL or HL. Mainly they offered credits towards their degree programmes where students achieved IB DP Mathematics SL, Mathematics HL, or Further Mathematics HL in certain grades. The number of credits offered usually depends on the grade achieved: more credits are awarded for higher grades (e.g. three credits for grade 5 and six credits for grade 6).

The IB DP is widely accepted for undergraduate admission in Singapore although specific grade requirements are not commonly published. Where subject-specific requirements are published, it is clear that programmes that would typically require a H1 Mathematics from home students accept Mathematics SL from the IB DP, whilst those which would require H2 Mathematics from home students specify IB DP Mathematics HL.

Table 25: Summary of progression options to undergraduate education

Qualification	Direct Entry	Credit / Advanced placement	University Admissions
IB DP	✓	✓ may be available at some institutions for those with HL programmes with grades of 5 or above	UK Admissions: Direct Entry - 36 to 39 points overall with 16-18 points at HL. USA Admissions: Credit may be received for scores of 5 or higher in HL programmes.
Advanced Placement		✓	USA Admissions: Course credit offered to skip introductory courses for AP scores of 3 to 5 (determined by each HEI).
GCE A level	✓		UK Universities: Three A levels at A*AA – AAB on average (entry requirements are set by the HEI)
SIPCAL	✓		Singapore Admissions: Two H2 passes (with General Paper (GP) or Knowledge & Inquiry (KI) attempted in the same sitting). Fulfillment of the Mother Tongue Language (MTL) requirement (various routes).
Gāokǎo	✓		Chinese Admissions: Based on where a student's score sits on different thresholds. For 2014, the science stream examination had the following thresholds: <ul style="list-style-type: none"> • Group 1 (prestigious universities): 423 • Group 2 (non-prestigious universities): 351
Alberta Diploma	✓		Canadian Admissions: Typically an average minimum mark of 60% in the Diploma.

6. Key Findings: Summary and Analysis

Drawing on the findings of the qualification-level contextual analysis and a comparative analysis of mathematics assessment, this study has found similarities between the IB DP mathematics programme and the international programmes reviewed.

6.1 Summary of Qualification Level Findings

The qualifications reviewed for this study are all taken at upper secondary school level and include a similar amount of prior study; 12 or 13 years of education. With the exception of the Gāokǎo, which requires a student to have fulfilled the high school graduation requirements, none of the qualifications have formal entry requirements (although five GCSE qualifications at grades A*-C are typically recommended by Pearson for admission to the GCE A Level Mathematics and Further Mathematics).

Nevertheless, prior learning of mathematics is assumed for all. The IB DP allows entry to students from either the IB MYP course or non IB courses, so although prior experience may vary, a lower secondary school level knowledge of mathematics is assumed. Similarly, the AP is taken alongside other high school classes in the USA, and it is expected that students would have lower or upper secondary school knowledge from their high school curriculum. Entry to the SIPCAL depends on the education route a student takes, however all students will have lower secondary prior experience and each SIPCAL programme assumes prior knowledge such as O Level Mathematics for the study of H1 and H2; and understanding of H2 content is assumed in the study of H3.

Each qualification, depending on a student's results, is accepted for undergraduate admissions or credit towards admission and study in their home country, and internationally, at institutional discretion. The IB DP, being an international award, has different levels of acceptance at universities internationally. The requisite grades for higher education study in mathematics or a related subject are usually set at the highest grade, or within the highest two grades, for each programme.

6.1.1 Mathematics in Context

Most of the qualifications reviewed in this study offer three or four mathematics programmes, with the exception of the Alberta Diploma, which offers 11 overall¹⁰¹; and the Gāokǎo providing two exam streams. The majority of the programmes were intended as general mathematics programmes, covering a range of mathematics topics; whilst the AP offered specialised programmes in calculus and statistics.

Mathematics is compulsory for the IB DP, Gāokǎo and Alberta Diploma¹⁰² only.

¹⁰¹ Most of these are internally assessed. The two programmes with a significant externally assessed component (50% though scheduled to reduce to 30% later in 2015), were reviewed within this report.

¹⁰² Mathematics programmes are available at 10-, 20- and 30-level programme within the Alberta Diploma, a 20-Level programme in mathematics, rather than the 30-level examined in this report, forms part of the Alberta Diploma requirements.

6.2 Programme-Level Analysis of Assessment

6.2.1 Key Similarities

The analysis found that the IB DP mathematics programmes each demonstrate similarities with one or more of the international programmes.

6.2.1.1 Aims

Themes commonly observed across the aims of the IB DP and the international programmes include developing knowledge and understanding of key mathematical principles, promoting positive attitudes towards the study of mathematics and the cultivation of problem solving skills. The mathematical programmes in the IB DP, the AP Calculus AB and Calculus BC, GCE A Level, and Alberta Diploma all share aims across the available mathematics programmes within the qualification. The Singaporean SIPCAL has shared aims for the H1 and H2 programmes, but sets different aims for the H3.

6.2.1.2 Purpose

In terms of programme purpose, four key themes emerged:

- Focus on the application of mathematics for everyday purposes, where students are unlikely to require knowledge of pure mathematics/calculus for their chosen area of undergraduate study or employment. This was specific to the IB DP Mathematical Studies SL and the Alberta Diploma Mathematics 30-2.
- Intended to provide a solid background in mathematics that would support undergraduate study in subjects such as social sciences, economics, and business. In particular the IB DP Mathematics SL, SIPCAL H1 Mathematics and, to some extent, the GCE A Level Mathematics (also considered below).
- Intended as preparation for higher education study of mathematics or degrees involving a substantial mathematics component such as engineering, computer science, and medicine. This was evident in the IB DP Mathematics HL, the Alberta Diploma Mathematics 30-1, the GCE A Level Mathematics and the SIPCAL H2 mathematics.
- The remaining programmes – the IB DP Further Mathematics HL, the GCE A Level Further Mathematics, and the SIPCAL H3 – are aimed at students with a particular passion for mathematics and would typically be taken in addition to the programmes mentioned above.

The Gāokāo was something of an exception to these themes: whilst the Arts Stream exam would be taken by students who intend to study arts or humanities subjects at undergraduate level; and the Science Stream by those looking to pursue science or mathematics-related studies, they test both application and pure mathematics.

The purpose of the mathematics programmes was important to consider given that it can be reasonably assumed that the stated purpose of a programme will be reflected in its content and assessment. When reviewing the external exam papers or past/sample questions, similar observations were made in that both the IB DP Mathematical Studies SL and Alberta Diploma Mathematics 30-2 grounded exam questions in real-life scenarios and, on balance, were considered to assess topics at a comparable level of demand. Overall the IB DP Mathematics SL assessed topics at a lower demand than the GCE A Level, with clearer similarities evident in comparison with the C1 and S1 papers typically taken within the AS. For the topics reviewed in this study, the IB DP Mathematics SL had a broadly similar distribution of low / medium / high demand questions as the AS level papers. In particular, where questions were broken down into sub-questions, students could achieve around two thirds of the total marks assigned for that question by answering the low-demand questions. By contrast both the IB DP HL programmes allocated a significant proportion of marks to questions considered to be of medium and high demand, with the proportion of marks assigned to high demand questions greater than in the GCE A Level.

6.2.1.3 Skills focus within the assessment objectives

Similarities were also observed in the IB DP assessment objectives and those prescribed in the international qualifications, in that they set out the broader cognitive skills required in the assessment and typically cover skills related to logical deduction, mathematical communication and interpretation. Similar to the qualification aims, the IB DP assessment objectives apply to most of the mathematics programmes with minor differences to those prescribed for the Mathematical Studies SL programme. The IB DP assessment objectives demonstrated the closest comparability with the GCE A Level and SIPCAL assessment objectives, focusing closely on the skills required to be assessed.

6.2.1.4 The use of core and elective content

Given the focus on assessment in this study, the content included in the mathematics programmes was not reviewed in detail; however a general overview provided contextual information for the assessment of each programme. The IB DP Mathematical Studies SL, Mathematical SL, and Further Mathematics HL include core content studied by all students undertaking those programmes, a content format also seen in the SIPCAL H1, H2, H3; Alberta Diploma Mathematics 30-1 and Mathematics 30-2; and both the Gāokǎo exam streams. The AP has a different approach with the entire programme being of a specialised nature but similar to the other programmes, all content was core. The IB DP Mathematics HL includes specialist electives, an option also found in the GCE A Level Mathematics and A Level Further Mathematics.

6.2.1.5 External Assessment

All of the programmes include external assessment in the format of one or more unseen tests, taken in an exam setting. For the IB DP Further Mathematics HL, the GCE A Level the AP programmes and the Gāokǎo, this external assessment accounts for 100% of the student's final grade. The other IB DP mathematics programmes all include internal assessment contributing toward 20% of the final grade whilst among the international

programmes examined, only those within the Alberta Diploma included internal assessment, at a higher weighting of 50%.

6.2.2 Key differences

6.2.2.1 Marking and grading

The most notable differences observed related to the marking approaches and grading employed across the different programmes. Across all mathematics programmes within the IB DP and GCE A Level, the mark schemes provide a significant level of guidance to examiners on how to apply marking principles to a variety of candidate responses including incomplete answers or alternative methods. Students are expected to show their workings out and mark schemes detail the number of marks associated with demonstrating an appropriate method. This approach ensures that students must demonstrate that they can apply the methods learned and the number of accuracy marks (that is, those available for a correct answer) are limited, and in some cases conditional on the method marks. Alternatively, in the Gāokǎo and AP mathematics programmes, a considerable proportion of the marks (50% in the case of the AP) are assigned to multiple choice questions, where no marks are available for method, thereby placing more weight on providing an accurate answer in order to receive marks. Furthermore whilst the Gāokǎo includes free response questions (extended and multi-part questions), the mark schemes comprised model answers and a breakdown of marks per sub-question, but no differentiation of marks for method and accuracy, indicating that no part marks would be awarded. The use of multiple choice and numerical response questions in the Alberta Diploma external examinations, enabling machine-marking, also means that there is no facility for method marks and accordingly 50% of students' overall grade will rely on accuracy (with the other 50% coming from internal assessment).

In terms of grading systems, there was considerable variation in the number and range of distinct grades and in the use/non-use of fail grades. Overall the IB DP mathematics programmes allow for the finest level of differentiation in student achievement by grade, which can be considered beneficial for facilitating identification of the highest levels of student achievement for competitive higher education admissions purposes. That said, both the Alberta Diploma and Gāokǎo do award a numerical score (out of 100 and 150 per subject respectively). Because of the variations in grading systems, little weight can be attributed to grade distribution for the purpose of comparing demand.

6.2.2.2 Question types

Furthermore, differences were observed in the number and type of questions used across the examination papers reviewed. Unlike the majority of the international programmes, the IB DP programmes do not include multiple choice questions, which can assess a wide range of mathematics topics in a single examination sitting. The Alberta Diploma and the AP examinations contain a large proportion of multiple choice questions, but whilst these allow for a large amount of material to be assessed, there is less scope to assess the candidate's ability to communicate mathematically, and their ability to apply logical deduction and inference skills; skills prioritised in the IB DP mathematics assessment in all programmes.

The IB DP Mathematics HL and Further Mathematics HL papers were observed to include a higher overall proportion of extended problems and multi-part questions with extended elements worth five marks or more in comparison to the GCE A Level programmes and the Gāokǎo streams.

6.2.2.3 Breadth and Depth of Assessed Content

There were some clear differences in the breadth and depth of content assessed though it is important to caveat this statement given that only external assessment materials were reviewed, and that the volume of study associated with each programme may vary, unfortunately with no comparable format available for analysis. Overall the AP exams were the most specialised, focussing on one area of mathematics (statistics or calculus); followed by the Alberta Diploma programmes, with each programme assessing three core areas where one is worth half of the marks for a given paper. The GCE A Level was also considered to be fairly specialised covering pure mathematics in depth with four of the six exam papers dedicated to the assessment of pure mathematics topics. The GCE A Level Further Mathematics by contrast had the greatest potential breadth because of the range of optional units available. Of the six exam papers needed to achieve the A Level Further Mathematics, only one (FP1, covering pure mathematics) would be sat by all students who would then choose one other pure mathematics paper (FP2 or 3) alongside any four papers relevant to statistics, mechanics or decision mathematics. The IB DP programmes all assess across the curriculum with a broader range of topics covered than within the GCE A Level Mathematics. The Gāokǎo assessed across a broad range of topics.

6.2.2.4 Calculator Usage

In terms of calculator usage, the IB DP Mathematics SL Paper 1, Mathematics HL Paper 1 and the AP Calculus AB and BC Part B free response questions are the only papers or sections to include non-calculator assessment, which can be considered beneficial in providing an assessment of candidate's accuracy and precision at mental mathematics and checking their answers manually without a calculator. Whereas all papers in the other international qualification exams require graphical calculators, the calculator papers in the IB DP Mathematics SL and HL allow for a high proportion of questions to be set to specifically assess skills involved in using graphical calculators. The technology use assessment objective is given a significantly higher weighting in the each of the IB DP Mathematics SL Paper 2 and Mathematics HL Paper 2 than is evident across the GCE A Level papers.

6.2.2.5 Aims and Assessment Objectives

Aims and assessment objectives, although used in broadly similar ways across assessment design, exhibited differences in the skills covered and how they are linked to the practical assessment. The IB DP aims place comparatively more weight than those of the other programmes on appreciating the international dimension of mathematics as well as promoting interdisciplinary study. The IB DP, SIPCAL and GCE A Level use a skills-based approach to assessment objectives whilst the Alberta Diploma employs content based outcomes in conjunction with underpinning cognitive levels and mathematical processes which are skills-based. The AP assessment goals combine both approaches including

reference to theoretical knowledge in addition to general mathematical skills. GCE A Level assessment objectives were found to place more emphasis than the IB DP programmes on mathematical models, proofs and solving substantial unstructured problems.

The IB DP assessment objectives weighting, as demonstrated in the Mathematics HL and Further Mathematics HL programmes, show variation in relation to the calculator use, whereas the GCE A Level weightings for each programmes' examination papers, highlight a stronger focus on employing the refining mathematical models in the statistics and mechanics papers. The SIPCAL was the only qualification reviewed which differentiated assessment objectives for its higher level programme, the H3, (from H1 and H2).

The IB DP programmes have a stronger emphasis on investigative mathematics than the international programmes, which is reflected in the internal assessment. The explorative aspect can be considered to be a strength of the IB DP, given that this element allows for application of advanced level maths skills to real world situations to a greater extent than the exam-based programmes such as the those in the GCE A Level, SIPCAL, Gāokǎo and AP. The IB DP internal assessment also allows for assessment of mathematical communication and the candidate's understanding of the relationship between mathematics and other disciplines than the other qualifications mentioned, corresponding with a key aim of the IB DP mathematics programmes.

6.3 Observations on Demand

As outlined above, the IB DP HL programmes (Mathematics and Further Mathematics) assign a significant proportion of marks to questions considered to be of high demand, more so than many of the other programmes reviewed in this study. What is clear however, is that assessment demand is achieved through different means. The IB DP, GCE A Level, and Alberta Diploma had questions at a variety of levels, enabling those at the lower levels to achieve some marks across the low-demand questions whilst allowing for differentiation of the more able students by including some more demanding questions. In contrast, the Gāokǎo questions were all considered to be of at least medium demand: this coupled with one of the shortest assessments in terms of duration, the use of extended problems, and the importance of accuracy to a student's overall marks presents a particularly demanding assessment.

Overall, it is clear that all IB DP mathematics programmes reviewed in this study assess mathematics at a level and breadth consistent with the individual purpose set for each of the four programmes; and consistent with those international programmes that are designed for similar purposes.

- The IB DP Mathematical Studies and the Alberta Diploma Mathematics 30-2
- The IB DP Mathematics SL and the Alberta Diploma Mathematics 30-1, with some clear aspects of GCE AS Level Mathematics
- The IB DP Mathematics HL and Further Mathematics HL, across the topics examined, assigned a greater proportion of marks to the high demand questions than was observed in A Level Mathematics paper though taking into consideration the breadth of the IB DP versus the depth of the A Level Mathematics, these could be considered

broadly comparable overall. The IB DP HL programmes could be considered similarly demanding to the Gāokǎo examinations though for markedly different reasons.

7. Reference list and bibliography

International Baccalaureate Organisation-Internal Documents

International Baccalaureate Diploma Programme:

- The conduct of IB Diploma Programme examinations May 2015 and November 2015
- Grade Descriptors
- Grade Award Manual 2014
- Regulations and policies documents.

Mathematical Studies SL:

- Examination Papers for 2014-2012 (time zone 1 and 2)
- Mark schemes for 2014-2012 examination papers (time zone 1 and 2)
- Teacher Support materials (for 2013-2012 exams)
- Guide for 2014 exams
- Subject Outline for 2014
- Formula booklet for 2014
- Notes on the Internal Assessment
- 2014 Maths Subject Report (time zone 1 and 2)
- Grade distribution data.

Mathematics HL:

- Examination Papers for 2014-2012 (time zone 1 and time zone 2)
- Mark schemes for 2014-2012 examination papers (time zone 1 and time zone 2)
- Teacher Support materials (for 2014-2012 exams)
- Guide for 2014 -2012 exams (and amendment for 2006 Guide)
- Subject Outline for 2014
- Formula booklet for 2014
- Notes on the Exploration
- 2014 Maths Subject Report (time zone 1 and 2)
- Grade distribution data
- Assessment Objectives
- Corrections List.

Mathematics SL:

- Examination Papers for 2014-2012 (time zone 1 and time zone 2)
- Mark schemes for 2014-2012 examination papers (time zone 1 and time zone 2)
- Teacher Support materials (for 2014-2012 exams)
- Guide for 2014 -2012 exams
- Subject Outline for 2014
- Formula booklet for 2014
- Notes on the Exploration
- 2014 Maths Subject Report (time zone 1 and 2)
- Grade distribution data
- Assessment Objectives.

Further Mathematics HL (2014) and Further Mathematics SL (2013-2012):

- Examination Papers for 2014-2012 (time zone 1 and time zone 2)
- Mark schemes for 2014-2012 examination papers (time zone 1 and time zone 2)
- Guide for 2014 -2012 exams
- Subject Outline for 2014
- Formula booklet for 2014
- 2014 Maths Subject Report (time zone 1 and 2)
- Grade distribution data
- Correction list
- 2014 Specimen papers.

International Baccalaureate Organisation-External Documents

International Baccalaureate Organization, (n.d.). *International Baccalaureate Diploma Programme: A guide to assessment*. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/assessmentbrief_web.pdf> [Accessed 7th April 2015].

International Baccalaureate Organization, 2014. *International Baccalaureate Diploma Programme Subject Brief: Mathematics: Further mathematics – Higher level*. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/5_furthermathhl.pdf> [Accessed 7th April 2015].

International Baccalaureate Organization, 2014. *International Baccalaureate Diploma Programme Subject Brief: Mathematics: Mathematical studies – Standard level*. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/5_mathstudiessl.pdf> [Accessed 7th April 2015].

International Baccalaureate Organization, 2014. *International Baccalaureate Diploma Programme Subject Brief: Mathematics: Mathematics – Higher level*. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/5_mathhl.pdf> [Accessed 7th April 2015].

International Baccalaureate Organization, 2014. *International Baccalaureate Diploma Programme Subject Brief: Mathematics: Mathematics – Standard level*. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/5_mathsl.pdf> [Accessed 7th April 2015].

International Baccalaureate Organization, 2014. *International Baccalaureate Diploma Programme: A guide to assessment*. [pdf]. Available at: <http://www.ibo.org/globalassets/publications/recognition/assessmentbrief_web.pdf> [Accessed 1st May 2015].

International Baccalaureate Organization, 2015. *Assessment and Exams*. [online] Available at: <<http://www.ibo.org/en/programmes/diploma-programme/assessment-and-exams/>> [Accessed 7th April 2015].

International Baccalaureate Organization, 2015. *Curriculum*. [online] Available at: <<http://www.ibo.org/en/programmes/diploma-programme/curriculum/>> [Accessed 7th April 2015].

International Baccalaureate Organization, 2015. *FAQs*. [online] Available at: <<http://www.ibo.org/en/university-admission/recognition-of-the-ib-diploma-by-countries-and-universities/faqs/>> [Accessed 13th April 2015].

International Baccalaureate Organization, 2015. *Guide to the International Baccalaureate Diploma Programme*. [pdf] Available at: <<http://www.ibo.org/globalassets/publications/recognition/dpguideforuniversities-en.pdf>> [Accessed 1st May 2015].

International Baccalaureate Organization, 2015. *International Baccalaureate Diploma Programme: Standard level and higher level courses*. [pdf] Available at: <<http://www.ibo.org/globalassets/publications/recognition/slhl-brief-en.pdf>> [Accessed 1st May 2015].

International Baccalaureate Organization, 2015. *Mathematics*. [online] Available at: <<http://www.ibo.org/en/programmes/diploma-programme/curriculum/mathematics/mathematics/>> [Accessed 7th April 2015].

International Baccalaureate Organization, 2015. *What is the DP?*. [online] Available at: <<http://www.ibo.org/en/programmes/diploma-programme/what-is-the-dp/>> [Accessed 8th April 2015].

Edexcel / Pearson GCE A level

Department for Education, 2010. *The Importance of Teaching. The Schools White Paper 2010*. [pdf] Norwich: The Stationary Office. Available at: <https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/175429/CM-7980.pdf> [Accessed 24th April 2015].

Department for Education, 2014. *Quality and Methodology Information*. [pdf]. Available at: <https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/365787/SFR42_2014_Qualityandmethodology.pdf> [Accessed 6th May 2015].

Department of Education, (n.d.). *Exam Results*. [online] Available at: <http://www.education.gov.uk/schools/performance/16to18_14/c14-2.html> [Accessed 25th June 2015].

Joint Council for Qualifications, 2010. *GCE Mathematics Aggregation Rules - Guidance for Centres. June 2008 onwards*. [pdf]. Available at: <<http://www.jcq.org.uk/exams-office/entries/gce-maths-information/notice-to-centres---grading-a-level-mathematics>> [Accessed 1st May 2015].

Joint Council for Qualifications, 2013. *GCE examinations. Entry, Aggregation and Certification. Procedures and Rules*. [pdf]. Available at: <http://www.rewardinglearning.org.uk/docs/quals/admin/procedures_rules_gce_gnvq_1314.pdf> [Accessed 6th May 2015].

Office of Qualifications and Examinations Regulation (Ofqual), 2014. Awarding organisations: understanding our regulatory requirements. [online] Available at: <<https://www.gov.uk/awarding-organisations-understanding-our-regulatory-requirements>> [Accessed 24th April 2015].

Office of Qualifications and Examinations Regulation (Ofqual), 2015. Get the facts: AS and A level reform. [online] Available at: <<https://www.gov.uk/government/publications/get-the-facts-gcse-and-a-level-reform/get-the-facts-as-and-a-level-reform>> [Accessed 24th April 2015].

Office of Qualifications and Examinations Regulation (Ofqual), 2015. Reform of AS and A level qualifications by Ofqual. [online] Available at: <<https://www.gov.uk/government/collections/reform-of-as-and-a-level-qualifications-by-ofqual>> [Accessed 24th April 2015].

Office of Qualifications and Examinations Regulation (Ofqual), 2015. Timeline of AS and A level reforms. [pdf]. Available at: <https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/404004/2015-02-13-timeline-of-as-and-a-level-reforms.pdf> [Accessed 24th April 2015].

Pearson Edexcel Limited, 2014. Grade Boundaries. Edexcel GCE AS/A level and Applied GCE. [pdf]. Available at: <http://qualifications.pearson.com/content/dam/pdf/Support/Grade-boundaries/A-level/1406_GCE_A_level_Grade_Boundaries.pdf> [Accessed 5th May 2015].

Pearson Education Limited, (n.d.). Frequently Asked Questions. [online] Available at: <<http://qualifications.pearson.com/content/dam/pdf/about/aa-faqs.pdf>> [Accessed 5th May 2015].

Pearson Education Limited, 2014. Understanding marking and grading. [online] Available at: <<https://secure.edexcel.com/i-am-a/student/results/Pages/raw-ums-marks.aspx>> [Accessed 5th May 2015].

Pearson Education Limited, 2015. About Edexcel. [online] Available at: <<http://qualifications.pearson.com/en/about-us/qualification-brands/edexcel/about-edexcel.html>> [Accessed 24th April 2015].

Pearson Education Limited, 2015. Exams Past papers. [online] Available at: <<http://qualifications.pearson.com/en/support/support-topics/exams/past-papers.html?Qualification-Family=A-Level>> [Accessed 29th April 2015].

Pearson Education Limited, 2015. Results and certification. Post-results services: Information for students. [online] Available at: <<http://qualifications.pearson.com/en/support/support-topics/results-certification/post-results-services-information-for-students/post-results-services-for-edexcel-a-level.html>> [Accessed 6th May 2015].

Pearson Education Limited, 2015. Results and certification. Understanding marks and grades. [online] Available at: <<http://qualifications.pearson.com/en/support/support-topics/results-certification/understanding-marks-and-grades.html>> [Accessed 1st May 2015].

Pearson Education Limited, 2015. Results and certification. Understanding your results: Information for students. [online] Available at:

<<http://qualifications.pearson.com/en/support/support-topics/results-certification/understanding-your-results-information-for-students/edexcel-a-level-results-explained.html/student>> [Accessed 5th May 2015].

Pearson Education Limited, 2015. What are 'R' exam papers?. [online] Available at:

<<https://knowledgebase.edexcel.com/?p=1749>> [Accessed 30th April 2015].

Pearson Education Limited, 2015. Work with us. Assessment Associate roles. [online]

Available at: <<http://qualifications.pearson.com/en/support/support-for-you/teachers/work-with-us/roles.html>> [Accessed 5th May 2015].

Pearson Education Ltd 2010. GCE Advanced Specifications Grade Statistics (Final) June 2010. [pdf] Pearson Education Ltd. Available at:

<<http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June-2010-Final-GCE-Advanced-including-Single-Awards-Applied-Subjects.pdf>> [Accessed 30th April 2015].

Pearson Education Ltd 2011. GCE Advanced Specifications Grade Statistics (Final) June 2011. [pdf] Pearson Education Ltd. Available at:

<<http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June-2011-Final-GCE-Advanced-including-Single-Awards-Applied-Subjects.pdf>> [Accessed 30th April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Decision Mathematics D1 Advanced Subsidiary Specimen Paper. [pdf] Pearson Education Ltd. Available at:

<http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/ARCHIVED%20319963_GCE_Decision_D1_D2_specimen_paper_mkscheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Decision Mathematics D2 Advanced /

Advanced Subsidiary Level Specimen Paper. [pdf] Pearson Education Ltd. Available at:

<http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/ARCHIVED%20319963_GCE_Decision_D1_D2_specimen_paper_mkscheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Mechanics M1 Advanced Subsidiary Specimen Paper. [pdf] Pearson Education Ltd. Available at:

<http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Exam%20materials/ARCHIVED%20319964_GCE_Mech_M1_M5_Specimen_Paper_mkscheme.pdf> [Accessed 27th April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Pure Mathematics C1 Advanced Subsidiary Specimen Paper. [pdf] Pearson Education Ltd. Available at:

<http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/ARCHIVED%20184697_GCE_Pure_Maths_C1_C4_Specimen_Paper_MkScheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Pure Mathematics C2 Advanced Subsidiary Specimen Paper. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/ARCHIVED%20184697_GCE_Pure_Maths_C1_C4_Specimen_Paper_MkScheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Pure Mathematics C3 Advanced Level Specimen Paper. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/ARCHIVED%20184697_GCE_Pure_Maths_C1_C4_Specimen_Paper_MkScheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Pure Mathematics C4 Advanced Level Specimen Paper. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/ARCHIVED%20184697_GCE_Pure_Maths_C1_C4_Specimen_Paper_MkScheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Statistics S1 Advanced Subsidiary Specimen Paper. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Exam%20materials/ARCHIVED%20319966_GCE_Statistics_S1_S3_specimen_paper_mkscheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Statistics S2 Advanced Level Specimen Paper. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Exam%20materials/ARCHIVED%20319966_GCE_Statistics_S1_S3_specimen_paper_mkscheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, (n.d.). Edexcel GCE Statistics S3 Advanced Level Specimen Paper. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Exam%20materials/ARCHIVED%20319966_GCE_Statistics_S1_S3_specimen_paper_mkscheme.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, 2007. Sample Assessment Materials. GCE Mathematics. [pdf] Pearson Education Ltd. Available at: <<http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/sam-gce-mathematics.pdf>> [Accessed 23rd April 2015].

Pearson Education Ltd, 2008. Advanced Extension Award Mathematics (2008). [online] Available at: <<http://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2008.html#tab-Teaching>> [Accessed 23rd April 2015].

Pearson Education Ltd, 2009. GCE. GCE Edexcel in Mathematics. Mathematical Formulae and Statistical Tables. [pdf] Pearson Education Ltd. Available at: <<http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/sam-gce-mathematics.pdf>> [Accessed 23rd April 2015].

on%20and%20sample%20assessments/N38210A-GCE-Mathematical-Formulae-Statistical-Tables.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, 2012. GCE Advanced Specifications Grade Statistics (Final) June 2012. [pdf] Pearson Education Ltd. Available at: <[http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June-2012-\(Final\)-GCE-Advanced-\(including-Single-Awards-Applied-Subjects\).pdf](http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June-2012-(Final)-GCE-Advanced-(including-Single-Awards-Applied-Subjects).pdf)> [Accessed 30th April 2015].

Pearson Education Ltd, 2013. GCE Advanced Specifications Grade Statistics (Final) June 2013. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June_2013_Final_GCE_Advanced_including_Single_Awards_Applied_Subjects.pdf> [Accessed 30th April 2015].

Pearson Education Ltd, 2013. Specification GCE Mathematics. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2013/Specification%20and%20sample%20assessments/UA035243_GCE_Lin_Maths_Issue_3.pdf> [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. GCE Advanced Specifications Grade Statistics (Provisional) June 2014. [pdf] Pearson Education Ltd. Available at: <[http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June_2014_\(Provisional\)_GCE_Advanced_\(including_Single_Awards_Applied_Subjects\).pdf](http://qualifications.pearson.com/content/dam/pdf/Support/Grade-statistics/A-level/June_2014_(Provisional)_GCE_Advanced_(including_Single_Awards_Applied_Subjects).pdf)> [Accessed 30th April 2015].

Pearson Education Ltd, 2014. Grade Boundaries Edexcel GCE AS/A level and Applied GCE June 2014. [pdf] Pearson Education Ltd. Available at: <http://qualifications.pearson.com/content/dam/pdf/Support/Grade-boundaries/A-level/1406_GCE_A_level_Grade_Boundaries.pdf> [Accessed 30th April 2015].

Pearson Education Ltd, 2015. Edexcel A level Mathematics (2008). [online] Available at: <<http://qualifications.pearson.com/en/qualifications/edexcel-a-levels/mathematics-2008.html>> [Accessed 23rd April 2015].

QAA, SCQF, CCEA Accreditation, Ofqual, CQFW & QQI, 2014. Qualifications Can Cross Boundaries- a guide to comparing qualifications in the UK and Ireland. [pdf]. Available at: <http://www.qaa.ac.uk/en/publications/documents/qualifications-can-cross-boundaries.pdf> [Accessed 11th August 2015].

Edexcel / Pearson GCE A level Examination Papers, Mark Schemes, and Examiner Reports for 2014-2012

All of the below documents were accessed on 23rd April 2015 at:

<<http://qualifications.pearson.com/en/qualifications/edexcel-a-levels/mathematics-2008.coursematerials.html#filterQuery=Pearson-UK:Category%2FExam-materials>>

Pearson Education Ltd, 2014. Edexcel GCE Core Mathematics C1 Advanced Subsidiary (6663). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Core Mathematics C2 Advanced Subsidiary (6664). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Core Mathematics C3 Advanced (6665). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Decision Mathematics D1 Advanced/Advanced Subsidiary (6689). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Decision Mathematics D2 Advanced/Advanced Subsidiary (6690). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Further Mathematics FP1 Advanced/Advanced Subsidiary (6667). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Further Pure Mathematics FP2 Advanced/Advanced Subsidiary (6668). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Further Pure Mathematics FP3 Advanced/Advanced Subsidiary (6669). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Mechanics M1 Advanced/Advanced Subsidiary (6677). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Mechanics M2 Advanced/Advanced Subsidiary (6678). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Mechanics M3 Advanced/Advanced Subsidiary (6679). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Mechanics M4 Advanced/Advanced Subsidiary (6680). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Mechanics M5 Advanced/Advanced Subsidiary (6681). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Statistics S1 Advanced/Advanced Subsidiary (6683). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Statistics S2 Advanced/Advanced Subsidiary (6684). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Statistics S3 Advanced/Advanced Subsidiary (6691). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Edexcel GCE Statistics S4 Advanced/Advanced Subsidiary (6686). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Core Mathematics C1 Advanced Subsidiary (6663). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Decision Mathematics D1 Advanced/Advanced Subsidiary (6689). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Decision Mathematics D2 Advanced/Advanced Subsidiary (6690). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Further Pure Mathematics FP3 Advanced/Advanced Subsidiary (6669). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Mechanics M1 Advanced/Advanced Subsidiary (6677). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Mechanics M2 Advanced/Advanced Subsidiary (6678). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Mechanics M3 Advanced/Advanced Subsidiary (6679). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Mechanics M4 Advanced/Advanced Subsidiary (6680). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Mechanics M5 Advanced/Advanced Subsidiary (6681). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Statistics S1 Advanced/Advanced Subsidiary (6683). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Statistics S2 Advanced/Advanced Subsidiary (6684). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Statistics S3 Advanced/Advanced Subsidiary (6691). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Statistics S4 Advanced/Advanced Subsidiary (6686). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Core Mathematics C2 Advanced Subsidiary (6664). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Core Mathematics C3 Advanced (6665). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Further Mathematics FP1 Advanced/ Advanced Subsidiary (6667). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE Further Pure Mathematics FP2 Advanced/Advanced Subsidiary (6668). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Examiner Report Summer 2014 Pearson Edexcel GCE in Core Mathematics 1 (6663). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Core Mathematics C1 Advanced Subsidiary (6663). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Core Mathematics C2 Advanced Subsidiary (6664). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Core Mathematics C3 Advanced (6665). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Decision Mathematics D1 Advanced/Advanced Subsidiary (6689). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Decision Mathematics D2 Advanced/Advanced Subsidiary (6690). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Further Mathematics FP1 Advanced/ Advanced Subsidiary (6667). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Further Pure Mathematics FP2 Advanced/Advanced Subsidiary (6668). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Further Pure Mathematics FP3 Advanced/Advanced Subsidiary (6669). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE in Core Mathematics 1 (6663). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Mechanics M1 Advanced/Advanced Subsidiary (6677). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Mechanics M2 Advanced/Advanced Subsidiary (6678). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Mechanics M3 Advanced/Advanced Subsidiary (6679). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Mechanics M4 Advanced/Advanced Subsidiary (6680). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Mechanics M5 Advanced/Advanced Subsidiary (6681). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Statistics S1 Advanced/Advanced Subsidiary (6683). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Statistics S2 Advanced/Advanced Subsidiary (6684). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Statistics S3 Advanced/Advanced Subsidiary (6691). Pearson Education Ltd [Accessed 23rd April 2015].

Pearson Education Ltd, 2014. Mark Scheme (Results) Summer 2014 Pearson Edexcel GCE Statistics S4 Advanced/Advanced Subsidiary (6686). Pearson Education Ltd [Accessed 23rd April 2015].

Gāokǎo

Baidu, 2015. 2014 年上海市高考数学试卷(理科). [online] Available at: <<http://wenku.baidu.com/view/8914033f1eb91a37f1115cac.html>> [Accessed 27th April 2015].

Baidu, 2015. 2014 年上海市高考数学试卷(理科). [online] Available at: <[http://wenku.baidu.com/view/4205b9c2dd88d0d232d46a40?fr=prin\(文科\)](http://wenku.baidu.com/view/4205b9c2dd88d0d232d46a40?fr=prin(文科))> [Accessed 27th April 2015].

CHSI, 2012. 2012 年全国各省市高考录取分数线. [online] Available at: <<http://gaokao.chsi.com.cn/z/2012gkbfslq/fsx.jsp>> [Accessed 20th April 2015].

CHSI, 2013. 2013 年全国各省市高考录取批次线. [online] Available at: <<http://gaokao.chsi.com.cn/z/2013gkbfslq/fsx.jsp>> [Accessed 20th April 2015].

CHSI, 2014. 2014 年全国各省市高考录取控制分数线. [online] Available at: <<http://gaokao.chsi.com.cn/z/gkbfslq/pcx.jsp>> [Accessed 20th April 2015].

Ministry of Education of the People's Republic of China, (n.d). 普通高等学校招生全国统一考试问答. [online] Available at:

<http://www.moe.gov.cn/publicfiles/business/htmlfiles/moe/moe_515/200508/11715.html> [Accessed 22nd April 2015].

Ministry of Education of the People's Republic of China, 2014. 聚焦高考改革十大亮点.

[online] Available at:

<<http://www.moe.gov.cn/publicfiles/business/htmlfiles/moe/s8300/201409/174625.html>> [Accessed 27th April 2015].

Ministry of Education of the People's Republic of China, 2015. 高考命题从分到统的历史逻辑.

[online] Available at:

<<http://www.moe.gov.cn/publicfiles/business/htmlfiles/moe/s8623/201503/184748.html>> [Accessed 27th April 2015].

SINA, 2015. 2014 年全国高考考试大纲(新课标版). [online] Available at:

<<http://edu.sina.com.cn/gaokao/2014-02-20/0834409973.shtml>> [Accessed 27th April 2015].

Tencent, 2015. 2014 年高考各科试题解析. [online] Available at:

<<http://edu.qq.com/zt2014/gkstts/>> [Accessed 27th April 2015].

上海市人民政府, 2009. 关于印发《上海市普通高中学业水平考试实施细则（试行）》的通知.

[online] Available at:

<<http://www.shanghai.gov.cn/shanghai/node2314/node2319/node12344/userobject26ai19875.html>> [Accessed 27th April 2015].

上海市教育委员会, 2011. 2011 年上海市普通高校招生考试问答. [online] Available at:

<<http://www.shmeea.edu.cn/node2/node118/node119/node175/node181/userobject1ai15476.html>> [Accessed 22nd April 2015].

上海市教育委员会, 2013. 上海市教育委员会关于完善上海市普通高中：学业水平考试制度的通知. [online] Available at: <<http://www.shmec.gov.cn/attach/xxgk/6435.doc>> [Accessed 21st April 2015].

上海市教育委员会, 2014. 上海市教育委员会关于做好 2015 年上海市普通高校招生报名工作的通知. [online] Available at: <<http://www.shmec.gov.cn/attach/xxgk/7163.doc>> [Accessed

27th April 2015].

上海市教育委员会, 2015. 2015 年上海市普通高校春季招生统一考试暨上海市普通高中学业水平考试考场规则（笔试. [online] Available at:

<<http://www.shmeea.com.cn/node2/node118/node545/userobject1ai18139.html>> [Accessed 22nd April 2015].

上海市教育考试院, 2015. 上海招考热线. [online] Available at:

<<http://www.shmeea.com.cn/node2/node118/node119/index.html>> [Accessed 16th May 2015].

中华人民共和国教育部, 2014. 改变用一个分数评价录取学生. [online] Available at: <<http://www.moe.gov.cn/publicfiles/business/htmlfiles/moe/s8298/201409/175293.html>> [Accessed 16th May 2015].

学信网 (阳光高考 教育部高校招生阳光工程指定平台), 2014. 山西 : 2015 年普通高等学校招生全国统一考试《大纲》、《说明》内容介绍及征订办法. [online] Available at: <<http://gaokao.chsi.com.cn/gkxx/zc/ss/201411/20141119/1407341075.html>> [Accessed 21st April 2015].

新郎教育, 2015. 上海公布普通高中学业水平考试实施办法. [online] Available at: <<http://edu.sina.com.cn/gaokao/2015-04-27/1556466862.shtml>> [Accessed 15th May 2015].

陈鹏, 2015. 明年 25 个省将统一命题 高考命题重回 “全国卷” 时代. [online] Available at: <http://news.xinhuanet.com/politics/2015-04/03/c_127653654.htm> [Accessed 16th May 2015].

Alberta Diploma

Alberta Government, 2013. Mathematics 30-2: 2013 Released Diploma Examination Items. [pdf]. Crown in Right of the Province of Alberta. Available at: <<http://education.alberta.ca/media/7427680/07%20math30-2%20released%202013-14-20130819.pdf>> [Accessed 22nd April 2015].

Alberta Government, 2014. Diploma Examination Multiyear Reports. [pdf]. Available at: <[http://education.alberta.ca/ProvincialTesting/reports/multiyear/Province/Diploma%20Multiyear%20Province%20Report%20\(Table\).pdf](http://education.alberta.ca/ProvincialTesting/reports/multiyear/Province/Diploma%20Multiyear%20Province%20Report%20(Table).pdf)> [Accessed 22nd April 2015].

Alberta Government, 2014. Education: Annual Report 2013-2014. [pdf]. Crown in Right of the Province of Alberta. Available at: <<http://education.alberta.ca/media/8883032/abedar2013-14.pdf>> [Accessed 22nd April 2015].

Alberta Government, 2014. Field Testing. [pdf]. Available at: <http://education.alberta.ca/media/6446764/10-dip-gib-2014-15_fieldtesting.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Guide to Education: ECS to Grade 12 2014-2015. [pdf]. Crown in Right of the Province of Alberta. Available at: <http://education.alberta.ca/media/8765464/guide_to_education_2014.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Marks, Results, and Appeals. [pdf]. Available at: <http://education.alberta.ca/media/6446760/09-dip-gib-2014-15_marks-results-appeals.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Mathematics 30-1: 2014 -15 Information Bulletin - Diploma Examinations Program. [pdf]. Available at: <http://education.alberta.ca/media/15045940/10-math30-1-infobulletin_20150429.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Mathematics 30-1: 2014 Released Diploma Examination Items. [pdf]. Crown in Right of the Province of Alberta. Available at: <http://education.alberta.ca/media/9451970/07_math30-1_released_2014-15.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Mathematics 30-1: 2014-2015 Assessment Standards and Exemplars - Diploma Examinations Program. [pdf]. Available at: <http://education.alberta.ca/media/6729366/14_math30-1_as_e_bulletin_2014-15.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Mathematics 30-2: 2014 -15 Information Bulletin - Diploma Examinations Program. [pdf]. Available at: <http://education.alberta.ca/media/15045941/07-math30-2-infobulletin_20150429.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Mathematics 30-2: 2014-2015 Assessment Standards and Exemplars - Diploma Examinations Program. [pdf]. Available at: <http://education.alberta.ca/media/6758262/13_math30-2_standardsexemp2014-15.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Security of Diploma Examination Materials. [pdf]. Available at: <http://education.alberta.ca/media/6446744/04-dip-gib-2014-15_security-examination-rules.pdf> [Accessed 22nd April 2015].

Alberta Government, 2014. Student Learning Opportunities: High School Completion Rates - Measure History. Province: Alberta. Report V 1. [pdf]. Available at: <http://ideas.education.alberta.ca/media/83072/may2014_high_school_completion_rate__3_year_.pdf> [Accessed 22nd April 2015].

Alberta Government, 2015. General Information Bulletin: Introduction to the Diploma Examinations Program. [pdf]. Available at: <http://education.alberta.ca/media/6902694/02-dip-gib-2014-15_introveisions.pdf> [Accessed 22nd April 2015].

SIPCAL

Ministry of Education, Singapore, (n.d.). Subjects Offered at 'A' Level. [pdf]. Available at: <http://www.moe.gov.sg/cpdd/alevel2006/pdf/subject_synopses.pdf> [Accessed 1st May 2015].

Ministry of Education, 2008. Frequently Asked Questions. [online] Available at: <<http://www.moe.gov.sg/cpdd/alevel2006/faqs.htm>> [Accessed 7th May 2015].

Ministry of Education Singapore, 2015. Bringing out the Best in Every Child: Education in Singapore. [pdf]. Available at: <<http://www.moe.gov.sg/about/files/moe-corporate-brochure.pdf>> [Accessed 1st May 2015].

Ministry of Education, Singapore, 2015. Integrated Programmes (IP). [online] Available at: <<http://www.moe.gov.sg/education/secondary/other/integrated-programme/>> [Accessed 18th May 2015].

Singaporean Examinations and Assessment Board (n. d.). Singaporean-Cambridge GCE N(T)-, N(A)-, O- & A-Level Examinations. Instructions for Private Candidates. [pdf]

Singaporean Examinations and Assessment Board. Available at:

<<https://www.seab.gov.sg/content/privateExamInstructions/2015InstructionsForPrivateCandidates.pdf>> [Accessed 1st May 2015].

University of Cambridge International Examinations (CIE), Ministry of Education Singapore and Singapore Examinations and Assessment Board (SEAB), 2013. Mathematics Higher 1 (Syllabus 8864). [pdf] MOE & UCLES. Available at:

<https://www.seab.gov.sg/content/syllabus/alevel/2015Syllabus/8864_2015.pdf> [Accessed 1st May 2015].

University of Cambridge International Examinations (CIE), Ministry of Education Singapore and Singapore Examinations and Assessment Board (SEAB), 2013. Mathematics Higher 2 (Syllabus 9740). [pdf] MOE & UCLES. Available at:

<https://www.seab.gov.sg/content/syllabus/alevel/2015Syllabus/9740_2015.pdf> [Accessed 1st May 2015].

University of Cambridge International Examinations (CIE), Ministry of Education Singapore and Singapore Examinations and Assessment Board (SEAB), 2013. Mathematics Higher 3 (Syllabus 9824). [pdf] MOE & UCLES. Available at:

<https://www.seab.gov.sg/content/syllabus/alevel/2015Syllabus/9824_2015.pdf> [Accessed 1st May 2015].

Advanced Placement (AP)

The College Board, (n.d). Statistics: Course Description: Effective Fall 2010. [pdf]. Available at: <<https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-statistics-course-description.pdf>> [Accessed 17th April 2015].

The College Board, 2010. AP Calculus AB Student Score Distributions – Global AP Exams – May 2010. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/2010_CalculusAB_Score_Dist.pdf> [Accessed 22nd April 2015].

The College Board, 2010. AP® Calculus BC Student Score Distributions – Global AP Exams – May 2010. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/2010_CalculusBC_Score_Dist.pdf> [Accessed 27th April 2015].

The College Board, 2010. AP® Statistics Student Score Distributions – Global AP Exams – May 2010. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/2010_Statistics_Score_Dist.pdf> [Accessed 24th April 2015].

The College Board, 2011. AP Calculus AB Student Score Distributions – Global AP Exams – May 2011. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/2011_Calculus_AB_Score_Dist.pdf> [Accessed 22nd April 2015].

The College Board, 2011. AP® Calculus BC Student Score Distributions – Global AP Exams – May 2011. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/2011_Calculus_BC_Score_Dist.pdf
> [Accessed 27th April 2015].

The College Board, 2011. AP® Statistics Student Score Distributions – Global AP Exams – May 2011. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/2011_Statistics_Score_Dist.pdf>
[Accessed 24th April 2015].

The College Board, 2012. AP Calculus AB Student Score Distributions – Global AP Exams – May 2012. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_AB_ScoringDist.pdf
> [Accessed 22nd April 2015].

The College Board, 2012. AP® Calculus BC Student Score Distributions – Global AP Exams – May 2012. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_BC_ScoringDist.pdf
> [Accessed 22nd April 2015].

The College Board, 2012. AP® Statistics Student Score Distributions – Global AP Exams – May 2012. [pdf]. Available at:

<http://apcentral.collegeboard.com/apc/public/repository/ap12_statistics_ScoringDist.pdf>
[Accessed 22nd April 2015].

The College Board, 2012. Calculus: Calculus AB Calculus, BC Course Description Effective Fall 2012. [pdf]. Available at: <<http://media.collegeboard.com/digitalServices/pdf/ap/ap-calculus-course-description.pdf>> [Accessed 20th April 2015].

The College Board, 2013. AP Calculus AB Student Score Distributions – Global AP Exams – May 2013. [pdf]. Available at:

<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13_calculus_ab_scoring_dist.pdf> [Accessed 21st April 2015].

The College Board, 2013. AP® Calculus AB 2013 Free-Response Questions. [pdf].

Available at:

<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13_frq_calculus_ab.pdf>
[Accessed 21st April 2015].

The College Board, 2013. AP® Calculus AB 2013 Scoring Guidelines. [pdf]. Available at:

<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13_calc_ab_scoring_guidelines.pdf> [Accessed 21st April 2015].

The College Board, 2013. AP® Calculus BC Student Score Distributions – Global AP Exams – May 2013. [pdf]. Available at:

<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13_calculus_BC_Score_Dist.pdf> [Accessed 22nd April 2015].

The College Board, 2013. AP® Statistics Student Score Distributions – Global AP Exams – May 2013. [pdf]. Available at:

<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13_statistics_ScoringDist.pdf> [Accessed 22nd April 2015].

The College Board, 2014. AP® Bulletin for AP Students and Parents 2014-2015. [pdf]. Available at: <https://secure-media.collegeboard.org/digitalServices/pdf/professionals/2014-15_ap_student_bulletin.pdf> [Accessed 28th April 2015].

The College Board, 2014. AP® Calculus AB 2014 Free-Response Questions. [pdf]. Available at: <http://media.collegeboard.com/digitalServices/pdf/ap/ap14_frq_calculus_ab.pdf> [Accessed 21st April 2015].

The College Board, 2014. AP® Calculus AB 2014 Scoring Guidelines. [pdf]. Available at: <http://media.collegeboard.com/digitalServices/pdf/ap/ap14_calc_ab_scoring_guidelines.pdf> [Accessed 21st April 2015].

The College Board, 2014. AP® Calculus AB Student Score Distributions – Global AP Exams – May 2014. [pdf]. Available at: <<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap14-calculus-ab-score-dist.pdf>> [Accessed 21st April 2015].

The College Board, 2014. AP® Calculus AB. [pdf]. Available at: <<https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-course-overviews/ap-calculus-ab-course-overview.pdf>> [Accessed 27th April 2015].

The College Board, 2014. AP® Calculus BC Student Score Distributions – Global AP Exams – May 2014. [pdf]. Available at: <<http://media.collegeboard.com/digitalServices/pdf/ap/ap14-calculus-bc-scoring-global-combined-score-subscore.pdf>> [Accessed 22nd April 2015].

The College Board, 2014. AP® Calculus BC. [pdf]. Available at: <<https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-course-overviews/ap-calculus-bc-course-overview.pdf>> [Accessed 27th April 2015].

The College Board, 2014. AP® Statistics Student Score Distributions – Global AP Exams – May 2014. [pdf]. Available at: <<http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap14-statistics-score-dist.pdf>> [Accessed 22nd April 2015].

The College Board, 2014. AP® Statistics. [pdf]. Available at: <<https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-course-overviews/ap-statistics-course-overview.pdf>> [Accessed 27th April 2015].

The College Board, 2015. About AP Scores. [online] Available at: <<https://apscore.collegeboard.org/scores/about-ap-scores>> [Accessed 20th April 2015].

The College Board, 2015. AP and College. [online] Available at: <<https://apstudent.collegeboard.org/creditandplacement/ap-and-college>> [Accessed 20th April 2015].

The College Board, 2015. AP Around the World. [online] Available at: <<https://apstudent.collegeboard.org/creditandplacement/ap-and-college/ap-around-the-world>> [Accessed 20th April 2015].

The College Board, 2015. AP Capstone. [online] Available at: <<https://lp.collegeboard.org/ap-capstone/how-ap-capstone-works>> [Accessed 1st May 2015].

The College Board, 2015. AP Courses. [online] Available at: <<https://apstudent.collegeboard.org/apcourse>> [Accessed 22nd April 2015].

The College Board, 2015. AP Central. [online] Available at: <<http://apcentral.collegeboard.com/home>> [Accessed 20th April 2015].

The College Board, 2015. AP Exam Scores. [online] Available at: <<https://aphighered.collegeboard.org/exams/scoring>> [Accessed 27th April 2015].

The College Board, 2015. AP[®] Bulletin for AP Students and Parents 2014-2015: California Supplement. [pdf]. Available at: <<https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-bulletin-ca-supplement-2014-15.pdf>> [Accessed 28th April 2015].

The College Board, 2015. How AP Develops Courses and Exams. [online] Available at: <<https://advancesinap.collegeboard.org/ap-course-exam-development>> [Accessed 27th April 2015].

The College Board, 2015. How to Earn Credit for Your Scores. [online]. Available at: <<https://apstudent.collegeboard.org/creditandplacement/how-to-earn-credit-for-your-scores>> [Accessed 28th April 2015].

The College Board, 2015. Registering for Exams. [online] Available at: <<https://apstudent.collegeboard.org/takingtheexam/registering-for-exams>> [Accessed 28th April 2015].

The University of Texas at Austin, (2015). College of Natural Sciences: AP Tests in Calculus. [online] Available at: <<https://cns.utexas.edu/mpa-advising/ap-tests-in-calculus>> [Accessed 19th May 2015].

Washington State University, (n.d.). AP Credit Chart. [online] Available at: <<https://admission.wsu.edu/apply/as/freshmen/ap-ib-and-running-start/ap-credit-chart/>> [Accessed 19th May 2015].

General Sources

Nuffield Foundation, 2010. An international comparison of upper secondary mathematics education: 24 Country Profiles. [pdf] Published by: Nuffield Foundation. Available at: <http://www.nuffieldfoundation.org/sites/default/files/files/Country_profiles_outlier_NuffieldFoundation18_04_11.pdf> [Accessed 1st April 2015].

Ofqual, 2012. International Comparisons in Senior Secondary Assessment Full Report. [pdf] Published by: Office of Qualifications and Examinations Regulation in 2012. Available at: <https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/372211/2012-06-12-international-comparisons-in-senior-secondary-assessment.pdf> [Accessed 1st April 2015].

UCAS, 2006. Expert Group Report For Awards Seeking Admission to the UCAS Tariff: The International Baccalaureate. [pdf]. Available at:
<<https://www.ucas.com/sites/default/files/ibreport.pdf>> [Accessed 1st April 2015].

UK NARIC, 2015. International Comparisons:UK file. [online-subscription only] Available at:
<<https://www.naric.org.uk/Product/International%20Comparisons/information.aspx?countryID=175>> [Accessed 10th August 2015].

Appendix 1: Glossary of Terms

Term	Definition applied in the context of this report
General Terms	
Qualification	For the purposes of this study, “qualification” has been used to refer to the overall qualification awarded (e.g. IB Diploma Programme, Alberta Diploma, GCE A Level).
Programme	For the purposes of this study, “programme” has been used to refer to the distinct mathematics course/assessment reviewed (GCE A Level Mathematics, [IB DP] Mathematical Studies SL, AP Calculus AB).
Guided learning hours	Guided learning hours, as defined by Ofqual (n.d) are, “...a measure of the time that a typical learner will require in direct guidance – from a teacher, tutor or other identified guidance provider – to complete their programme of learning successfully. The guidance must be related to the programme that the learner is taking, and can include time in lectures, classes, tutorials, one-to-one meetings, workshops or assessment sessions. The guidance may be delivered through virtual means, such as online, or through video and teleconferencing. If the guidance is in the form of supervised study, then the supervising person must be available to provide support for the specific subject. Guided learning does not include marking assessments when the learner is not present; work experience when an assessor or centre/employer learning provider is not present; or custodial supervision”.
Assessment objectives	Statements referring to key skills and competencies to be evaluated in a programme's assessment, which may include coursework and examinations.
Assessment Methods	The type and style of assessment within a programme (i.e. Coursework, examination, test conditions, etc.).
Assessment item	A question within an examination paper
Marking	The method and criteria used to award marks to assessment items.
Mark Scheme	A detailed framework for assigning marks, where a specific number of marks is given to individual components of the answer. http://www.qaa.ac.uk/about-us/glossary?Category=M-O#142
Question Types	
Multiple choice questions	Questions in which the candidates are expected to select the best possible answer (or answers) out of a range of possible answers (known as distractors).
Free-response questions	Overarching term used to encompass question types below.
Short answer questions	In the context of this study, these questions include: <ul style="list-style-type: none"> • One-step equations • Numerical response questions [as used in the Alberta Diploma Mathematics 30-1 and 30-2] • Gap fill [as used in the Gāokāo].
Multiple-part questions	These questions are broken down into two or more constituent parts - sub-questions. Sub-questions are typically each given a separate mark allocation. For example, the following question has been taken from the IB DP Mathematical Studies SL Paper 1 (2014)

1. [Maximum mark: 17]

As part of his IB Biology field work, Barry was asked to measure the circumference of trees, in centimetres, that were growing at different distances, in metres, from a river bank. His results are summarized in the following table.

Distance, x (metres)	5	12	17	21	24	30	34	44	47
Circumference, y (centimetres)	82	76	70	68	67	60	62	50	50

- (a) State whether *distance from the river bank* is a continuous **or** discrete variable. [1]
- (b) **On graph paper**, draw a scatter diagram to show Barry's results. Use a scale of 1 cm to represent 5 m on the x -axis and 1 cm to represent 10 cm on the y -axis. [4]
- (c) Write down
- the mean distance, \bar{x} , of the trees from the river bank;
 - the mean circumference, \bar{y} , of the trees. [2]
- (d) Plot and label the point $M(\bar{x}, \bar{y})$ on your graph. [2]
- (e) Write down
- the Pearson's product-moment correlation coefficient, r , for Barry's results;
 - the equation of the regression line y on x , for Barry's results. [4]
- (f) Draw the regression line y on x on your graph. [2]
- (g) **Use the equation of the regression line** y on x to estimate the circumference of a tree that is 40 m from the river bank. [2]

They can also involve more substantial, extended problems, which may not be broken down, attracting a high number of marks.

For example, the following question has been taken from the IB DP Mathematics HL Paper 3 Calculus (2014).

1. [Maximum mark: 16]

Consider the functions f and g given by $f(x) = \frac{e^x + e^{-x}}{2}$ and $g(x) = \frac{e^x - e^{-x}}{2}$.

- (a) Show that $f'(x) = g(x)$ and $g'(x) = f(x)$. [2]
- (b) Find the first three non-zero terms in the Maclaurin expansion of $f(x)$. [5]
- (c) Hence find the value of $\lim_{x \rightarrow 0} \frac{1 - f(x)}{x^2}$. [3]
- (d) Find the value of the improper integral $\int_0^{\infty} \frac{g(x)}{[f(x)]^2} dx$. [6]

Appendix 2: Programme Cohort Size for 2014

Table 26: Grade distribution cohort by qualification

	Number of students taking award in 2014
IB DP Mathematics HL Timezone 1	4,604
IB DP Mathematics HL Timezone 2	7,347
IB DP Further Mathematics HL / SL	149
IB DP Mathematical Studies SL Timezone 1	20,851
IB DP Mathematical Studies SL Timezone 2	12,121
IB DP Mathematics SL Timezone 1	18,057
IB DP Timezone SL Timezone 2	18,839
Edexcel GCE Advanced Mathematics	42,993
Edexcel GCE Advanced Further Mathematics	7,061
Edexcel GCE Advanced Pure Mathematics	54
Edexcel GCE Advanced Further Mathematics (Additional)	58
Advanced Placement Calculus AB	89,577
Advanced Placement Calculus BC	112,384
Advanced Placement Statistics	41,596
Alberta High School Diploma Mathematics 30-1	21,314
Alberta High School Diploma Mathematics 30-2	11,934

Appendix 3: Grade Distribution by Programme

IB DP Programmes

Figure 8: Percentage grade distribution of IB DP Mathematics Studies SL Timezone 1

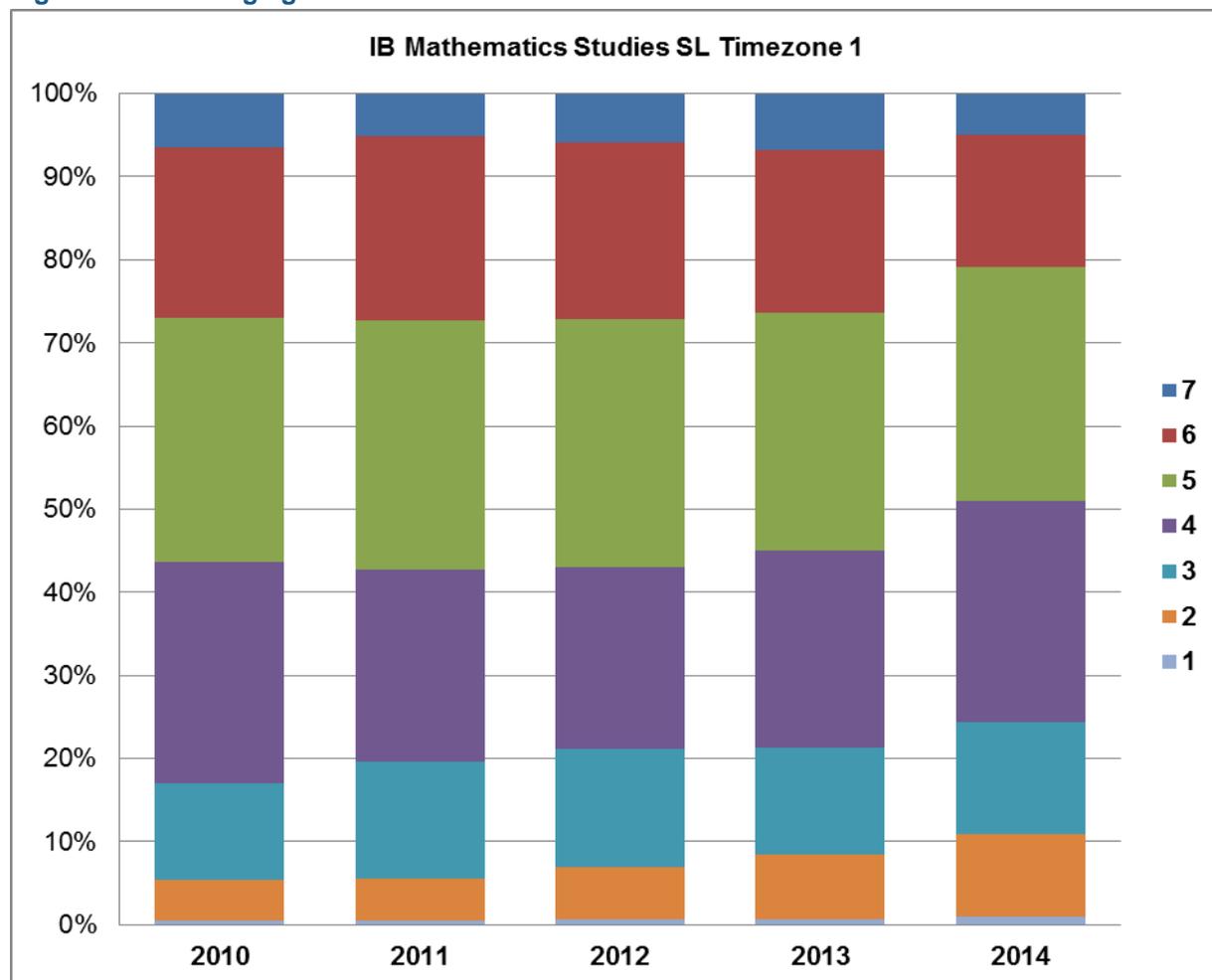


Figure 9: Percentage grade distribution of IB DP Mathematics SL Timezone 1

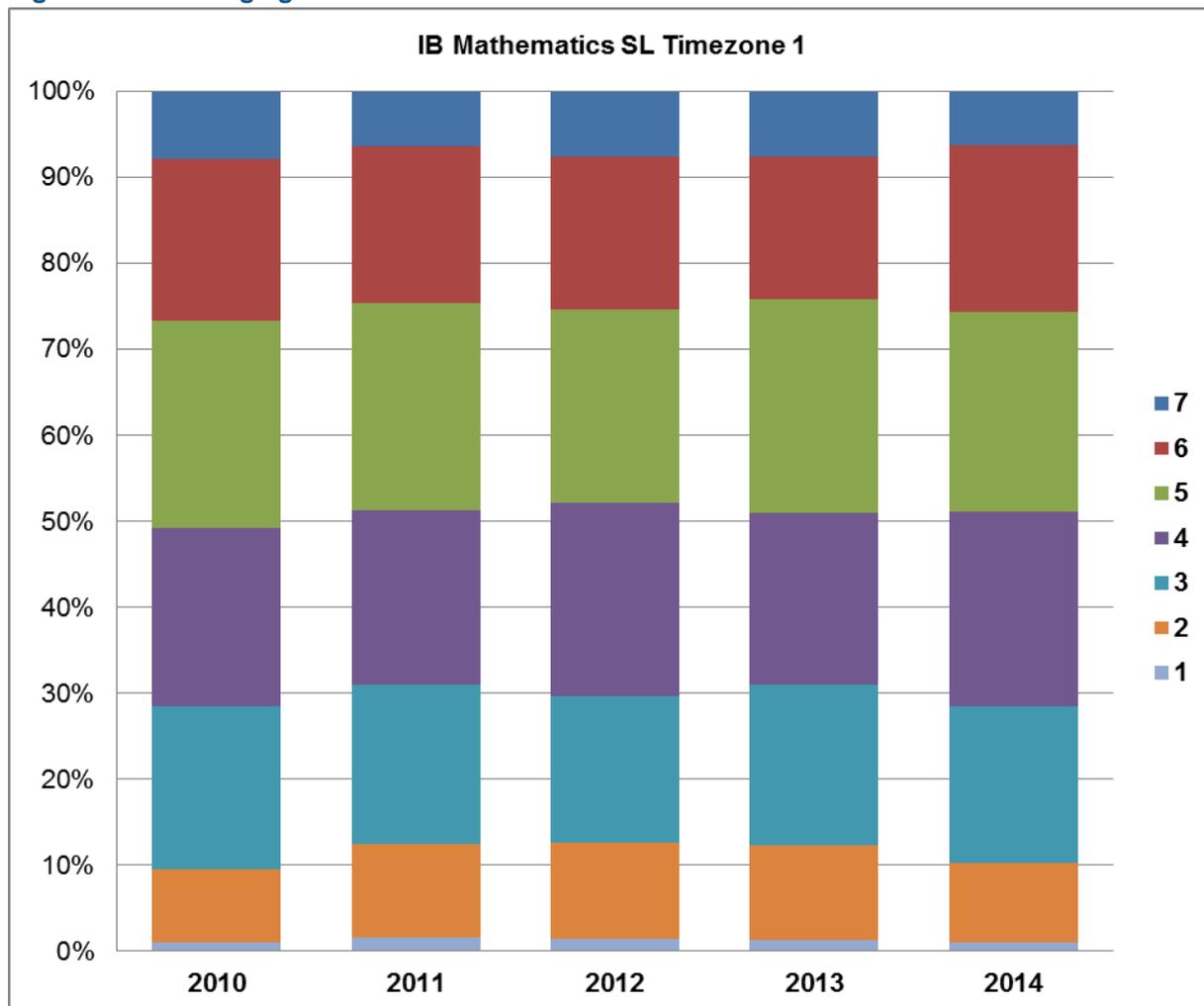


Figure 10: Percentage grade distribution of IB DP Mathematics HL Timezone 1

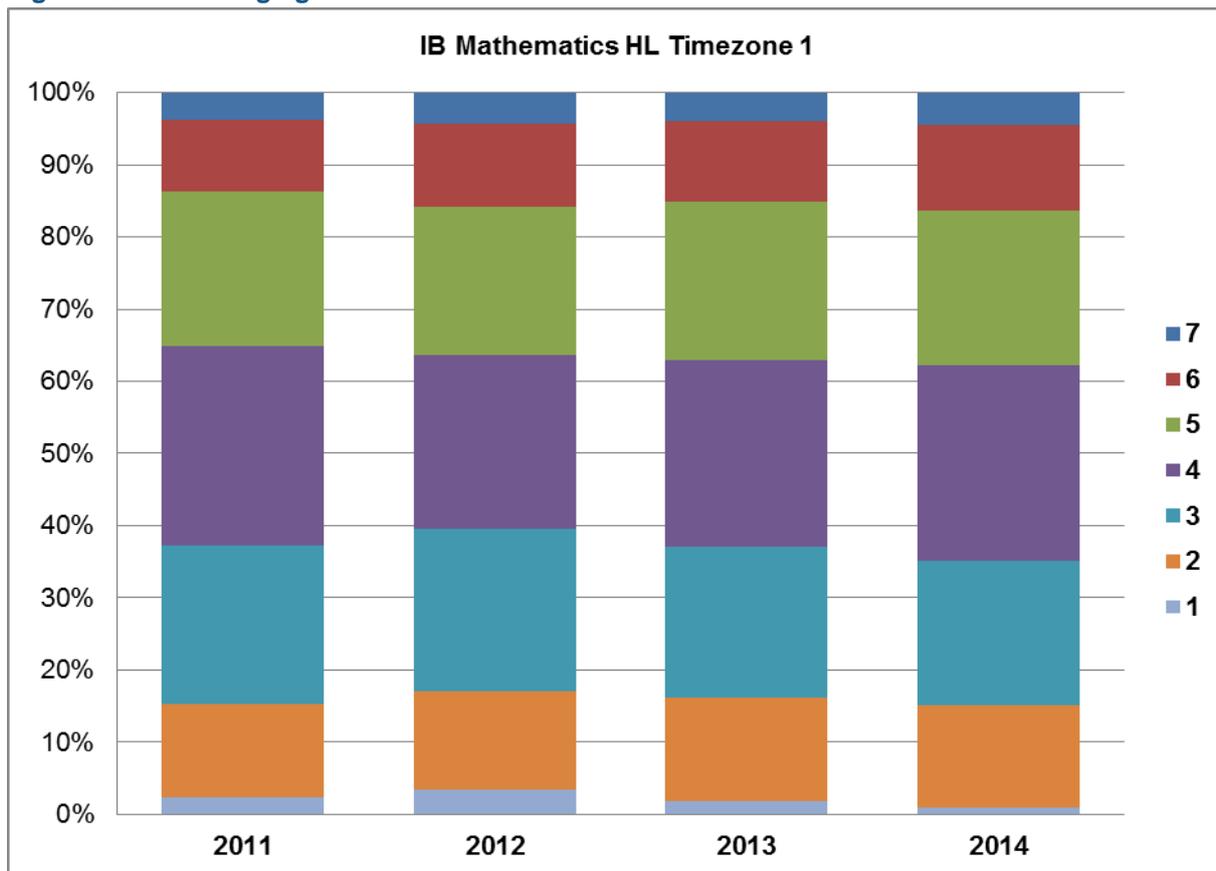
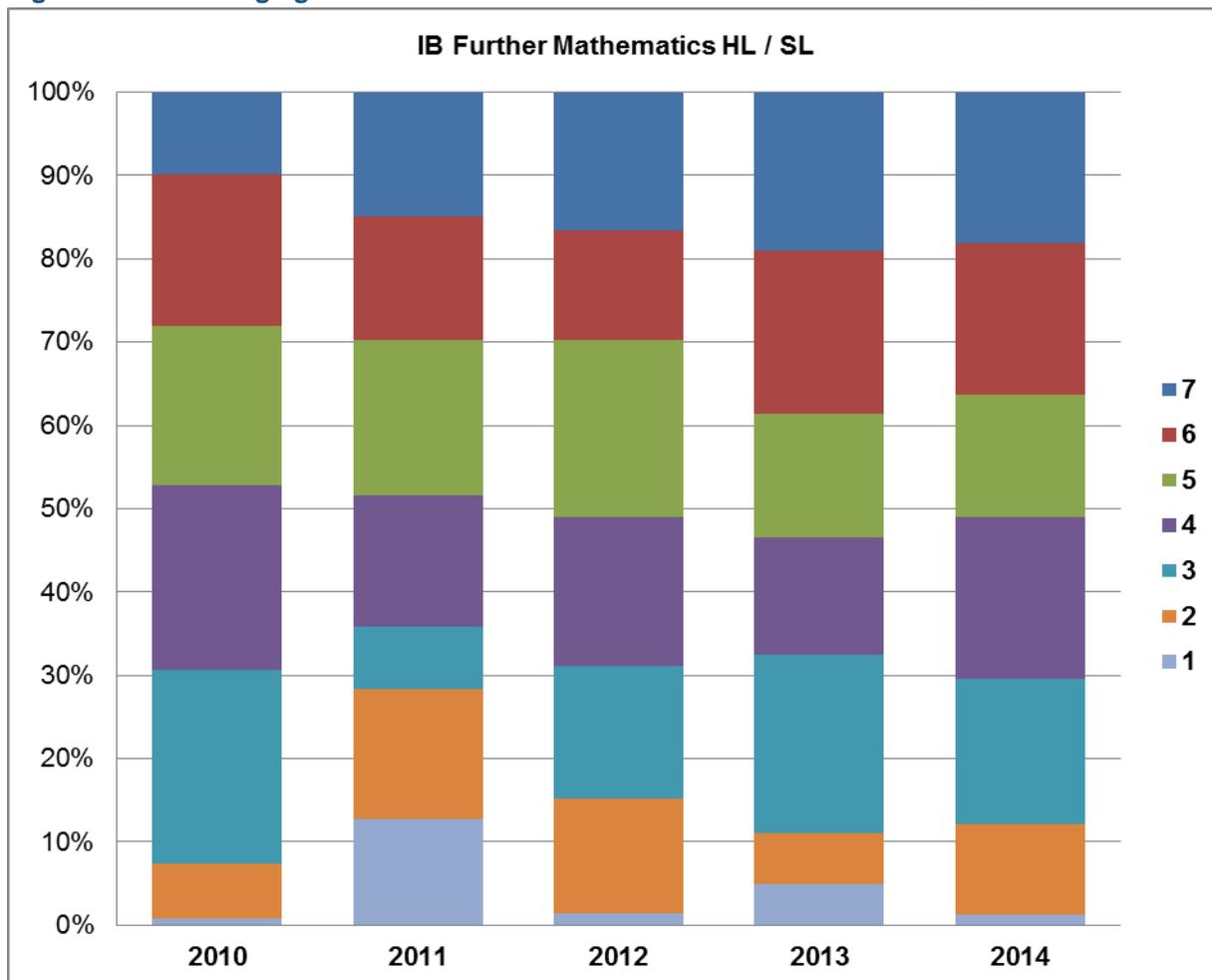
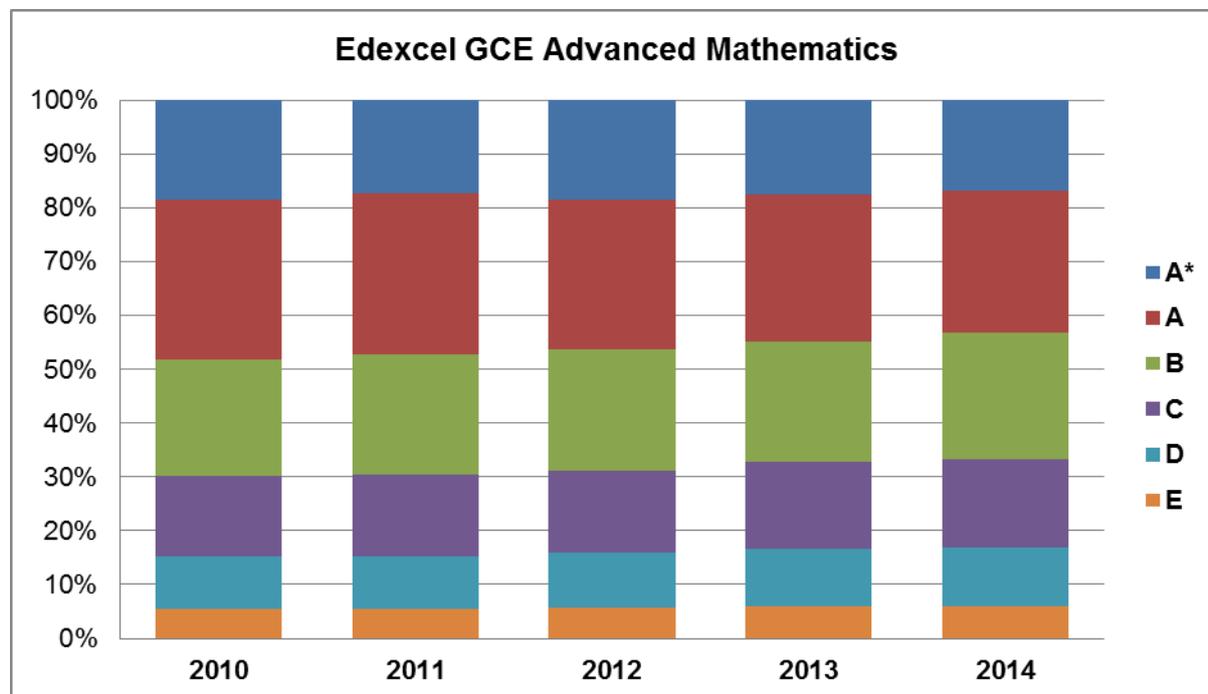


Figure 11: Percentage grade distribution of IB DP Further Mathematics HL / SL

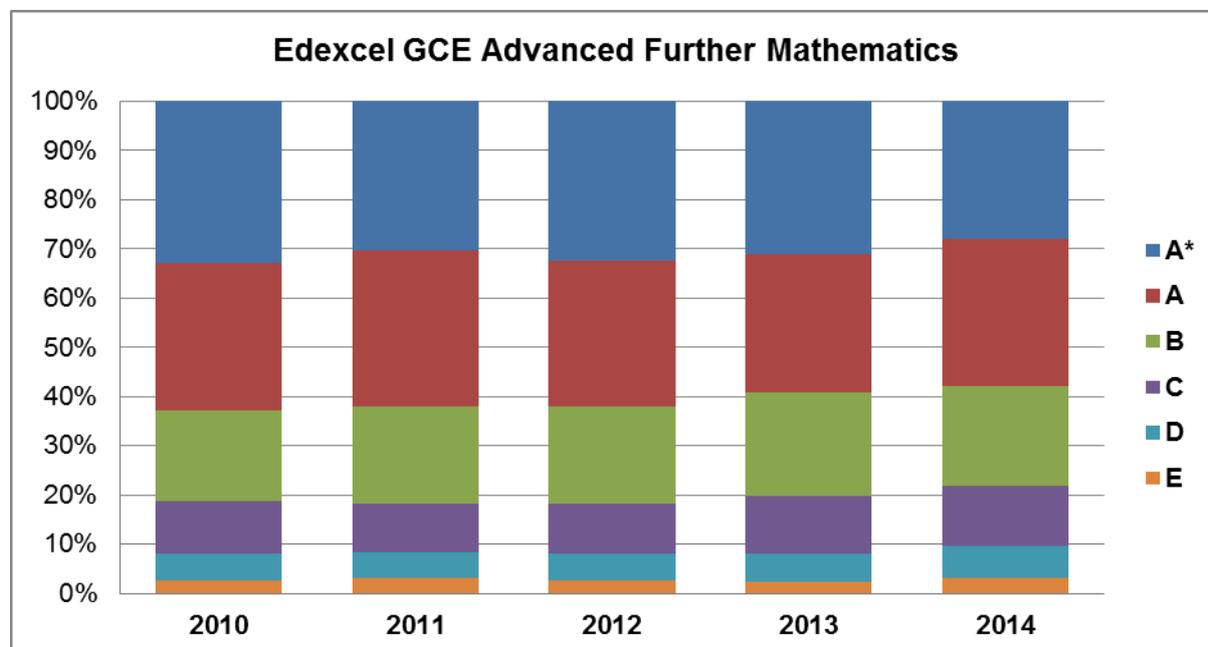


International Qualification Programmes

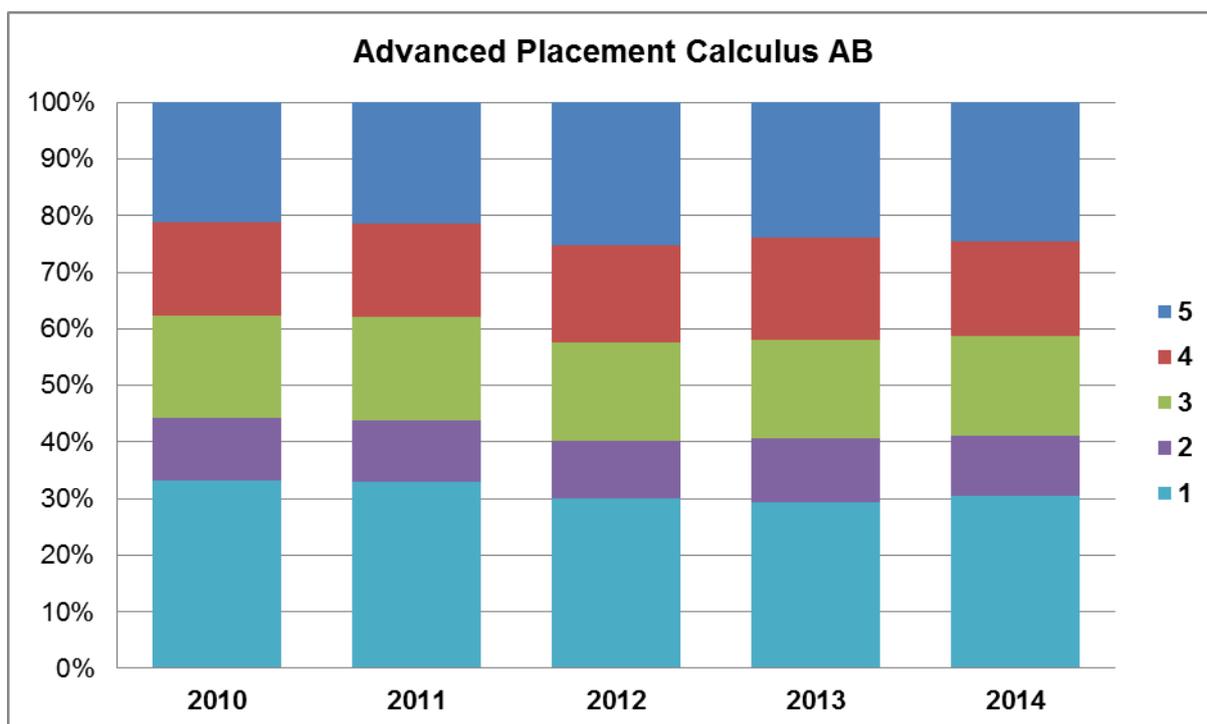
GCE A Level Mathematics



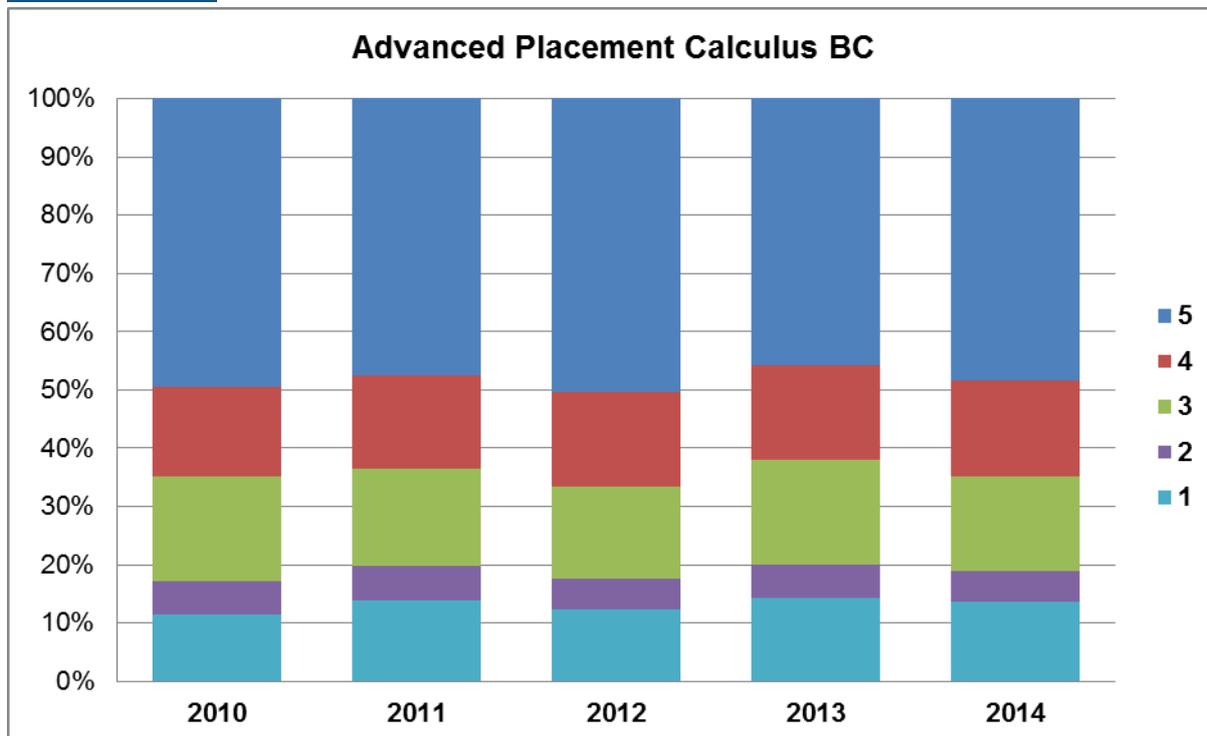
GCE A Level Further Mathematics



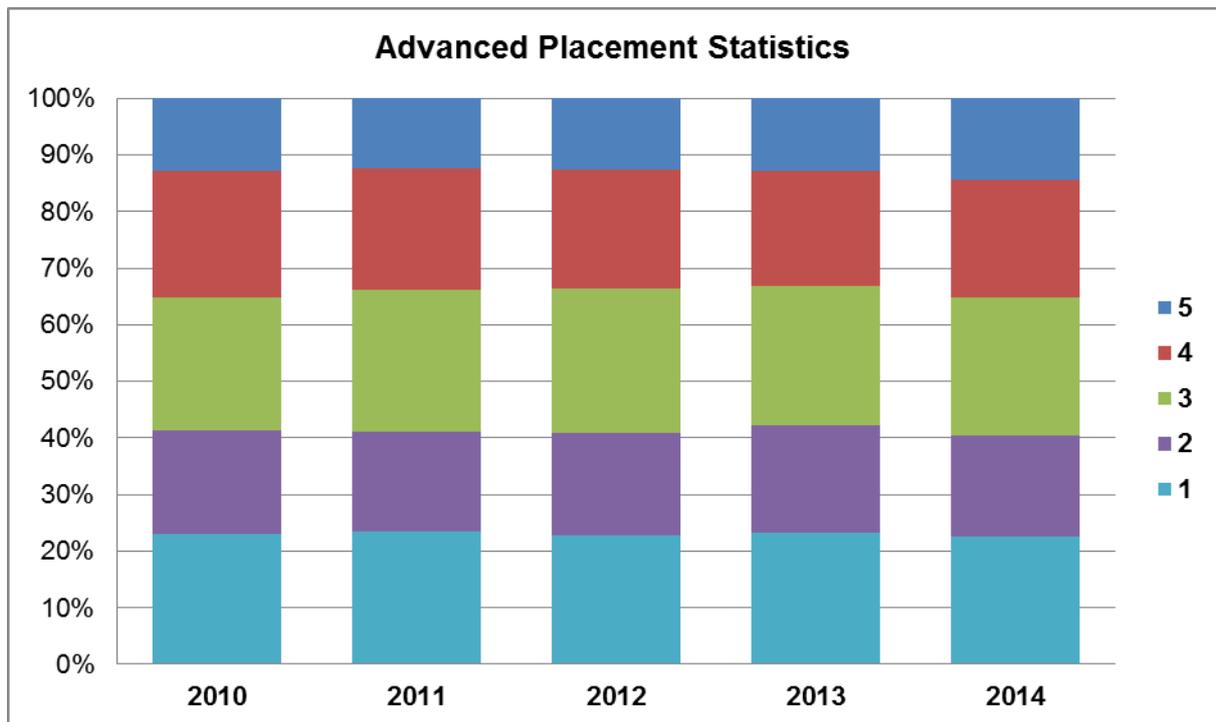
AP Calculus AB



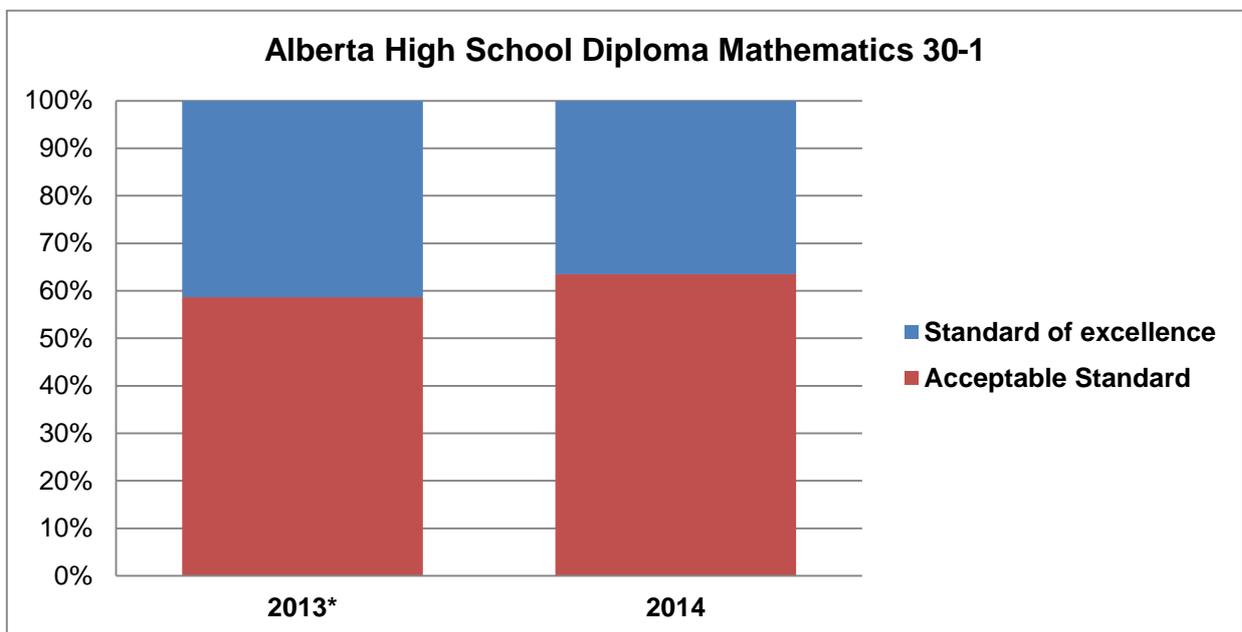
AP Calculus BC



AP Statistics



Alberta Mathematics 30-1



Alberta Mathematics 30-2

