DP Country Alignment Studies: Alignment of DP Mathematics with the Mathematics Subjects in Eleven Comparison Programmes/Standards

Submitted to the IB by Ecctis

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Acronyms

AA	mathematics: analysis and approaches
AF	Advanced Functions
AHL	additional higher level
AI	mathematics: applications and interpretation
BGE	Basic General Education
BHSC	Brazilian High School Curriculum
BNCC	(Brazilian) National Common Curricular Base
CAS	creativity, activity, service
CCSS	Common Core State Standards
CCSSM	Common Core State Standards for Mathematics
CV	Calculus and Vectors
DM	Mathematics of Data Management
DP	Diploma Programme
FB	French Baccalauréat
FI	Formative Itinerary
FM	Foundation Mathematics
FNCC	Finnish National Core Curriculum
FRM	Further Mathematics
GCE	General Certificate of Education
GM	General Mathematics
GSPI	General Subjects Pathway I
GSPII	General Subjects Pathway I
GUSE	General Upper Secondary Education
H2F	Higher 2 Further Mathematics
HL	higher level
IB	International Baccalaureate

IBO	International Baccalaureate Organisation
JHSC	Japanese High School Curriculum
KHSCG	South Korean High School Certificate of Graduation
MAT	Mathematics and Technology
MBG	Mexican Bachillerato General
MCF	Mathematics Curriculum Framework
MEXT	Ministry of Education, Culture, Sports, Science and Technology
MM	Mathematical Methods
MOE	Ministry of Education
MT	Mathematical Thinking
NCC	National Core Curriculum
OSSD	Ontario Secondary School Diploma
SB	Spanish Bachillerato
SC	specific competence
SEAB	Singapore Examinations and Assessment Board
SGA	Singaporean GCE A Level
SL	standard level
SM	Specialist Mathematics
SMC	Science and Mathematics Course
SSPI	Specialised Subjects Pathway I
SSPII	Specialised Subjects Pathway II
TOK	theory of knowledge
UP	University Preparation
USA	United States of America
VCAA	Victorian Curriculum and Assessment Authority
VCE	Victorian Certificate of Education
21CC	21st Century Competencies

Contents

1. Executive Summary

1.1 Project Aims and Context

The International Baccalaureate (IB) Organization is a not-for-profit educational foundation offering four programmes across the world. One of them – the Diploma Programme (DP) – is a two-year upper secondary programme, primarily intended to prepare students for university matriculation and higher education.

Ecctis was commissioned by the IB to conduct a series of in-depth studies that assessed the level of alignment between the DP and comparison points within the upper secondary education systems of Australia (Victoria), Canada (Ontario), the United States, Singapore, South Korea, Finland, France, Spain, Brazil, Mexico, and Japan.[1] The studies identified areas of similarity and difference between the DP and these educational systems, providing valuable insights to inform the IB's development of tools and resources for IB teachers, supporting their navigation between the DP and local curriculum of each country. The studies also contributed to supporting fair recognition of the DP by institutions, employers, and other key stakeholders – ultimately supporting the progression and mobility of DP graduates. For all countries, the studies assessed alignment at both the programme and subject levels, with a particular focus on mathematics and science. For some countries, additional subjects were included in the analysis, as outlined in the table below:

Table I: Additional subjects included in the country studies.

Country	Additional DP Subject(s)	Country	Additional DP Subject(s)
Australia	History	Brazil	Language A: language and literature History Philosophy Brazilian social studies
United States	English		
France	Philosophy Theory of knowledge (TOK)	Mexico	Language A: language and literature
Spain	Economics Business management	Japan	Language A: literature Language B History

This report forms one of the key deliverables of the project and is designed to address the research questions pertaining to how DP mathematics aligns with the mathematics curricula in each of the comparison programmes/standards examined in these studies, namely:

- Australia: Victorian Certificate of Education (VCE)
- Canada: Ontario Secondary School Diploma (OSSD)
- Finland: Finnish National Core Curriculum (FNCC)/General Upper Secondary Education (GUSE)
- Singapore: Singaporean GCE A Level (SGA)
- South Korea: Korean High School Certificate of Graduation (KHSCG)
- US: Common Core State Standards (CCSS)
- France: French Baccalauréat (FB)

[1] The IB DP Country Alignment reports can be accessed at: www.ibo.org/research/curriculum-research/dp-studies/dp-country-alignment-studies-2023/

- Spain: Spanish Bachillerato (SB)
- Brazil: Brazilian High School Curriculum (BHSC)
- Mexico: Mexican Bachillerato General (MBG)
- Japan: Japanese High School Curriculum (JHSC).

1.2 Research Questions and Methods

All comparative studies in this series have been framed by responses to Research Questions (RQs), both at programme and subject levels. For this study, the RQs were the following:

RQ1: To what degree do the DP mathematics curricula align with the upper secondary mathematics curricula of the eleven comparison programmes? In what way are the curricula similar and in what way are they different? To what degree do the subjects[2] align with regard to:
1.1: Content
- Topics (i.e. scope of content area, breadth, depth)
- Learning activities (i.e. difficulty, demand).

1.2: Expected learning outcomes
- Knowledge
- Competencies (i.e. subject-specific, 21st century competencies).

To answer the above RQs, Ecctis developed and applied a bespoke methodology which involved the comparative analysis of key components of the DP and the comparison subjects, including: learning outcomes, content, and demand.

Where appropriate, Ecctis complemented its standard comparative methodology with a comprehensive mapping method, extracting themes from the DP to evaluate their presence in the comparison point(s). Additionally, to assess demand at subject level, Ecctis designed and deployed an expert panel approach, scoring each individual subject against a common set of demand criteria.[3]

[2] With regard to subjects within scope, see Table below.
[3] Each individual subject was scored for: cognitive skills evidenced in the learning outcomes (based on the Revised Bloom's Taxonomy), depth of knowledge (adapted from Webb's Depth of Knowledge levels), volume of work (a tri-factor score considering breadth, depth and allocated timeframe), and outstanding areas of subject demand (stretch areas).

1.3 Key Findings

Regarding subjects compared in the subject-level comparative analysis, the following tables present the agreed scope:

Table II: DP and comparison subjects (from the 2022 studies).

		Comparison subjects per country and comparison programme					
Country	**International**	**Australia**	**Canada**	**Finland**	**Singapore**	**South Korea**	**USA**
Programme / Standards	**DP**	**Victoria (VCE)**	**Ontario (OSSD)**	**Finland (FNCC)**	**Singapore (SGA)**	**South Korea (KHSCG)**	**Common Core State Standards (CCSS)**
MATHEMATICS							
Subjects	Mathematics: analysis and approaches (AA) SL and HL Mathematics: applications and interpretation (AI) SL and HL	Foundation Mathematics (Units 1 and 2) General Mathematics (Units 1 and 2) Further Mathematics (Units 3 and 4) Mathematical Methods (Units 1, 2, 3 and 4) Specialist Mathematics (Units 1, 2, 3 and 4)	Grade 9 De-streamed Grade 10 Foundations of Mathematics Grade 10 Principles of Mathematics Grade 11 Functions Grade 11 Functions and Applications Grade 12 Advanced Functions Grade 12 Calculus and Vectors Grade 12 Mathematics of Data Management	Basic math syllabus Advanced math syllabus	Higher 1 (H2) mathematics Higher 2 (H2) mathematics Higher 2 further (H2F) mathematics Higher 3 (H3) mathematics	<u>Pathway</u>: Mathematics, Mathematics I, Mathematics II, Probability and Statistics, Geometry, and Calculus. <u>Other electives</u>: Mathematical Inquiry Task, Economic Mathematics, and Artificial Intelligence Mathematics	Common Core State Standards for Mathematics (high school) (CCSSM)

Table III: DP and comparison subjects (from the 2023, 2024, and 2025 studies)

<table>
<tr><th></th><th></th><th colspan="4">Comparison subjects per country and comparison programme</th><th></th></tr>
<tr><th>Country</th><th>International</th><th>France</th><th>Spain</th><th>Brazil</th><th>Mexico</th><th>Japan</th></tr>
<tr><th>Programme / Standards</th><th>DP</th><th>France (FB)</th><th>Spain (SB)</th><th>Brazil (BHSC)</th><th>Mexico (MBG)</th><th>Japan (JHSC)</th></tr>
<tr><td colspan="6">MATHEMATICS</td><td></td></tr>
<tr><td rowspan="2">Subjects</td><td>Mathematics: analysis and approaches (AA) SL and HL</td><td rowspan="2">Mathematics</td><td rowspan="2">Math I and II</td><td rowspan="2">Mathematics and Technology
• Basic General Education
• Formative Itinerary</td><td rowspan="2"><u>Mathematical Thinking</u>

Compulsory units
• Mathematical Thinking I, II, and III
• Selected Topics in Mathematics I and II

Optional units
• Probability and Statistics I and II
• Differential Calculus and Integral Calculus
• Financial Mathematics I and II
• Drawing I and II</td><td rowspan="2">General subjects
• Mathematics I
• Mathematics II
• Mathematics III
• Mathematics A
• Mathematics B
• Mathematics C
• Basic Inquiry
• Inquiry

Specialised subjects
(For the Science and Mathematics Course)
• Mathematics I
• Mathematics II
• Advanced Mathematics</td></tr>
<tr><td>Mathematics: applications and interpretation (AI)
SL and HL</td></tr>
</table>

Detailed findings from the subject-level analysis are summarised in the 5. Key Findings section and fully detailed in the respective country sections in the main body of this report:

- Australia: Victorian Certificate of Education (VCE)
- Canada: Ontario Secondary School Diploma (OSSD)
- Finland: Finnish National Core Curriculum (FNCC)/General Upper Secondary Education (GUSE)
- Singapore: Singaporean GCE A Level (SGA)
- South Korea: Korean High School Certificate of Graduation (KHSCG)
- USA: Common Core State Standards (CCSS).
- France: Baccalauréat (FB)
- Spain: Bachillerato (SB).
- Brazil: Brazilian High School Curriculum (BHSC)
- Mexico: Mexican Bachillerato General (MBG)
- Japan: Japanese High School Curriculum (JHSC)

1.4 Cross-cutting Findings

In addition to the insights gained from the analysis of each comparison subject against the DP mathematics subjects, the study also adopted a cross-cutting perspective of the mathematics curricula across all countries included in the studies. This horizontal analysis identified key similarities, differences, and trends relating to learning outcomes, content and demand. The findings are summarised in the visuals and bullet points that follow.

1.4.1 Learning Outcomes

The table below displays the learning outcome themes extracted from the DP mathematics subject group and summarises their presence in the mathematics curricula of the eleven comparison programmes/standards.

Table IV: Presence of the DP's mathematics learning outcomes in the comparison curricula.

Learning outcome themes extracted from the DP mathematics subject group	Presence of DP themes in the learning outcomes of the comparison curricula										
	VCE (Victoria)	OSSD (Ontario)	FNCC (Finland)	SGA (Singapore)	KHSCG (South Korea)	CCSS (US)	FB (France)	SB (Spain)	BHSC (Brazil)	MBG (Mexico)	JHSC (Japan)
1. Wider contexts											
2. Transferable learning skills											
3. Inquiry-based approaches											
4. Understanding and application											
5. Links and generalisations											
6. Critical/creative thinking skills											
7. Communicating mathematics											
8: Technology											

Key:

	This theme is well-evidenced in the learning outcomes of the comparison curriculum.		*This theme is partially evidenced in the learning outcomes of the comparison curriculum.*		*This theme is not evidenced in the learning outcomes of the comparison curriculum.*

- Overall, most comparison curricula have a high degree of alignment with the DP's mathematics learning outcomes. Rarely are the DP's learning outcome themes absent from the comparison curricula and, from the other perspective, very few themes emerge from the comparison curricula that are significantly different to the DP's. The curricula with the strongest alignment are the OSSD, FNCC, SB and BHSC, closely followed by the SGA.

- The themes of *Transferable learning skills*, *Understanding and application*, *Making links and generalisations*, *Critical and creative thinking skills*, *Communication*, and *Using technology* are usually well-evidenced in the comparison curricula.

- Although present in some curricula, the themes of *Linking mathematics to wider contexts* and *Using inquiry-based approaches* are the least consistently evidenced in the learning outcomes of the comparison curricula.

- In terms of themes or emphases featured in the comparison programmes but not in the DP, the skill of modelling has greater emphasis in the learning outcomes of some of the comparison curricula; though it can be noted that this is focused on in the other sections of the DP syllabi. There is also a stronger emphasis in some comparison curricula on encouraging students to engage in certain types of deeper thinking during their mathematical learning – e.g. developing metacognition skills or developing social-emotional learning skills. The latter is not a feature of the DP mathematics' learning outcomes but is an undercurrent of the wider philosophical underpinnings of the DP (e.g. in the learner profile). Lastly, some curricula more strongly emphasis heuristic or computational thinking in their learning outcomes.

1.4.2 Content

The following tables summarise the presence of DP mathematics SL and AHL content in the mathematics curricula of the eleven comparison programmes/standards. Here, all the mathematics content within each comparison curriculum is considered. To view the mathematics content within specific subjects, and how it aligns with DP mathematics subjects, refer to the relevant country in Section 4. Subject-Level Alignment.

Tables V and VI: Presence of DP mathematics content in the mathematics curricula of the comparison programmes/standards.

Mathematics: analysis and approaches topics		Presence in comparison curricula										
		VCE	OSSD	FNCC	SGA	KHSCG	CCSS	FB	SB	BHSC	MBG	JHSC
SL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
AHL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
SL/HL	Mathematics exploration											

Mathematics: applications and interpretation topics		Presence in comparison curricula										
		VCE	OSSD	FNCC	SGA	KHSCG	CCSS	FB	SB	BHSC	MBG	JHSC
SL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
AHL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
SL/HL	Mathematics exploration											

Key:

	Strong presence of this topic in the comparison curricula		*Partial presence of this topic in the comparison curricula*		*Little or no presence of this topic in the comparison curricula*

1.4.2.1 Structure

- The organisation of the mathematics subject area into AA and AI is unique to the DP. None of the comparison curricula offer two subjects that cover the same broad range of topics but differ in their thematic focus.

- The size and thematic focus of mathematics subjects, courses, and electives in the comparison curricula vary significantly. While most subjects, like those in the DP, VCE, SGA, FNCC, CCSS, FB, and SB, cover a broad range of mathematical areas, other curricula, like the KHSCG, OSSD, BHSC, MBG, and JHSC organise all, or some, of their upper secondary mathematics content into courses/electives/units that each focus on a narrower range of mathematical areas. Topics like calculus, functions, and statistics are often taught in separate, standalone courses.

- The DP is unique in having five main topics which are studied by all students, regardless of subject and course choice. In other mathematics curricula, the main topics can greatly vary depending on the subject chosen, or the number of courses/electives taken.

- Except for the NGSS, the comparison curricula offer students the option to study mathematics at different levels. Often, the comparison curricula cater to a wider range of mathematical ability than the DP, with most offering options pitched at a more foundational level than DP mathematics SL and a few also offering more opportunity for specialisation.

- In the DP and most comparison curricula, once chosen, all the content within a specific mathematics course is compulsory. The few exceptions to these are in the FNCC, VCE, and JHSC.

1.4.2.2 Content Alignment

- Content from each of the DP's five main topics of *Number and algebra*, *Functions*, *Geometry and trigonometry*, *Statistics and probability*, and *Calculus* is found in all comparison curricula, except the CCSS and BHSC, which do not cover any calculus content.

- The comparison curricula tend to align more strongly with AA SL content than AI SL content. With a few exceptions, AA SL content is strongly present across the comparison curricula. AI SL content is strongly present in some curricula, however, it has more of a partial presence in several others. Notably, the AI SL sub-topic of Voronoi diagrams is not present in any of the comparison curricula and hypothesis testing is also often absent.

- To varying degrees, all the comparison curricula (except the BHSC) contain some AHL content. However, this content is not strongly present across the comparison curricula. The specific AHL topics and sub-topics which are covered vary – i.e. there are very few AHL sub-topics that are consistently covered in all, or most, of the curricula. That said, the comparison curricula (excluding the BHSC and MBG) typically include some vectors content, which aligns with *Geometry and trigonometry* AHL content. Certain areas of AI AHL content are particularly rare in the comparison curricula. Indeed, there is often limited or no alignment with AI *Statistics and probability* AHL content. Other AI areas which are often not present in other curricula include eigenvalues and eigenvectors, graph theory, adjacency matrices, slope fields, and phase portraits. As with SL content, the comparison curricula tend to align more strongly with AA AHL content than AI AHL content.

- Only some of the comparison curricula have alignment with the DP's mathematical exploration, namely the VCE, OSSD, KHSCG, BHSC and JHSC. The strongest similarities to this DP component are observable in the KHSCG and JHSC.

- Regarding content not covered by the DP, there is no large mathematical area which is commonly covered by the comparison curricula and not by the DP. However, it can be noted that it is common for other curricula to include different geometry content, such as conics, as well as polar coordinates and limits of sequences. One notable difference, that is not common across the curricula, is the coverage of algorithms and programming in the FB and BHSC.

1.4.3 Demand

The following tables provide a visual representation of the demand scores that the expert panel awarded to the DP mathematics subjects and respective subjects in each of the eleven comparison programmes/standards.

Key:

VCE: FM: Foundation Mathematics GM: General Mathematics FRM: Further Mathematics MM: Mathematical Methods SM: Specialist Mathematics	OSSD: F&A: Functions and Applications F: Functions AF: Advanced Functions CV: Calculus and Vectors DM: Mathematics of Data Management UP: University Preparation pathway	CCSS: CCSSM: Common Core State Standards for Mathematics	FNCC/GUSE: B: Basic A (C): Advanced, compulsory modules only A (C+O): Advanced, compulsory and optional modules combined
SGA: H1: Higher 1 H2: Higher 2 H2F: Higher 2 Further H3: Higher 3	KHSCG: Pathway: Mathematics, Mathematics I, Mathematics II, Calculus, Probability and Statistics, and Geometry	FB: Math P: Mathematics *Première* Math T: Mathematics *Terminale*	SB: Mathematics I Mathematics II
BHSC BGE: Basic General Education FI: Formative Itinerary	MBG C: Compulsory units C+O: Compulsory and optional units	JHSC GSPI: General Subjects Pathway I GSPII: General Subjects Pathway II SSPI: Specialised Subjects Pathway I SSPII: Specialised Subjects Pathway II	

Table VII: The demand scores of the mathematics subjects in the DP and comparison programmes (2022 studies; Australia, Canada and Finland).

Demand Category	Scores																	
	DP (IB)				VCE (Australia)					OSSD (Canada)						FNCC/GUSE (Finland)		
	AA SL	AA HL	AI SL	AI HL	FM	GM	FRM	MM	SM	F&A 11	F 11	AF 12	CV 12	DM 12	UP	B	A (C)	A (C+O)
Revised Bloom's Cognitive Skills	3	3	3	3	1	2	2	2	2.5	3	3	3	3	3	3	3	3	3
Depth of Knowledge	2	3	2	3	0	1	1.5	2	3	1	1	2	2	1.5	2	1	2	2.5
Volume of Work	2	3	2	3	0	1	1	1	2	0	0	0.5	0.5	0.5	0.5	0	1	1.5
Outstanding Demand Areas	1	3	1	3	0	0	1	1	3	0	0	0	1	1	2	0	0	1

Table VIII: The demand scores of mathematics subjects in the DP and comparison programmes/standards (2022 studies; Singapore, South Korea and US).

Demand Category	Scores									
	DP (IB)				SGA (Singapore)				KHSCG (S. Korea)	CCSS (US)
	AA SL	AA HL	AI SL	AI HL	H1	H2	H2F	H3	Pathway	CCSSM
Revised Bloom's Cognitive Skills	3	3	3	3	3	3	3	3	3	3
Depth of Knowledge	2	3	2	3	1.5	3	3	3	2.5	2
Volume of Work	2	3	2	3	1	1	2	3	0.5	0
Outstanding Demand Areas	1	3	1	3	0	1	3	3	1	0

Table IX: The demand scores of mathematics subjects from the DP and comparison programmes (2023 and 2024 studies; France, Spain, Brazil, Mexico and Japan).

Demand Category	Scores															
	DP (IB)				FB (France)		SB (Spain)		BHSC (Brazil)		MBG (Mexico)		JHSC (Japan)			
	AA SL	AA HL	AI SL	AI HL	Math P	Math T	Math I	Math II	BGE	FI	C	C+O	GSPI	GSPII	SSPI	SSPII
Revised Bloom's Cognitive Skills	3	3	3	3	2.5	2.5	3	3	2.5	3	2	2.5	3	3	3	3
Depth of Knowledge	2	3	2	3	1.5	3	1.5	3	1.5	2	1.5	2	2	3	3	3
Volume of Work	2	3	2	3	1	1	2	3	0	0	0	0.5	1	1.5	1.5	2
Outstanding Demand Areas	1	3	1	3	1	3	1	3	0	1	0	0	1	2	3	3

- Most comparison curricula have subjects that score the same, or similarly, as DP mathematics subjects for Bloom's cognitive skills, receiving the highest scores for their evidence of analysis, critical thinking, and creativity in their learning outcomes.

- All comparison curricula offer at least one subject/pathway which scores the same as, or similar to, DP mathematics SL for depth of knowledge. In contrast, only the VCE, SGA, FB, SB, and JHSC have subjects/pathways which score the same as DP mathematics HL subjects in this category. It can be noted that both the FNCC and KHSCG receive very similar scores where specific courses are taken. The OSSD, VCE, and FNCC all include at least one subject or course which scores one point less than DP mathematics SL in this category. This would also be true for the KHSCG and JHSC, depending on the electives chosen.

- Apart from some subjects/pathways in the VCE, SGA, SB, and JHSC, the subjects in the comparison curricula score lower for volume of work than DP subjects, often significantly so. Many subjects/courses have a generous or standard amount of time allocated to teach the syllabus and don't comprise a moderate or heavy workload. However, teaching hours are not described in the SGA or CCSS, meaning that some caution is necessary when interpreting the judgements for these.

- Apart from the CCSS and MBG, all the comparison curricula offer at least one subject/course that features at least one area of outstanding demand – the same as DP mathematics SL. In contrast, only the VCE, SGA, FB, SB, and JHSC offer subjects/pathways that have a comparable number of areas of outstanding subject demand as DP mathematics HL.

- Overall, most comparison curricula offer a subject/pathway which is similar in demand to DP mathematics subjects at SL. However, only the VCE, SGA, SB, and JHSC offer subjects/pathways which closely align with DP mathematics subjects at HL – though it can be noted FB also has some alignment. Generally, volume of work is the primary category where the comparison subjects score significantly lower than DP subjects, followed by outstanding areas of demand.

2. Introduction

2.1 Context and Scope

The International Baccalaureate (IB) Organization is a not-for-profit educational foundation offering four programmes across the world, including the Primary Years Programme (PYP), Middle Years Programme (MYP), Diploma Programme (DP) and the Career-related Programme (CP). The DP – the IB's two-year upper secondary programme – is conceived as a preparatory programme for university matriculation and higher education, aimed at developing students with “excellent breadth and depth of knowledge” who “flourish physically, intellectually, emotionally and ethically”.[4]

Ecctis was commissioned by the IB to conduct a series of in-depth studies to assess the level of alignment between the DP and comparison points within the upper secondary education systems of Australia (Victoria), Canada (Ontario), the United States, Singapore, South Korea, Finland, France, Spain, Brazil, Mexico, and Japan.[5] More specifically, the studies identified areas of similarity and difference between the DP and these educational systems at the programme level by comparing philosophical underpinnings, structures, entry requirements, assessment methods, and learning pathways. The studies also examined how the DP compared with selected benchmarks in terms of intended student learning outcomes at the subject level. For all countries, the studies focused on mathematics and sciences, with some also including additional subjects, as outlined in the table below:

Table 1: Additional subjects included in the country studies

Country	Additional DP Subject(s)	Country	Additional DP Subject(s)
Australia	History	Brazil	Language A: language and literature History Philosophy Brazilian social studies (BSS)
United States	English		
France	Philosophy Theory of knowledge (TOK)	Mexico	Language A: language and literature
Spain	Economics Business management	Japan	Language A: literature Language B History

Ultimately, this series of comparative studies informed the IB's development of tools and resources for IB teachers, helping them navigate between the DP and the local curriculum in the target countries. The studies also contributed to further supporting fair recognition of the DP by institutions, employers, and other key stakeholders, supporting progression and mobility for DP graduates.

This report forms one of the key deliverables of the project and is designed to address the research questions related to the extent of alignment between the DP mathematics subjects and the mathematics subjects within each of the eleven comparison programmes/standards examined in these studies, namely:

[4] International Baccalaureate. (2022). *Diploma Programme*. Available from: https://www.ibo.org/programmes/diploma-programme/

[5] The full reports can be accessed at: www.ibo.org/research/curriculum-research/dp-studies/dp-country-alignment-studies-2023/

- Australia: Victorian Certificate of Education (VCE)
- Canada: Ontario Secondary School Diploma (OSSD)
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- US: Common Core State Standards (CCSS)
- France: French Baccalauréat (FB)
- Spain: Spanish Bachillerato (SB)
- Brazil: Brazilian High School Curriculum (BHSC)
- Mexico: Mexican Bachillerato General (MBG)
- Japan: Japanese High School Curriculum (JHSC).

2.2 Research Questions

All comparative studies in this series have been framed by responses to Research Questions (RQs), both at programme and subject levels. For this study, the RQ is the following:

Table 2: Research questions

RQ1: To what degree do the DP mathematics curricula align with the upper secondary mathematics curricula of the eleven comparison programmes? In what way are the curricula similar and in what way are they different? To what degree do the subjects[6] align with regard to: 1.1: Content • Topics (i.e. scope of content area, breadth, depth) • Learning activities (i.e. difficulty, demand). 1.2: Expected learning outcomes • Knowledge • Competencies (i.e. subject-specific, 21st century competencies).

Regarding subjects to be compared in the subject-level comparative analysis, the following tables indicate the agreed scope:

[6] With regard to subjects within scope, see Table below.

Table 3: DP mathematics subjects and comparison subjects (2022 studies)

		Comparison subjects per country and comparison programme					
Country	**International**	**Australia**	**Canada**	**Finland**	**Singapore**	**South Korea**	**USA**
Programme / Standards	**DP**	**Victoria (VCE)**	**Ontario (OSSD)**	**Finland (FNCC)**	**Singapore (SGA)**	**South Korea (KHSCG)**	**Common Core State Standards (CCSS)**
Subjects	mathematics: analysis and approaches (AA) SL and HL mathematics: applications and interpretation (AI) SL and HL	Foundation Mathematics (Units 1 and 2) General Mathematics (Units 1 and 2) Further Mathematics (Units 3 and 4) Mathematical Methods (Units 1, 2, 3 and 4) Specialist Mathematics (Units 1, 2, 3 and 4)	Grade 9 De-streamed Grade 10 Foundations of Mathematics Grade 10 Principles of Mathematics Grade 11 Functions Grade 11 Functions and Applications Grade 12 Advanced Functions Grade 12 Calculus and Vectors Grade 12 Mathematics of Data Management	Basic math syllabus advanced math syllabus	Higher 1 (H2) mathematics Higher 2 (H2) mathematics Higher 2 further (H2F) mathematics Higher 3 (H3) mathematics	Pathway: Mathematics, Mathematics I, Mathematics II, Probability and Statistics, Geometry, and Calculus. Other electives: Mathematical Inquiry Task, Economic Mathematics, and Artificial Intelligence Mathematics	Common Core State Standards for Mathematics (high school) (CCSSM)

Table 4: DP and comparison subjects (2023, 2024, and 2025 studies).

<table>
<tr><th></th><th></th><th colspan="4">Comparison subjects per country and comparison programme</th><th></th></tr>
<tr><th>Country</th><th>International</th><th>France</th><th>Spain</th><th>Brazil</th><th>Mexico</th><th>Japan</th></tr>
<tr><th>Programme / Standards</th><th>DP</th><th>France (FB)</th><th>Spain</th><th>Brazil (BHSC)</th><th>Mexico (MBG)</th><th>Japan (JHSC)</th></tr>
<tr><td colspan="6">MATHEMATICS</td><td></td></tr>
<tr><td rowspan="2">Subjects</td><td>mathematics: analysis and approaches (AA)
SL and HL</td><td rowspan="2">Mathematics
• Première
• Terminale</td><td rowspan="2">• Math I
• Math II</td><td rowspan="2">Mathematics and Technology (MAT)
• Basic General Education (BGE)
• Formative Itinerary (FI)</td><td rowspan="2"><u>Mathematical Thinking</u>

Compulsory units
• Mathematical Thinking I, II, and III
• Selected Topics in Mathematics I and II

Optional units
• Probability and Statistics I and II
• Differential Calculus and Integral Calculus
• Financial Mathematics I and II
• Drawing I and II</td><td rowspan="2">General subjects
• Mathematics I
• Mathematics II
• Mathematics III
• Mathematics A
• Mathematics B
• Mathematics C
• Basic Inquiry
• Inquiry

Specialised subjects (For the Science and Mathematics Course)
• Mathematics I
• Mathematics II
• Advanced Mathematics</td></tr>
<tr><td>mathematics: applications and interpretation (AI)
SL and HL</td></tr>
</table>

2.3 Report Structure

In responding to the above RQ, this report includes the following sections:

- 3. Methodology: this section provides a brief overview of the methodology applied in this study. This includes details of how the document selection and identification of comparison points for the study took place; a definition of 'alignment'; an outline of the methodology used for comparisons at subject level; and an outline of the methodology used to assess demand.

- 4. Subject-Level Alignment: this section presents the synthesised analysis from the subject-level comparisons between the DP mathematics subjects and the mathematics subjects in the eleven comparison programmes. For each comparison subject, this includes the comparative analysis on their learning outcomes, content, and demand.

- 5. Key Findings: this section outlines the key findings from the subject-level comparisons undertaken between the DP mathematics subjects and those of each of the eleven comparison programmes. In doing so, it provides a top-level conclusion on subject-level alignment for each programme, and a succinct summary of key similarities and key differences.

- 6. Cross-cutting findings: this section outlines the key cross-cutting findings from the various subject-level comparisons carried out between the DP mathematics subjects and those of each of the eleven comparison programmes. In doing so, it unpicks the main trends identified at the learning outcomes, content and demand levels, highlighting noticeable similarities and differences.

- 7. Bibliography: this section references all sources cited in the study, including the documents used for programme descriptions and subject-level analyses.

3. Methodology

3.1 Document Selection and Identification of Comparison Points

To undertake the comparative analyses at subject level, the following core documentation was reviewed (supplemented by additional documentation – detailed in the Bibliography – where relevant and available):

IB Documentation

- DP: From principles into practice
- Programme standards and practices
- DP subject guides:
  - mathematics: analysis and approaches
  - mathematics: applications and interpretation.

Victoria Documentation

- Documentation by the Victorian Curriculum and Assessment authority (VCAA)
- VCE Mathematics Study Design.

Ontario Documentation

- Ontario Curriculum and Resources (website)
- The Ontario Curriculum – subject guides:
  - mathematics grade 9 (de-streamed, online)
  - mathematics (grade 10)
  - mathematics (grades 11 and 12).

Finland Documentation

- Finnish National Agency for Education – National Core Curriculum for General Upper Secondary Education
- Subject syllabi:
  - Basic syllabus in mathematics
  - Advanced syllabus in mathematics.

Singapore Documentation

- Ministry of Education (MOE) – Singapore (website)
- MOE – Singapore's Post-Secondary Education booklet
- The Singapore Curriculum – Pre-University subject syllabi:
  - Higher 1 Mathematics
  - Higher 2 Mathematics
  - Higher 2 Further Mathematics
  - Higher 3 Mathematics.

South Korea Documentation

- MOE – The National Curriculum for Primary and Secondary Schools
- MOE – Education in Korea (website)
- MOE – Mathematics Curriculum.

CCSS Documentation

- The 'About the Standards' section of the CCSS website
- The CCSS for Mathematics (high school standards).

France Documentation

- French Baccalaureate curriculum (website)
- The French Baccalaureate – subject programmes – Mathematics *Général Première Terminale*

Spain Documentation

- Government of Spain, Ministry of Education and Vocational Training (website), including information about Key Competences of the SB,[7] as well as the specific competences (SCs), evaluation criteria and content covered in each of the following subjects:
    - mathematics I
    - mathematics II
- Where possible, due to the less detailed nature of the SB curriculum, this was complemented by a review of publicly available official textbooks. References to these can be found in the Bibliography section of this report.

Brazil Documentation

- Ministry of Education updates to the National Curriculum Guidelines for High School Education
- National Common Curriculum Base (BNCC) for Mathematics and Technology
- Curricular References for the Preparation of Formative Itineraries
- Rio de Janeiro Referential Curriculum (RJRC)
- Rio de Janeiro specialisation pathways for formative itineraries in Mathematics and Technology

Mexico Documentation

- Learning Progressions for Mathematical Thinking
- Programmes of Study (Mathematics):
    - Mathematical Thinking I, II & III
    - Selected Topics in Mathematics I & II
    - Differential Calculus
    - Drawing I & II
    - Financial Mathematics I & II
    - Integral Calculus
    - Probability and Statistics I & II

JHSC Documentation

- High School Course of Study (Announced in 2018)
- Commentary on the High School Curriculum Guidelines - Mathematics

[7] Government of Spain, Ministry of Education and Vocational Training. (2022) Royal Decree 243/2022. https://www.boe.es/buscar/act.php?id=BOE-A-2015-37; https://educagob-educacionyfp-gob-es.translate.goog/curriculo/curriculo-lomloe/menu-curriculos-basicos/bachillerato/competencias-clave.html?_x_tr_sl=es&_x_tr_tl=en&_x_tr_hl=en-US&_x_tr_pto=wapp

3.1.1 Learning Outcomes Comparison

For the Learning Outcomes comparisons, as not all subjects in each of the qualifications reviewed explicitly name learning outcomes in their syllabi, Ecctis used the following categories of the curriculum documentation for comparison:

Table 5: Learning outcomes for comparison of the DP mathematics and respective comparison subjects

MATHEMATICS		
Programme/ Standards	**Subject**	**Categories utilised as learning outcomes**
DP	mathematics: analysis and approaches mathematics: applications and interpretation	DP mathematics subject group – aims and assessment objectives
Australia, Victoria (VCE)	Foundation Mathematics (Units 1 and 2) General Mathematics (Units 1 and 2) Further Mathematics (Units 3 and 4) Mathematical Methods (Units 1, 2, 3 and 4) Specialist Mathematics (Units 1, 2, 3 and 4)	Aims and Outcomes
Canada, Ontario (OSSD)	Grade 9 Mathematics De-streamed Grade 10 Foundations of Mathematics Grade 10 Principles of Mathematics Grade 11 Functions and Applications Grade 11 Functions Grade 12 Advanced Functions Grade 12 Calculus and Vectors Grade 12 Mathematics of Data Management	Mathematical Process Expectations and Grade 9 Expectations by Strand (A and AA)*
Finland (FNCC)	Basic mathematics Advanced mathematics	Task of the subject Transversal competences General objectives
Singapore (SGA)	Mathematics H1 Mathematics H2 Mathematics H2F Mathematics H3	Mathematics Curriculum Framework, Mathematics and 21CC, Syllabus Aims
South Korea (KHSCG)	Mathematics pathway	Subject introductions (titled ‘Character’) and objectives
USA (CCSS)	Mathematics (high school)	Standards for Mathematical Practice
France (FB)	Mathematics (*Première*) Mathematics (*Terminale)*	Major Intentions, which include the six ‘Math Skills’ which are to be developed
Spain (SB)	Mathematics I Mathematics II	Specific competences and evaluation criteria
Brazil (BHSC)	Mathematics and Technology (Basic General Education)	Specific competencies and skills in the BNCC
	Mathematics and Technology (Formative Itinerary)	Skills in the curricular references for formative itineraries
Mexico (MBG)	Mathematical Thinking (compulsory and optional units)	Learning Progressions Extended Disciplinary Competencies Generic Competencies
Japan (JHSC)	Mathematics I Mathematics II Mathematics III Mathematics A	General Subjects - Mathematics - subject area objectives and individual subject objectives

MATHEMATICS		
Programme/ Standards	**Subject**	**Categories utilised as learning outcomes**
Japan (JHSC)	Mathematics B	
	Mathematics C	
	Basic Inquiry-Based Study of Science and Mathematics	General Subjects - Science and Mathematics - subject area objectives and individual subject objectives
	Inquiry-Based Study of Science and Mathematics	
	Mathematics I (SMC)	Specialised Subjects - Science and Mathematics – subject area objectives and individual subject objectives
	Mathematics II (SMC)	
	Advanced Mathematics (SMC)	

*'Mathematical Process Expectations' were common to all mathematics subjects in the OSSD curriculum, while the 'Grade 9 – Expectations by Strand' are specific to the grade 9 mathematics subject. For the latter, both strands A and AA were included for review, as these were deemed to provide important complementary information on non-content specific skills.

Although not all labelled as learning outcomes per se, the above categories were chosen as they were deemed to provide the most complete picture of the skills and knowledge that students should obtain upon completion of each subject.

For more information on the mapping process, see the Measuring Alignment section.

3.1.2 Content Comparison

For BHSC Mathematics and Technology (MAT) subjects, it can be noted that the analysis drew upon a range of sources to establish a more detailed picture of the subject content used in high school education. The primary document used for basic general education (BGE) subjects was the BNCC (National Common Curricular Base). The BNCC merges skills and content into specific skills for Mathematics and Technology. The specific skills emphasise skills rather than describing content through topics and subtopics. As such, to provide a more concrete picture of the content covered, the Rio de Janeiro Referential Curriculum (RJRC) for high school was used to complement the BNCC for the content analysis of BGE mathematics.

The RJRC was also used to analyse the content of a formative itinerary (FI) specialising in Mathematics and Technology. While the formative itineraries offered can vary from state to state, the RJRC provided insight into the type of mathematics content that these can cover. The analysis reviewed the specialisation pathways offered in Rio de Janeiro for a formative itinerary in Mathematics and Technology.[8]

The following table shows the documentation and sections that were used to inform the content analysis of each BHSC Mathematics and Technology (MAT) subject.

[8] Formative itineraries which integrated MAT with other areas of knowledge were not reviewed.

Table 6: Documentation and sections used for the content analysis of BHSC subjects.

	Document: BNCC	Document: Curricular references for FI	Document: RJRC
BHSC subjects	Section	Section	Section
BHSC MAT (BGE)	Specific competencies and skills for MAT	N/A	Mathematics (for BGE)
BHSC MAT (FI)	N/A	Specific skills associated with structuring axis	MAT Specialisation Pathways (for the FI)

3.2 Measuring Alignment (Similarities and Differences)

Alignment is a key concept for this series of studies, which aims to unpick the level of alignment between the DP and each of the eleven comparison programmes. Although Ecctis has sought to represent the alignment findings as straightforwardly as possible in this report, alignment is not a simple concept, so it is important to establish Ecctis' approach in this regard.

Alignment, as a term, is often used in education circles to refer to *internal* coherence between learning outcomes, assessment methods, teaching practices and other features of teaching and learning. This report does not consider *internal* alignment, but what might appropriately be labelled *external* alignment. Alignment of this type looks at the extent to which a programme (in this case, the DP) aligns with other educational programmes (in this case, the eleven comparison programmes). This form of external alignment is particularly key to understand for an organisation like the IB which operates in so many international contexts, often alongside national curricula, where teachers and students may seek to move back and forth between IB and national streams of education.

Within this narrower definition of *external* alignment, the idea is still broad and could be seen from any number of perspectives. In this series of studies, the IB has specifically asked Ecctis to consider alignment from the specific perspectives outlined by the RQs. The RQs thereby define the limits of the type of alignment that have been considered within the subject reports. Namely:

- At the subject level (in selected subjects):
  - Alignment of learning outcomes
  - Alignment of content
  - Alignment of demand.

To form a comprehensive picture of alignment, Ecctis' approach has used multiple repeating steps within each report:

- Analysing to what extent each comparison programme has similarities with the DP.
- Analyse to what extent each comparison programme lacks features contained within the DP.
- Analyse to what extent the DP lacks features contained within each comparison programme.

In this respect, alignment is a measure of the extent to which there are similarities and differences between key selected criteria of two educational programmes. High alignment indicates significant similarities, with few differences in key areas, whereas low alignment results from many differences in important aspects, with perhaps only few or non-impactful similarities. Alignment judgements in this study took a holistic view of similarities and differences and the likely impact these have on what skills and knowledge students possess upon completion of a subject and programme of study. As such, the study did not use fixed quantitative criteria to differentiate high from low alignment, but rather produced informed, holistic judgements drawing on an outcomes-focused perspective.

3.2.1 Mapping

To accurately measure the alignment of the DP subjects to each comparison programme's subjects, it was necessary to map the similarities and differences across the selected alignment criteria. This necessitates identification of the same structural features in the DP and in the comparison programmes so that a mapping process can be undertaken.

Mapping, in this case, refers to detailed analysis of a feature of an education programme (generally as represented within that programme's documentation). Specifically, mapping applies the same analytical method to two separate sets of data (for example, the learning outcomes of two different curricula), enabling similarities and differences between those two data sets to be understood through the different results of applying the same mapping method to both. Another important feature of mapping is that there is a paper trail of the analysis, as the approach is methodical, testable, and repeatable.

Section 3.3 below provides further detail on how mapping has been applied in this study at subject level. For more information on how mapping has been applied in this series of studies at programme level, refer to sections 3.2.1 of the country alignment reports.

3.3 Method: Subject-Level Comparison

Different methods were used to analyse the alignment at subject level regarding learning outcomes, subject content, and demand. Each approach is outlined below.

3.3.1 Learning Outcomes

To analyse the alignment of learning outcomes at the subject level, the process began by extracting six to eight themes from the DP's subject-level learning outcomes for each subject being analysed, encompassing both skills and knowledge areas. This thematic code was then mapped onto the learning outcomes of the DP subject and the comparison subjects from the eleven comparison programmes.

The top-level results of the mapping process are represented with a table per subject. Following the tables, a written commentary is provided regarding the presence of DP knowledge areas and skills (represented by themes) in each comparison subject and any significant knowledge areas and skills found in the respective comparison subject but not in the DP.

3.3.2 Content

To compare the content of the DP subjects and the comparison subjects, these are presented in a simple tabular format. Additionally, content mapping took place through a simple process of establishing whether each content sub-topic covered by the DP subject in question has 'clear alignment' with any content in the comparison subjects. The mapping spreadsheets demonstrate the full logic of all judgements.

A commentary is provided on DP subject content not found to have alignment points in the comparison subjects and on comparison subject content topics not found to have alignment points in the DP subjects.

3.3.3 Demand

Comparing the demand of subject curricula is perhaps the most complex mapping and alignment analysis within this report. Ecctis' approach views demand from multiple perspectives to capture its relationship to skills as well as to the detail and scope of content.

To allow for a comprehensive assessment of the level of demand of the DP selected subjects against the respective comparison points, Ecctis has created a Demand Profile for each subject in the study. Each Demand Profile comprises four criteria designed to judge complexity, depth, breadth, workload levels and potential for intellectual stretch. These criteria have been applied uniformly across all subjects in the study, using an expert panel-approach (as outlined below).

Demand Profile – Subject-level Judgement

The Demand Profile is comprised of four scores (each between zero and three) based on specific criteria. Each score within each category has a specific definition which is listed in Appendix A. A panel of subject, teaching, and curriculum design experts analysed each subject curriculum and arrived at a consensus on which score descriptor in each category best matched with the curriculum in question. The categories which comprise the Demand Profile are as follows:

- **Revised Bloom's Cognitive Skills** score (0-3): this is an overall score of course demand, based entirely on a review of learning outcomes. Levels have been defined based on increasing emphasis of higher order cognitive skills taken from Bloom's Revised Taxonomy.[9]

- **Depth of Knowledge** (adapted from Webb's) score (0-3): this is an overall score evaluating the depth of knowledge or complexity of knowledge and skills required by curriculum standards and expectations. The score is focused on subject content and learning outcomes, complemented by assessment where relevant/possible. Levels have been defined based on the level of detail studied per topic, as well as the levels of thinking described in Webb's Depth of Knowledge framework.[10]

- **Volume of Work** score (0-3): this is a trifactor score, considering:

[9] Ruhl, C. (2021). *Bloom's taxonomy of learning*. Simply Psychology. Available from: www.simplypsychology.org/blooms-taxonomy.html

[10] Webb, N. L. (2002). *Depth-of-knowledge levels for four content areas*. Language Arts. Available from: Microsoft Word - Webb DOK all content.doc (pbworks.com)

a. breadth of content – i.e. how many topics and sub-topics are covered
b. depth of content – i.e. the extent to which the topics and sub-topics are focused upon, amplified and explored.[11]
c. specified timeframe – i.e. the time allocated for studying the subject.

The three factors – breadth, depth, and time – were all considered in defining the levels.

- **Outstanding Areas of Subject Demand** score (0-3): this score reflects the number of content areas viewed as more challenging and/or conducive to intellectual stretching of students. Levels have been defined on a scale of increasing number of 'stretch areas'.

Demand Panel: Expert Judgement Procedure

Demand analysis and judgements against the above criteria rested with a panel of experts comprised of both curriculum and teaching experts – i.e. international education researchers experienced in comparative secondary curriculum evaluation – and subject experts – i.e. researchers and consultants with a subject specialism in the relevant subject areas. For both expert types, teaching experience, understanding of appropriate national/international teaching contexts, and experience of curriculum and learning outcomes comparisons were prioritised.[12]

Five expert panels were hosted to judge the demand levels of the mathematics subjects in this series of studies, details of each panel discussion are shown below:

1. DP subjects and the subjects from the Victorian (Australia), Ontario (Canada) and CCSS (US) curricula
2. Subjects from the Finnish (FNCC), Singaporean (SGA) and South Korean (KHSCG) curricula
3. Subjects from the French (FB) and Spanish (SB) curricula
4. Subjects from the Brazilian (BHSC) and Mexican (MBG) curricula
5. Subjects from the Japanese (JHSC) curricula.

For each of these, the composition of each panel was as follows.

[11] Note: 'depth of content' primarily describes what is on the curriculum (i.e. the level of detail comprised in each topic), whereas 'depth of knowledge' describes what the students need to be able to do (i.e. how complex and extensive the thinking processes involved are).

[12] To minimise potential biases and subjectivity, Ecctis' recruitment procedure excluded candidates with experience of teaching any of the comparison qualifications in this study.

Figure 1: Demand panels details

Mathematics panel 1
DP, VCE, OSSD, CCSS

Length: one day
Preparation time: four days
Format: remote
Number of subjects discussed: 18
Number of panellists: 6

Composition:
- two Mathematics experts with experience of teaching across multiple education systems
- two Mathematics experts with experience of teaching upper secondary Mathematics in the UK
- two curriculum experts with a background in learning outcomes analysis and teaching at higher education level

Mathematics panel 2
FNCC, SGA, KHSCG

Length: half day
Preparation time: four days
Format: remote
Number of subjects discussed: 8
Number of panellists: 6

Composition:
- two Mathematics experts with experience of teaching across multiple education systems
- two Mathematics experts with experience of teaching upper secondary Mathematics in the UK
- two curriculum experts with a background in learning outcomes analysis and teaching (one at secondary level and one at higher education level)

Mathematics panel 3
FB and SB

Length: half day
Preparation time: one day
Format: remote
Number of subjects discussed: 4
Number of panellists: 4

Composition:
- one Mathematics experts with experience of teaching across multiple education systems
- two Mathematics experts with experience of teaching upper secondary Mathematics in the UK
- one curriculum expert with a background in learning outcomes analysis and teaching (at higher education level)

Mathematics panel 4:
BHSC and MBG

Length: quarter day
Preparation time: one and a quarter days
Format: remote
Number of subjects discussed: 4
Number of panellists: 6

Composition:
- one Mathematics expert with experience of teaching across multiple education systems
- two mathematics experts with experience of teaching upper secondary Mathematics in the UK
- two STEM experts with experience of teaching upper secondary in the UK
- one curriculum expert with a background in learning outcomes analysis and teaching (at higher education level)

Mathematics panel 5
JHSC

Length: quarter day
Preparation time: one and a quarter days
Format: remote
Number of subjects discussed: 4
Number of panellists: 3

Composition:
- one Mathematics expert with experience of teaching across multiple education systems
- two mathematics experts with experience of teaching upper secondary Mathematics in the UK

All panellists were provided with the relevant extracts from the appropriate qualifications' specifications, including (where available):

- Learning outcomes and aims of the qualification
- Assessment structure
- Information about guided learning hours or curriculum time
- Assessment objectives
- Content.

The experts were also provided with a document containing:

- An introduction to the comparative analysis task
- Descriptions of the demand taxonomies
- The demands instrument (used to record findings).

Panellists conducted between one and four days of panel preparation, reviewing the appropriate curriculum documentation in detail and scoring each subject against the demand criteria provided (the template utilised for this has been included in Appendix C). Following this preparation, participants then took part in their respective panels, which were all hosted remotely on Microsoft Teams. The first mathematics panel lasted a day, others half or a quarter day, due to DP scores already being set and generally a lower number of subjects being discussed.

All judgements resulted in scores from 0-3 for each demand criterion mentioned above, with each score for each criterion being pulled into each course's demand profile. The panel approach was used to debate the findings and scores reached by each member of the panel and arrive at an evidence-based consensus on every demand score for every subject.[13]

Visually, each demand profile is represented by radar diagrams to facilitate demand comparison between subjects.

NB: all demand scores produced should be interpreted as approximate judgements given the varying degrees of documentation and detail available for each curriculum, as well as likely variation on how the curricula are implemented in practice.

[13] Note: each score was debated by the panel until a unanimous agreement was reached.

4. Subject-Level Alignment

This section focuses on answering RQ1 and the sub-questions associated to it, namely:

Table 7: Research questions

RQ1: To what degree do the DP mathematics curricula align with the upper secondary mathematics curricula of the eleven comparison programmes? In what way are the curricula similar and in what way are they different? To what degree do the subjects align with regard to: 1.1: Content • Topics (i.e. scope of content area, breadth, depth) • Learning activities (i.e. difficulty, demand). 1.2: Expected learning outcomes • Knowledge • Competencies (i.e. subject-specific, 21st century competencies).

Each sub-section below includes a brief overview of the respective comparison programme/standards and the comparison subjects. This is followed by an overview of the findings from the comparative analysis between the DP subjects and the comparison points regarding learning outcomes, content, and demand.

4.1 DP Mathematics Overview

The Diploma Programme (DP) was established in 1968 as a two-year pre-university programme for 16–19-year-old students.[14]

Students who aim to achieve the Diploma award must generally select one subject from each of the six subject groups:

- Studies in language and literature
- Language acquisition
- Individuals and societies
- Sciences
- Mathematics
- The arts.[15]

Students who do not wish to take a subject from the arts subject group may opt to study an additional sciences, individuals and societies, or languages course instead.

All subjects are studied concurrently over the two-year duration of the programme and most subjects can be taken at either SL or HL. In terms of teaching hours, the DP's documentation recommends 150 teaching hours for individual subjects at SL and 240 teaching hours are at HL.[16] In addition to the six subjects taken from these groups, DP students also need to complete three further curriculum components. Together, these three components comprise the DP 'core'. Theory of knowledge (TOK) allows students to reflect on the nature of

[14] International Baccalaureate. (2015). *Diploma Programme. From principles into practice*. p. 5.
[15] International Baccalaureate. (2025). *DP curriculum*. Available from: DP curriculum - International Baccalaureate®
[16] Ibid.

knowledge by considering their subjects from a broader perspective.[17] The extended essay is a self-directed piece of research which results in a 4000-word essay.[18] Creativity, activity, service (CAS) is not formally assessed but requires that students undertake a creative endeavour, take part in something physically active, and participate in a voluntary or unpaid activity.[19]

To achieve the IB Diploma a student must take at least three HL subjects.[20] The maximum number of subjects that can be taken at higher level is four. HL subjects are intended to prepare learners for the discipline specialisation of higher education, whilst the SL subjects balance this by broadening the range of subjects studied.[21]

DP students must choose one mathematics subject. The DP mathematics subject group offers the below subjects.

Mathematics: analysis and approaches[22]
Mathematics: analysis and approaches (AA) is a subject option from the mathematics group in the DP curriculum and is offered at both SL and HL. This subject is intended for students who are interested in both real and abstract applications of mathematical concepts and enjoy problem solving and generalisation. SL is suitable for students who want to study a good level of mathematics, but not at an advanced level. Therefore, SL prepares students for further study in areas involving mathematical elements, such as geography. HL is suitable for students who want an in-depth study of mathematics and enjoy solving challenging problems. Therefore, HL prepares students for further study in mathematics, as well as other areas with a strong mathematical focus, such as physics and engineering.

Mathematics: applications and interpretation[23]
Mathematics: applications and interpretation (AI) is a subject option from the mathematics group in the DP curriculum and is offered at both SL and HL. This subject is intended for students who are interested in exploring more practical applications of mathematics and would enjoy using mathematical models and technology. SL is most suitable for those who want to obtain a good level of knowledge of mathematics, with a focus on real-world applications. Therefore, SL prepares students for further study in areas with some practical mathematics elements, such as biology and business. HL is suitable for students wishing to gain more in-depth knowledge of mathematics, with a focus on real-world situations and the applications of mathematics.

[17] International Baccalaureate. (2025). *DP core*. Available from: [DP core - International Baccalaureate®]
[18] Ibid.
[19] Ibid.
[20] International Baccalaureate. (2025). *DP curriculum*.
[21] International Baccalaureate. (2015). *Diploma Programme. From principles into practice*. p.6.
[22] International Baccalaureate. (2019). *Mathematics: analysis and approaches guide.*
[23] International Baccalaureate. (2019). *Mathematics: applications and interpretation guide.*

The visuals below show the structure of each DP mathematics subject. Students taking a HL subject study both the standard level (SL) and additional higher level (AHL) content. Students studying a SL subject only study SL content.

Figure 2: DP mathematics: analysis and approaches content visualiser

	Standard level (SL) topics	Additional higher level (AHL) topics
Topic 1 Number and algebra	1.1 Standard form; 1.2 Arithmetic sequences and series; 1.3 Geometric sequences and series; 1.4 Financial applications and geometric sequences and series; 1.5 Integer exponents and intro to logarithms; 1.6 Simple proof; 1.7 Rational exponents and laws of logs; 1.8 Sum of infinite convergent geometric sequences; 1.9 Binomial theorem (natural number)	1.10 Counting principles and extended binomial theorem; 1.11 Partial fractions; 1.12 Complex numbers intro; 1.13 Polar and Euler form; 1.14 Complex roots, De Moivre's theorem and powers/roots of complex numbers; 1.15 Proof by counter example, contradiction, and induction; 1.16 Solutions of systems of linear equations
Topic 2 Functions	2.1 Gradients and equations of straight lines; 2.2 Intro to functions; 2.3 Graphing functions; 2.4 Key features of graphs; 2.5 Composite, identity, and inverse functions; 2.6 Quadratic functions; 2.7 Solving quadratic equations and inequalities & the discriminant; 2.8 Reciprocal and rational functions; 2.9 Exponential and logarithmic functions; 2.10 Graphical and analytical solutions; 2.11 Transformations	2.12 Polynomial functions; 2.13 Harder rational functions; 2.14 Odd, even, and inverse functions; 2.15 Graphical and analytical solutions of inequalities; 2.16 Further graphs, including modulus and solutions
Topic 3 Geometry and trigonometry	3.1 Geometry recap; 3.2 Trigonometry recap; 3.3 Applications and diagrams; 3.4 Circles and radians; 3.5 Definitions, exact values, and sine rule for ambiguous case; 3.6 Identities and relationships; 3.7 Functions and transformations of sin, cos, and tan; 3.8 Solving trigonometric equations graphically and analytically	3.9 Reciprocal trigonometric ratios, identities, and inverse functions; 3.10 Compound angle identities and double angle for tan; 3.11 Symmetry properties; 3.12 Intro to vectors; 3.13 Scalar product and application; 3.14 Vector equation of a line and application; 3.15 Coincident, parallel, skew, and intersecting lines; 3.16 Cross product of vectors; 3.17 Planes; 3.18 Intersections and angles (planes)
Topic 4 Statistics and probability	4.1 Sampling; 4.2 Presenting data (tables, histograms, cumulative freq.); 4.3 Measures of central tendency and dispersion; 4.4 Correlation and regression line; 4.5 Intro to probability; 4.6 Diagrams, conditional probability, combined or independent events; 4.7 Discrete random variables; 4.8 Binomial distribution; 4.9 Normal distribution; 4.10 Equation of regression line of x on y; 4.11 Formulae for conditional probabilities and independent events; 4.12 Standardisation of normal variables (z-values)	4.13 Bayes' theorem; 4.14 Continuous random variables
Topic 5 Calculus	5.1 Intro to limits and derivatives; 5.2 Increasing and decreasing functions; 5.3 Derivative of $f(x)=ax^n$. 5.4 Tangents and normal; 5.5 Definite integrals; 5.6 More derivatives and use of product, chain, and quotient rules; 5.7 The second derivative; 5.8 Maximum, minimum and inflection points, and optimization; 5.9 Kinematic problems; 5.10 Indefinite integrals and integration by inspection and substitution; 5.11 Definite integrals and area of a curve	5.12 Continuity, differentiability, limits, and higher derivatives; 5.13 Evaluation of limits and L'hopitals rule; 5.14 Implicit differentiation; 5.15 Further derivatives and indefinite integrals; 5.16 Integration by substitution and by parts; 5.17 Volumes of revolution; 5.18 First order differential equations; 5.19 Maclaurin Series
The toolkit and mathematical exploration	The exploration is a piece of written work that involves investigating an area of mathematics.	

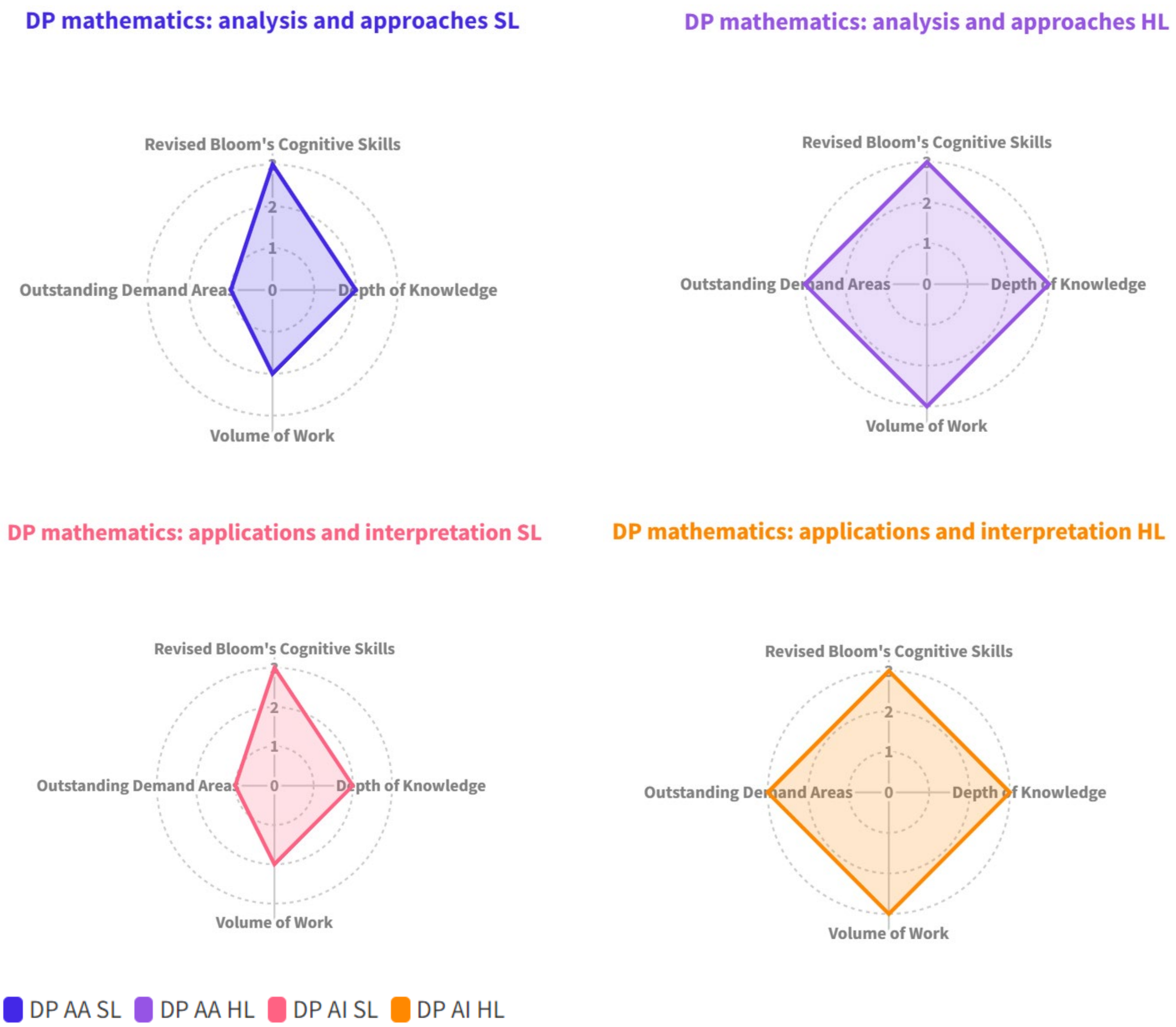
Figure 3: DP mathematics: applications and interpretation content visualiser

	Standard level (SL) topics	Additional higher level (AHL) topics
Topic 1 Number and algebra	1.1 Standard form; 1.2 Arithmetic sequences and series; 1.3 Geometric sequences and series; 1.4 Financial applications of geometric sequences and series; 1.5 Integer exponents and intro to logarithms; 1.6 Approximation, estimation, bounds and errors; 1.7 Amortization and annuities using technology; 1.8 Using technology to solve systems of equations and polynomials	1.9 Laws of logarithms; 1.10 Rational exponents; 1.11 The sum of infinite geometric sequences; 1.12 Complex numbers; 1.13 Euler and Polar form; 1.14 Matrices; 1.15 Eigenvalues and eigenvectors
Topic 2 Functions	2.1 Gradients and equations of straight lines; 2.2 Intro to functions; 2.3 Graphing functions; 2.4 Key features of graphs; 2.5 Modelling with functions; 2.6 Modelling skills	2.7 Composite and inverse functions; 2.8 Transformations; 2.9 Modelling further functions; 2.10 Using logarithms to scale numbers and linearize data
Topic 3 Geometry and trigonometry	3.1 Geometry recap; 3.2 Trigonometry recap; 3.3 Applications and diagrams; 3.4 Circles, sectors, and arcs; 3.5 Equations of perpendicular bisectors; 3.6 Voronoi diagrams	3.7 Radians; 3.8 Sin, Cos, Tan definitions, and Pythagorean identity; 3.9 Matrix transformations; 3.10 Vectors introduction and notation; 3.11 Vector equation of a line; 3.12 Vector application to kinematics; 3.13 Scalar and cross product; 3.14 Graph theory, simple and directed graphs, and subgraphs; 3.15 Adjacency matrices and weighted adjacency tables; 3.16 Decision math
Topic 4 Statistics and probability	4.1 Sampling; 4.2 Presenting data (tables, histograms, cumulative freq.); 4.3 Measures of central tendency and dispersion; 4.4 Correlation and regression line; 4.5 Intro to probability; 4.6 Diagrams, conditional probability, combined or independent events; 4.7 Discrete random variables; 4.8 Binomial distribution; 4.9 Normal distribution; 4.10 Spearman's rank; 4.11 Hypothesis testing, Chi-Squared and T-tests	4.12 Collecting and organising data and testing for reliability and validity; 4.13 Regression, residuals, coefficient of determination; 4.14 Linear transformations, linear combinations, unbiased estimations; 4.15 Central Limit theorem; 4.16 Confidence intervals; 4.17 Poisson distribution; 4.18 Further hypothesis testing; 4.19 Transition matrices and Markov chains
Topic 5 Calculus	5.1 Intro to limits and derivatives; 5.2 Increasing and decreasing functions; 5.3 Derivative of $f(x)=ax^n$; 5.4 Tangents and normal; 5.5 Definite integrals; 5.6 Maximum and minimum points; 5.7 Optimisation; 5.8 Area using trapezoidal rule	5.9 More derivatives and the chain, product, and quotient rule; 5.10 Second derivatives; 5.11 Finding further integrals and integration by inspection and substitution; 5.12 Area of a region and volumes of revolution; 5.13 Kinematic problems; 5.14 Differential equations; 5.15 Slope fields and their diagrams; 5.16 Euler's method and numerical solutions to differential equations and coupled systems; 5.17 Phase portraits; 5.18 Simple second order differential equations
The toolkit and mathematical exploration	The exploration is a piece of written work that involves investigating an area of mathematics.	

The analysis compares every mathematics subject in the comparison curriculum to each DP mathematics subject, at both SL and HL, i.e. the comparison subject has been assessed for alignment with AA SL, AI SL, AA HL, and AI HL. For some points of comparison, the analysis does not need to distinguish between DP mathematics subjects and/or level. Indeed, DP learning outcomes are the same for all mathematics courses, therefore alignment is between the comparison subject/s and DP mathematics learning outcomes in general. Furthermore, as shown below, at the respective levels, AA and AI score the same for all demand categories, hence comparison focuses on alignment with SL and HL.

In order to assess demand alignment between DP mathematics and the comparison subjects, demand profiles were created for AA SL, AA HL, AI SL, and AI HL. Each of these were scored (0-3) in four demand categories. Below, the profiles are presented as radar diagrams. These demand profiles are used for comparison in the demand section of each country.

Figure 4: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  The DP mathematics learning outcomes apply to all subjects; hence the scores are the same for AA (SL and HL) and AI (SL and HL). These outcomes were given a score of 3 on the basis that they strongly evidenced the development of critical and creative thinking skills through their focus on reasoning, inquiry-based approaches, reflection, generalisation, unfamiliar contexts, and consideration of wider implications.

- Regarding the scores for **Depth of Knowledge**:
  Both AA and AI at SL were deemed to merit a score of 2 for depth of knowledge. Both subjects were judged to cover the topics of 'Number and algebra', 'Functions', 'Geometry and trigonometry', 'Statistics and probability', and 'Calculus' in considerable detail, building in complexity and requiring a substantial amount of pre-requisite knowledge. At HL, both DP mathematics subjects were awarded a score of 3 for depth of knowledge. The subjects were judged to cover topics in a high level of detail, with many sub-topics having high complexity and requiring a large amount of pre-requisite knowledge.

- Regarding the scores for **Volume of Work**:
  The DP SL mathematics subjects were each deemed to comprise of a moderate-heavy volume of work and were given a score of 2. The panel concluded that the teaching time allotted to cover the different concepts was short (150 hours) but acknowledged that some sub-topics contained basic concepts and recapped prior learning, hence 2 was deemed an appropriate score. For HL, both DP mathematics subjects were considered to have a heavy volume of work, due to the short amount of time allocated (240 hours) and the level of complexity of the content, which combined merited a score of 3.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  Both DP mathematics subjects at SL scored 1 in this category for containing one outstanding area – the mathematical exploration. This element of the SL subjects was considered to apply skills typically needed in higher education, such as extended writing and presentation of mathematical concepts, student-led exploration, and academic writing skills. In addition to the mathematical exploration, both subjects at HL had further areas of outstanding demand. For AA HL, some of the identified areas were proof by induction, complex numbers (De Moivre's theorem), vectors (cross product, equations of planes and intersections), continuous random variables (probability density functions), and Maclaurin series. For AI HL, some identified areas were eigenvalues and eigenvectors, nonlinear regression, Markov chains, second order differential equations, slope fields, Euler's method, and phase portraits. Overall, there was a high number of outstanding areas of demand and a score of 3 was awarded to both HL subjects.

4.2 Australia (Victoria)

The Victorian Certificate of Education (VCE) is awarded to students who successfully complete the last two years (11 and 12) of secondary education in the state of Victoria, Australia. This is typically achieved by successfully completing 16-24 units of study across a number of subjects. Each subject contains four units that last for one semester each and have a minimum scheduled classroom instruction time of 50 hours.[24] For most students, the VCE is completed over a two-year period. Students typically study Units 1 and 2 of each subject in their first year, and Units 3 and 4 in their second year.[25]

The VCE is flexible, meaning that students can take longer than two years to complete studies if needed. Alternatively, the VCE course can also be completed in a shorter or earlier timeframe – some students may opt to start studying the VCE in year 10, while some may study Units 3 and 4 in year 11.[26] Additionally, those who are identified as outstanding students may opt to extend their studies by undertaking a university subject within the VCE.[27]

Students have the option to study Unit 1 or Unit 2 of a subject as stand-alone units. However, students must enrol in Units 3 and 4 of the same subjects as a sequence. This sequence needs to be completed in the same year for the study score to be calculated.[28]

To be awarded the VCE, candidates must successfully complete 16 units, including:

- three units from the English group, two of which must be a Unit 3 and 4 sequence.
- at least three additional Unit 3 and 4 sequences.

As illustrated by the above, students are not required to study any mathematics as part of the VCE. For students who wish to study mathematics, the VCE offers five subjects as described below.

Foundation Mathematics Units 1 and 2[29]
Foundation Mathematics (FM) only comprises two units, Units 1 and 2, and is studied in the first year. FM is aimed at students who do not intend to study mathematics in the second year to obtain a Unit 3 and 4 sequence. However, if supplementary learning is undertaken, students may be able to progress on to Further Mathematics Units 3 and 4.

General Mathematics Units 1 and 2[30]
General Mathematics (GM) only comprises two units, Units 1 and 2, and is studied in the first year. GM offers a range of optional topics which can be chosen to suit students' interests and provide preparation for a range of Unit 3 and 4 sequences. Depending on the topics chosen, GM may complement and support learning in Mathematical Methods and Specialist

[24] Victorian Curriculum and Assessment Authority. (n.d.). *How VCE Works – The Facts.* Available from: https://www.vcaa.vic.edu.au/studentguides/myvce/Pages/HowVCEWorks.aspx
[25] Ibid.
[26] Ibid.
[27] Ibid. No definition of what constitutes an 'outstanding student' is provided.
[28] Ibid.
[29] Victorian Curriculum and Assessment Authority. (2015). *Victorian Certificate of Education. Mathematics Study Design.* Available from: VCE Mathematics Study Design (vcaa.vic.edu.au)
[30] Ibid.

Mathematics, however, if GM units are taken alone then students can only progress onto Further Mathematics Units 3 and 4.

Further Mathematics Units 3 and 4[31]
Further Mathematics (FRM) is only comprised of two units, Units 3 and 4, to be taken in the second year. These units assume knowledge from GM (though can also be accessed with Mathematical Methods) and are designed to provide general preparation for employment or further study, particularly where data analysis, recursion and number patterns are important.

Mathematical Methods Units 1, 2, 3, and 4[32]
Mathematical Methods (MM) is comprised of four units, Units 1 and 2 – to be studied in the first year, and Units 3 and 4 – to be studied in the second year. MM Units 1 and 2 are designed as preparation for Units 3 and 4 and altogether prepare students for further studies in areas such as science, humanities, economics, and medicine. Taking MM also enables students to study Specialist Mathematics.

Specialist Mathematics Units 1, 2, 3, and 4[33]
Specialist Mathematics (SM) is comprised of four units, Units 1 and 2 – available to be taken in the first year, and Units 3 and 4 – to be studied in the second year. SM Units 3 and 4 should be taken in conjunction with MM Units 3 and 4. The SM areas of study extend concepts learnt in MM and introduce additional advanced topics. Taking Units 3 and 4 also assumes knowledge of MM Units 1 and 2 and the prescribed topics of SM Units 1 and 2. Taking these units prepares students for further study in areas with an advanced mathematical component, such as mathematics or physics.

4.2.1 Learning Outcomes

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for AA and AI. The VCE's mathematics learning outcomes are represented by three broad aims, as well as unit-specific outcomes in each area of study of a course. The unit-specific outcomes include details of the key knowledge and key skills students should gain in that unit, which are necessary for the achievement of each outcome 1, 2, and 3. Though there is some repetition in the outcomes and key skills between units, often they vary and sometimes they are unique to the unit. Therefore, overarching aims and unit-specific outcomes were both reviewed to determine the presence of DP themes in the VCE.

The following table demonstrates the learning outcome themes that were extracted from the DP mathematics curricula and indicates if and where they were judged to have presence within the learning outcomes of the VCE mathematics curricula.

[31] Ibid.
[32] Ibid.
[33] Ibid.

Table 8: Presence of the DP mathematics subject group learning outcome themes in the VCE mathematics curriculum

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the VCE	
1. Being aware of, and engaging with, mathematics in its wider context		Mostly not present in the aims or outcomes
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Not present in the aims or outcomes
3. Using inquiry-based approaches		Present in the aims and outcomes which describe analysing and investigation. However, in unit-specific outcomes, FM demonstrates this theme the least
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present in the aims which state that the study will develop mathematical concepts, knowledge, and skills, as well as the ability to apply mathematics in a range of contexts
5. Making links and generalisations		Weakly present. Some evidence of linking to everyday life in FM and some evidence of generalisation in SM
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Present in the aims describing contexts requiring modelling, problem-solving, analysing, and investigation. Evidence of reasoning is only weakly present in some units (MM and SM)
7. Communicating mathematics clearly and in various forms		Not present in the aims but present in some unit-specific outcomes
8. Knowing how technology and mathematics influence each other and using technology to develop ideas and solve problems		Present in the aims and unit outcomes, technology to be used as a tool for working mathematically

Key:

	This theme is well-evidenced in the learning outcomes of the VCE.		*This theme is partially evidenced in the learning outcomes of the VCE.*		*This theme is not evident in the learning outcomes of the VCE.*

<u>*Presence of the DP's Learning Outcome Themes*</u>

The three overall aims of the Mathematics Study Design are short in detail but do offer some insight into the key outcomes for VCE mathematics. Similarly to the DP, the aims state that VCE mathematics will develop mathematical concepts, knowledge, and skills, as well as the ability to apply mathematics in a range of contexts. Furthermore, both critical thinking and inquiry approach themes are present, as the aims describe contexts requiring modelling, problem-solving, analysing, and investigation. As the third aim, technology is similarly a theme in VCE mathematics outcomes and is to be used as a tool for working mathematically. No other DP themes are present in the overarching aims (communication, learning skills, wider context, or making links), thus they do not present the same holistic development as the DP.

There are no significant themes present in the VCE aims which are not present in the DP. However, it can perhaps be noted that specific critical-thinking skills/processes, such as performing analysis and modelling, are referred to more frequently and have more of a presence in the outcomes than in the DP. This is true for all VCE subjects, except Foundation Mathematics which contained minimal evidence of modelling and analysis.

The outcomes in the units below provide more detail about the skills to be developed and will also be reviewed for their presence of DP themes.

Foundation Mathematics Units 1 and 2

There is some presence of the DP's learning outcome themes in FM, though few are strongly evidenced, and others are absent. Similarly to the DP, FM outcomes describe the need for students to be able to successfully communicate maths in various forms by requiring students to produce tables, graphs and diagrams (with and without technology), and also to be able to interpret, represent and discuss mathematics in different contexts. Furthermore, FM outcomes have a similar focus on technology, as students are expected to be able to perform calculations, produce representations, perform analysis, and solve problems using technological tools. However, at this level, the outcomes do not include the use of technology to develop mathematical ideas, hence this investigative aspect that the DP involves is not included. Indeed, taking an inquiry-based approach is generally not a present theme in these units, as no outcomes or skills require students to conduct investigations, make conjectures, test conclusions or to explore abstract scenarios. Similarly to the DP, FM outcomes include the development of critical thinking skills and processes through requiring students to perform some analysis and solve problems. However, use of reasoning, including inferring and constructing arguments, is not specified clearly in FM outcomes and skills, thus this area does not appear to have the same focus as the DP. Furthermore, there is only limited evidence of links to other disciplines (Pythagoras and art) and no evidence with regard to making generalisations or engaging with mathematics in its wider context (beyond links to everyday life and some real-life applications). Finally, there is no development of learning skills such as the ability to work both collaboratively and independently, reflect, or to develop a positive and curious attitude towards mathematics studies.

General Mathematics Units 1 and 2

There is some presence of the DP's learning outcome themes in GM, though not all are strongly evidenced, and others are not present at all. Similar to the DP, GM expects students to develop critical thinking skills through problem-solving and modelling in a variety of contexts, although at this level these contexts are described as practical rather than 'non-routine' or 'abstract'. Furthermore, inquiry-based approaches are somewhat present as the skills detail that students should be able to conduct investigations in a variety of contexts and use technology to develop mathematical ideas. Additionally, technology is also a key theme in GM, as students must use it in a variety of ways similar to the DP, including to produce results, problem solve, and understand how mathematics and technology differ. Further themes/skills are present in GM but are only referred to within the context of some specific content. For example, the use of reasoning is only explicitly mentioned with regard to describing a problem-solving process and skills involving communication refer mostly to modelling and statistical representations. Furthermore, making links to other disciplines is restricted to technology. Hence, unlike the DP, these skills/themes are not clearly intended to be developed throughout all the topics within the unit. Finally, in contrast to the DP, GM does not include outcomes or skills which are aimed at student awareness and engagement with maths in its wider context or the development of learning skills such as reflection, collaboration, independence, and positive disposition.

Further Mathematics Units 3 and 4

Some of the DP's themes are present in the outcomes and skills of FRM. Similarly to the DP, FRM includes outcomes and skills directed at the development of critical thinking skills, such as being able to solve extended problems, analyse information, and use modelling. Furthermore, the outcomes expect students to be able to communicate in various forms such as through producing representations (with and without technology), interpretation of information, and reports of investigations. Also similar is their technology-focused outcomes and skills which describe that students will be able to use technology in a variety of ways, specifically to develop ideas and carry out analysis for problem-solving and investigations. The outcomes for these units also require students to be able to investigate a range of problems and scenarios, with and without technology, thus this is comparable to the DP's theme of expecting students to be able to take an inquiry-based approach. However, despite these similarities, there are DP themes which are not present in the units. FRM outcomes do not include being aware of, or engaging with, maths in its wider context, such as exploring different global and local issues or perspectives or critically questioning the implications of mathematics. Also, the units do not focus on developing learning skills such as the ability to collaborate, work independently, reflect, or to build curiosity and confidence. Furthermore, reasoning is not a general skill to be developed over the unit but is only mentioned in the context of describing the process taken to problem-solve in technology. Finally, a theme not present is the development of students' powers of abstraction and generalisation and the ability to make links to other disciplines beyond technology.

Mathematical Methods Units 1, 2, 3 and 4

The DP's learning outcome themes are somewhat present in the outcomes of MM. Looking at the outcomes and key skills, it is evident that students are similarly expected to develop critical thinking skills and processes through problem-solving, modelling, analysing, inferring, and reasoning. At this level, it is expected that the students work with familiar, unfamiliar, and non-routine contexts. Furthermore, students need to be able to use technology to calculate, produce graphs/tables, analyse, problem-solve and develop ideas, as well as understand the similarities and differences of technology and mathematics. Communication of mathematics is also a present theme in these units, as key skills describe the production of tables, graphs, diagrams, and also the communication of reasoning and conclusions using words and mathematical expressions. There is also evidence of students being expected to have the skills to work using an investigative approach, as the outcomes and key skills describe students being able to use mathematical processes in scenarios requiring investigative techniques and approaches. However, some of DP's themes are not present or are only somewhat present. Not present is the DP's theme of students being aware of, and engaging with, mathematics in its wider context, as these units do not include any key skills or outcomes related to questioning the implications of mathematics or looking at global and historical perspectives. Furthermore, there is no focus on students making links to other disciplines or everyday life and local contexts. Another theme not present is the development of learning skills, such as the ability to work collaboratively, reflect on work, be resilient, or to have curiosity and enthusiasm. Finally, there is only limited evidence of skills aimed at the development of abstraction and generalisation skills, as these are only present where the content requires them – rather than being skills to be continuously developed in all topics.

Specialist Mathematics Units 1, 2, 3 and 4

The DP's learning outcome themes are somewhat present in SM. A theme which is strongly evidenced is that of using critical thinking skills through problem-solving, use of reasoning, inferring conclusions, and applying analysis when problem solving, taking investigation approaches or modelling. There is also evidence of students being expected to have the skills to work using an inquiry approach, as the outcomes and key skills describe students being able to use mathematical processes in scenarios requiring investigative techniques and approaches. In addition, the key skills clearly demonstrate that students are required to be able to communicate by producing representations (tables, graphs, diagrams), conclusions using mathematical and everyday language, and through using technology. Furthermore, as can be seen in the other outcomes, technology is a key area of focus, indeed it is one of the three main outcomes described. In SM, students are expected to use technology to perform calculations, produce graphics, develop mathematical ideas, perform analysis, and understand how mathematics and technology relate to one another and the differences that are produced. Finally, from the learning outcomes and skills there is some evidence that students are expected to use abstraction and generalisation, although these are mostly linked to content where these skills are necessary, rather than being skills which are to be used throughout the whole unit. There is also some limited evidence of students making links through drawing upon various areas of knowledge for an investigation, however there is no expectation that students make links to other disciplines. In contrast to the DP, SM contains no outcomes or skills which are aimed at developing students' awareness and engagement with mathematics in a wider context or developing transferable learning skills such as resilience, collaboration, independence, and reflection.

Summary

The VCE's learning outcomes are structured differently to the DP, as they include specific outcomes for each mathematics course, whereas the DP has the same set of outcomes for all its mathematics subjects. There are some overlaps in the learning outcomes of the DP and VCE mathematics subjects. The strongest similarities found are with the themes of understanding mathematical concepts, applying critical thinking skills in analysis and problem-solving, using investigative skills, and the emphasis on the use of technology. However, not all extracted DP themes are present, as VCE mathematics outcomes do not focus on wider contexts of mathematics or transferable learning skills. Furthermore, DP themes which are present in the VCE are often not emphasised or described in similar ways to the DP. Generally lacking in emphasis is reasoning, using generalisation, making links, and communicating in various forms. Though references to these skills are made, they are often very specific in nature, rather than being presented as general expectations to be continually developed. Overall, SM is the most aligned with DP outcomes as it includes more focus on reasoning and making generalisations, closely followed by MM. In contrast, FM learning outcomes are the least aligned to the DP.

4.2.2 Content

This section compares and contrasts the content of the DP and VCE curricula falling within the category of mathematics. To support visual comparison at-a-glance, the VCE mathematics curriculum is presented below in a diagram which shows the key topics and sub-topics included in each subject.

Figure 5: VCE Units 1 and 2 mathematics content visualiser

Foundation Mathematics Units 1 and 2	**Area of Study 1: Space, shape and design**	Overview: In this area of study students cover the geometric properties of lines and curves, and shapes and objects, and their graphical and diagrammatic representations with attention to scale and drawing conventions used in domestic, societal, industrial and commercial plans, maps and diagrams.		
	Area of Study 2: Patterns and number	Overview: In this area of study students cover estimation, the use and application of different forms of numbers and calculations, and the representation of patterns and generalisations in number including formulas and other algebraic expressions in everyday contexts.		
	Area of Study 3: Data	Overview: In this area of study students cover collection, presentation and analysis of gathered and provided data from community, work, recreation and media contexts, including consideration of suitable forms of representation and summaries.		
	Area of Study 4: Measurement	Overview: In this area of study students cover the use and application of the metric system and related measurement in a variety of domestic, societal, industrial and commercial contexts, including consideration of accuracy.		
General Mathematics Units 1 and 2	**Area of Study 1: Algebra and structure**	Linear relations and equations		
	Area of Study 2: Arithmetic and number	Computation and practical arithmetic	Financial arithmetic	
	Area of Study 3: Discrete mathematics	Matrices	Graphs and networks	Number patterns and recursion
	Area of Study 4: Geometry, measurement, and trigonometry	Shape and measurement	Applications of trigonometry	
	Area of Study 5: Graphs of linear and non-linear relations	Linear graphs and models	Inequalities and linear programming	Variation
	Area of Study 6: Statistics	Investigating and comparing data distributions	Investigating relationships between two numerical variables	
Mathematical Methods Unit 1	**Area of Study 1: Functions and graphs**	In this area of study students cover the graphical representation of simple algebraic functions (polynomial and power functions) of a single real variable and the key features of functions and their graphs such as axis intercepts, domain (including the concept of maximal, natural or implied domain), co-domain and range, stationary points, asymptotic behaviour and symmetry. The behaviour of functions and their graphs is explored in a variety of modelling contexts and theoretical investigations.		
	Area of Study 2: Algebra	This area of study supports students' work in the 'Functions and graphs', 'Calculus' and 'Probability and statistics' areas of study, and content is to be distributed between Units 1 and 2. In Unit 1 the focus is on the algebra of polynomial functions of low degree and transformations of the plane.		

	Area of Study 3: Calculus	In this area of study students cover constant and average rates of change and an introduction to instantaneous rate of change of a function in familiar contexts, including graphical and numerical approaches to estimating and approximating these rates of change.	
	Area of Study 4: Probability and statistics	In this area of study students cover the concepts of event, frequency, probability and representation of finite sample spaces and events using various forms such as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams. This includes consideration of impossible, certain, complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable), including rules for computation of probabilities for compound events.	
Mathematical Methods Unit 2	**Area of Study 1: Functions and graphs**	In this area of study students cover graphical representation of functions of a single real variable and the key features of graphs of functions such as axis intercepts, domain (including maximal, natural or implied domain), co-domain and range, asymptotic behaviour, periodicity and symmetry.	
	Area of Study 2: Algebra	This area of study supports students' work in the 'Functions and graphs', 'Calculus' and 'Probability and statistics' areas of study. In Unit 2 the focus is on the algebra of some simple transcendental functions and transformations of the plane. This area of study provides an opportunity for the revision, further development and application of content prescribed in Unit 1, as well as the study of additional algebra material introduced in the other areas of study in Unit 2	
	Area of Study 3: Calculus	In this area of study students cover first principles approach to differentiation, differentiation and anti-differentiation of polynomial functions and power functions by rule, and related applications including the analysis of graphs.	
	Area of Study 4: Probability and statistics	In this area of study students cover introductory counting principles and techniques and their application to probability and the law of total probability in the case of two events.	
Specialist Mathematics Units 1 and 2	**Area of Study 1: Algebra and structure**	Logic and algebra	Transformations, trigonometry, and matrices
	Area of Study 2: Arithmetic and number	Number systems and recursion	Principles of counting
	Area of Study 3: Discrete mathematics	Graph theory	
	Area of Study 4: Geometry, measurement, and trigonometry	Geometry in the plane and proof	Vectors in the plane
	Area of Study 5: Graphs of linear and non-linear relations	Graphs of non-linear relations	Kinematics
	Area of Study 6: Statistics	Simulation, sampling and sampling distributions	

Figure 6: VCE Units 3 and 4 mathematics content visualiser

Further Mathematics Units 3 and 4	**Area of Study 1: Unit 3 – Core**	Data analysis	Recursion and financial modelling		
	Areas of Study 2: Unit 4 – Applications	Matrices	Networks and decision mathematics	Geometry and measurement	Graphs and relations
Mathematical Methods Units 3 and 4	**Area of Study 1: Functions and graphs**	In this area of study students cover transformations of the plane and the behaviour of some elementary functions of a single real variable, including key features of their graphs such as axis intercepts, stationary points, points of inflection, domain (including maximal, implied or natural domain), co-domain and range, asymptotic behaviour and symmetry. The behaviour of these functions and their graphs is to be linked to applications in practical situations.			
	Area of Study 2: Algebra	In this area of study students cover the algebra of functions, including composition of functions, simple functional relations, inverse functions and the solution of equations. They also study the identification of appropriate solution processes for solving equations, and systems of simultaneous equations, presented in various forms. Students also cover recognition of equations and systems of equations that are solvable using inverse operations or factorisation, and the use of graphical and numerical approaches for problems involving equations where exact value solutions are not required, or which are not solvable by other methods. This content is to be incorporated as applicable to the other areas of study			
	Area of Study 3: Calculus	In this area of study students cover graphical treatment of limits, continuity and differentiability of functions of a single real variable, and differentiation, anti-differentiation and integration of these functions. This material is to be linked to applications in practical situations.			
	Area of Study 4: Probability and statistics	In this area of study students cover discrete and continuous random variables, their representation using tables, probability functions (specified by rule and defining parameters as appropriate); the calculation and interpretation of central measures and measures of spread; and statistical inference for sample proportions.			
Specialist Mathematics Units 3 and 4	**Area of Study 1: Functions and graphs**	In this area of study students cover inverse circular functions, reciprocal functions, rational functions and other simple quotient functions, the absolute value function, graphical representation of these functions, and the analysis of key features of their graphs including intercepts, asymptotic behaviour and the nature and location of stationary points, points of inflection, periodicity, and symmetry			
	Area of Study 2: Algebra	In this area of study students cover the expression of simple rational functions as a sum of partial fractions; the arithmetic and algebra of complex numbers, including polar form; points and curves in the complex plane; introduction to factorisation of polynomial functions over the complex field; and an informal treatment of the fundamental theorem of algebra.			
	Area of Study 3: Calculus	In this area of study students cover advanced calculus techniques for analytic and numeric differentiation and integration of a range of functions, and combinations of functions; and their application in a variety of theoretical and practical situations, including curve sketching, evaluation of arc length, area and volume, differential equations and kinematics.			
	Area of Study 4: Vectors	In this area of study students cover the arithmetic and algebra of vectors, linear dependence and independence of a set of vectors, proof of geometric results using vectors, vector representation of curves in the plane and vector kinematics in one and two dimensions			
	Area of Study 5: Mechanics	In this area of study students cover an introduction to Newtonian mechanics, for both constant and variable acceleration			
	Area of Study 6: Statistics and probability	In this area of study students cover statistical inference related to the definition and distribution of sample means, simulations and confidence interval.			

4.2.2.1 Structure

Similarly to the DP, VCE mathematics is designed to span two years and offers different options for mathematics study. However, differently to the DP, the VCE structures its programme and content into yearly units. Like the DP, VCE mathematics offers an option for those wanting to study mathematics at a more advanced level, which is that of SM. Just as additional higher level (AHL) content cannot be studied without SL in DP mathematics subjects, SM is designed to be studied along with MM in Units 3 and 4 and expects students to have taken MM Units 1 and 2 and have relevant knowledge from SM Units 1 and 2. However, where the DP offers two HL options that each take a different focus (AA and AI), SM does not take a specific focus but combines elements of both pure and applied mathematics. Furthermore, content in the VCE units caters for a greater range of mathematical abilities by also offering courses at the lower end, such as FM. In addition to course options based on level, like the DP, VCE also offers courses which consider the relevant skills needed for future studies/careers, as SM and MM offer more opportunities to explore ‘pure’ mathematics and FRM dedicates more time to explore applications in statistics and finance. However, where AA and AI have a large amount of overlapping content, the difference between some VCE ‘pure-focused’ and ‘applied-focused’ units’ content is more pronounced – for instance, FRM and MM share very little content.

Like the DP, content within units is structured into main themes – Areas of Study – which are then broken down into smaller sections detailing what is to be studied in each area. However, where the DP’s main topics remain the same regardless of the subject and level, VCE’s main themes and content structure vary greatly between units. See below for more detail on each subject’s content structure.

Foundation Mathematics Units 1 and 2

FM content is structured into main themes of ‘Space, shape and design’, ‘Patterns and number’, ‘Data’, and ‘Measurement’. Thus, FM offers fewer main themes than the DP, which covers ‘Number and algebra’, ‘Functions’, ‘Geometry and trigonometry’, ‘Statistics and probability’, and ‘Calculus’ – five themes in total. Furthermore, the main topics named differ significantly to those in the DP, hence demonstrating that FM covers different material to the DP subjects.

General Mathematics Units 1 and 2

Differently to the DP, GM is structured into six, rather than five, main themes. These are ‘Algebra and structure’, ‘Arithmetic and number’, ‘Discrete mathematics’, ‘Geometry, measurement, and trigonometry’, ‘Graphs of linear and non-linear relations’, and ‘Statistics’. However, unlike the DP, not all these main themes are studied by students taking the course. The Areas of Study are broken down into topics, all of which are optional. Students must take at least eight (out of 13) topics, from at least three Areas of Study. Thus, this offers an element of flexibility not offered in AA and AI.

Further Mathematics Units 3 and 4

These units differ significantly in content structure to other units and the DP. Unit 3 has one Area of Study, ‘Core’, which has two large, non-optional topics: ‘Data analysis’ and ‘Recursion and financial modelling’. In Unit 4, the Area of Study is ‘Applications’, where two topics must be chosen from ‘Matrices’, ‘Networks and decision mathematics’, ‘Geometry and measurement’, and ‘Graphs and relations’. Thus, FRM has an element of flexibility not present in DP mathematics subjects. Moreover, FRM covers a much smaller range of mathematical areas than AA and AI and tends to focus on applied mathematics, hence this subject is most different to AA.

Mathematics Methods Units 1, 2, 3 and 4

In these units, content is structured into fewer main themes than the DP mathematics subjects. These include: ‘Functions and graphs’, ‘Algebra’, ‘Calculus’, and ‘Probability and statistics’. These themes are very similar to those of the DP – with the exception of ‘Geometry and trigonometry’ content being integrated into other themes rather than being stand-alone themes themselves. Overall, this demonstrates that the MM covers similar content areas to those covered in the AA and AI.

Specialist Mathematics 1, 2, 3 and 4

Units 1 and 2 are structured differently to Units 3 and 4. Areas of Study in Units 1 and 2 are ‘Arithmetic and number’, ‘Geometry, measurement and trigonometry’, ‘Graphs of linear and non-linear relations’, ‘Algebra and structure’, ‘Discrete mathematics’, and ‘Statistics’. However, unlike the DP subjects, these units contain both ‘prescribed’ topics and optional topics. Prescribed topics are ‘Number systems and recursion’, ‘Geometry in the plane and proof’, ‘Vectors in the plane’, and ‘Graphs of non-linear relations’. As well as these, at least four topics must be chosen from either GM or the following list: ‘Logic and algebra’, ‘Transformations, trigonometry and matrices’, ‘Principles of counting’, ‘Graph theory’, ‘Kinematics’, and ‘Simulation, sampling, and sampling distributions’. Thus, the structure of content in SM Units 1 and 2 allows for more flexibility than DP mathematics subjects. For Units 3 and 4, all content is non-optional, and includes ‘Functions and graphs’, ‘Algebra’, ‘Calculus’, ‘Vectors’, ‘Mechanics’, and ‘Probability and statistics’. Altogether, SM units include more main themes than AA and AI.

4.2.2.2 Content Alignment

To complement the analysis, the tables below represent a simplified summary of the VCE’s content alignment, at topic-level, with AA (SL and HL) and AI (SL and HL).

Table 9: Summary of the content alignment VCE subjects have with the main topics in AA

Mathematics: analysis and approaches topics		Presence in VCE				
		FM	GM	FRM	MM	SM
SL	1. Number and algebra					
	2. Functions					
	3. Geometry and trigonometry					
	4. Statistics and probability					
	5. Calculus					
AHL	1. Number and algebra					
	2. Functions					
	3. Geometry and trigonometry					
	4. Statistics and probability					
	5. Calculus					

Key:

	Strong presence of this topic in the VCE		*Partial presence of this topic in the VCE*		*Little or no presence of this topic in the VCE*

Table 10: Summary of the content alignment VCE subjects have with the main topics in AI

Mathematics: applications and interpretation topics		Presence in VCE				
		FM	GM	FRM	MM	SM
SL	1.Number and algebra		▒	■		■
	2. Functions				■	■
	3. Geometry and trigonometry			▒	▒	▒
	4. Statistics and probability			▒	▒	■
	5. Calculus				■	■
AHL	1. Number and algebra					■
	2. Functions				▒	■
	3. Geometry and trigonometry			▒		■
	4. Statistics and probability					▒
	5. Calculus					■

Key:

■	*Strong presence of this topic in the VCE*	▒	*Partial presence of this topic in the VCE*		*Little or no presence of this topic in the VCE*

*Where applicable, content alignments found in pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

Foundation Mathematics Units 1 and 2

All DP mathematics subjects

FM contains very minimal content alignment with the content in both AA and AI. Most of the topics and sub-topics in FM would be covered by, or be at the same level of, the prior-learning topics of the DP. There are a couple of partial alignments with introductory sub-topics in ‘Geometry and trigonometry’ and ‘Statistics and probability’ – for both AA and AI – and ‘Number and algebra’ – AI only, though these are due to overlaps with prior-learning content (such as finding measures of central tendency). Therefore, FM content does not align with the DP mathematics content, as it covers material more congruent with mathematic studies prior to upper secondary.

General Mathematics Units 1 and 2

Mathematics: analysis and approaches

GM contains small amounts of AA content in all topics apart from ‘Calculus’. The mapping of content shows that alignment is only found with a small number of the SL sub-topics AA shares with AI, hence no AA-specific content was covered, including AHL. Furthermore, some topic areas of GM, such as Area of Study 1: Algebra and structure, are more aligned with content that would be expected to be covered in learning prior to the DP. Where found, alignments tend to be partial, due to either containing less of the content in the sub-topic, or for taking a different approach. The topic GM has the most sub-topic alignments with (four) is ‘Number and algebra’, due to GM containing sequences and applications – though they somewhat differ by focusing on recursion. The topic which GM has the most strongly aligned sub-topics with is ‘Statistics and probability’, as GM includes sub-topics of presenting data, measures of central tendency and dispersion, and correlation and linear regression.

However, many sub-topics and larger topics are not covered. Some significant SL areas not included in GM are calculus, probability, laws of exponents and logarithms, quadratics and other functions, and trigonometric equations and identities. However, it can be noted that GM contains some significant sub-topics which are not included in AA content, which are found in Area of Study 3: 'Discrete mathematics' and Area of Study 5: 'Graphs of linear and non-linear relations'. 'Discrete mathematics' covers topics such as matrices (including matrix algebra and solving systems of equations) and 'Graphs and networks' (including minimum spanning trees and Prim's Algorithm). Furthermore, Area of Study 5 includes a sub-topic of inequalities and linear programming. However, it is important to reinstate that all topics are optional and therefore these topics may not be studied by some students taking GM.

In summary, GM content has weak to no alignment with SL, no alignment with AHL content and observes smaller content breadth than AA SL and AA HL.

Mathematics: applications and interpretation

GM contains some AI content, though very little in certain topics and none from 'Calculus'. The mapping of the content shows that GM has some alignments in the sub-topic areas present at the start of each topic, common to both AA and AI. However, unlike AA, further alignments beyond these are found when comparing GM content and AI content, with additional ones in the topics of 'Number and algebra', 'Functions', and 'Geometry and trigonometry'. Specifically, these alignments come from GM's inclusion of matrices, graphs and networks, and modelling. Matrices in GM are covered to the same depth as in AI, though are not extended to eigenvectors and eigenvalues. Modelling skills are covered to a similar extent to AI, though GM mostly focuses on linear models and linear regression. Likewise, GM has good alignment with AI's graphs and algorithm sub-topics, though GM only considers Prim's Algorithm and no others. The only significant sub-topic covered by GM and not AI is that of linear programming.

In summary, there is weak to no alignment with AI's SL content in most topics, and only occasional sub-topic alignment with AHL content in 'Number and algebra' and 'Geometry and trigonometry'. Overall, GM observes smaller content breadth than AI SL and AI HL.

Table 11: General Mathematics content which is not covered by DP mathematics subjects

Significant GM content not in AA (only)*	Significant GM content not in AI (only)*
○ Matrices** ○ Graphs and networks** ○ Variation** ○ Linear models (including piecewise)**	
Significant content not in either DP mathematics subject*	
○ Linear programming**	

*Significant content does not include topics which are typical to mathematical learning prior to upper secondary
** Optional content

Further Mathematics Units 3 and 4

The analysis of alignment below considers General Mathematics (GM) and Further Mathematics (FRM) together, as this is a typical combination of units.

Mathematics: analysis and approaches

FRM Units 3 and 4 content is mostly not aligned with AA, for both SL and HL. No AA AHL sub-topics are found to be present in FRM, nor any alignments from the topic of 'Calculus'. There are a few alignments with SL sub-topics within the other main topics. However, FRM largely contains content that is not focused on in AA. Unit 3 covers 'Core' topics to be taken by all studying FRM, which focuses on two key areas of 'Data analysis' and 'Recursion and financial modelling'. Within these are topics which are not covered by AA, such as a focus on modelling, performing residual analysis, data transformations, modelling with time series data, annuities and perpetuities, and balancing repayment loans. Thus, FRM content goes deeper into some areas of statistics and finance than AA. Furthermore, the optional topics in Unit 4 include other areas not covered by AA such as matrices, networks and decision mathematics, spherical geometry, and linear programming. Overall, FRM Units 3 and 4 are weakly aligned with AA SL content and have no alignment with AHL content – they generally focus on areas of applied mathematics and have far less content breadth.

Mathematics: applications and interpretation

FRM has slightly more alignments with AI content than AA, due to its similar focus on applications. However, FRM does not have enough sub-topic alignments with SL or AHL to conclude good alignment with either level. FRM has 4-7 sub-topic alignments in all main AI topics, except for 'Calculus', including occasional AHL sub-topics. AHL sub-topics alignments are matrices, graph theory, decision maths, and transition matrices – with also some elements of analysing the appropriateness of a model (linear models only). Significant alignments with SL content include financial applications, such as compound interest, amortization and annuities, and modelling skills. Though many AI topics are not included in FRM, there are a few areas not covered by AI which are in FRM. These include, data transformations, modelling time series data, spherical geometry, linear programming, and greater depth in financial applications. Overall, FRM is not closely aligned with either AI SL or AHL content but does involve some of the similar areas in statistics, finance, and decision maths. In terms of subject breadth, FRM is considerably narrower than AI SL and HL, especially considering that only two Unit 4 topics are studied.

Table 12: Further Mathematics content which is not covered by DP mathematics subjects

<table>
<tr><th>Significant FRM content not in AA (only)</th><th>Significant FRM content not in AI (only)</th></tr>
<tr><td>• Amortization and annuities
• Residual analysis and use of the coefficient of determination
• Matrices*
• Transition matrices*
• Graph theory and decision mathematics*
• Linear models* (including piecewise)</td><td></td></tr>
<tr><th colspan="2">Significant content not in either DP mathematics subject</th></tr>
<tr><td>• Modelling time series data
• Data transformation
• Recursion and further depth into financial applications</td><td>• Linear programming*
• Spherical geometry*</td></tr>
</table>

*Optional content

Mathematical Methods 1, 2, 3 and 4

Mathematics: analysis and approaches

A considerable amount of AA SL content is present in MM units, however most AHL content is not present. The mapping of content shows that MM has the most sub-topic alignments with 'Functions', with SL content being well-covered, along with a small amount of AHL. Though the content does not extend to most of AHL, MM differs to the scope of AA by also requiring modelling with the functions. SL 'Calculus' content is also strongly present in MM, with only the sub-topic of the second derivate being notably absent. Though most AHL content is not covered, MM does go into some depth by similarly including concepts of continuity and differentiability, evaluation of limits, and finding derivatives of some polynomials using a first principles approach.

Furthermore, most SL 'Geometry and trigonometry' content is present in MM, though AHL sub-topics are not covered, as MM does not extend to reciprocal trigonometric functions, include double angle identities, or (most significantly) cover any content related to vectors, which accounts for a considerable proportion of this topic. For 'Statistics and probability', MM covers some sub-topics relating to probability and discrete/continuous random variables, but a considerable number of sub-topics are not present. However, the lack of content alignment is partially a result of MM focusing on different areas such as sampling and population proportions. MM has the least alignment with 'Number and algebra', as a considerable amount of SL sub-topics are not included, nor are significant AHL areas of proof and complex numbers.

In summary, MM has strong alignment with AA SL content in 'Functions', 'Calculus', and 'Geometry and trigonometry', good alignment with 'Statistics and probability', and some alignment with 'Number and algebra'. Additionally, a few alignments with AHL content are found in all topics, though these were mostly in small sub-topics rather than large areas. Thus, content in MM units can be considered to have similar breadth and depth of AA SL, but not AA HL.

Mathematics: applications and interpretation

A considerable amount of AI SL content is present in MM units, however most significant AHL sub-topics are not included. The mapping of content shows that 'Functions' content is strongly present, as MM has alignment with most SL sub-topics and is partially aligned with AHL content, with no large areas excluded. Furthermore, 'Calculus' SL content is also strongly present and, though most AHL content is not covered, MM has some elements of depth by including concepts of continuity and differentiability, evaluation of limits, and finding derivatives using a first principles approach.

For 'Geometry and trigonometry', MM has alignment with less than half of the sub-topics (which are a mixture of SL and AHL). Significant areas not present in MM content are Voronoi diagrams and AHL content such as vectors, graph theory, and decision maths. Similarly, a considerable amount of 'Number and algebra' is not present, such as sequences, financial applications, matrices, and complex numbers. With regard to 'Statistics and probability', MM covers some sub-topics relating to probability, binomial and normal distributions, and discrete and continuous random variables, but a considerable number of sub-topics (mostly AHL and some SL) are not present.

In summary, for AI SL content, MM is strongly aligned with the topics of 'Functions' and 'Calculus', partially aligned with 'Statistics and probability' and 'Geometry and trigonometry', and not aligned with 'Number and algebra'. For AHL content, MM has some alignment with 'Functions', and a couple of sub-topic alignments with other topics. If MM is taken with GM, SL content would be largely covered in all topics, as would AHL content involving matrices and decision mathematics. However, this is not to say that MM content breadth and depth is less than AI SL, as it contained some areas not in AI (see table below). However, it can be noted that the breadth and depth of MM is considerably less than that of AI HL.

Table 13: Mathematical Methods content which is not covered by DP mathematics subjects

Significant MM content not in AA (only)	Significant MM content not in AI (only)
○ Modelling functions and skills ○ Matrix transformations ○ Definition and distribution of sample proportions, simulations, and confidence intervals	○ Binomial theorem ○ Counting principles ○ Solving inequalities ○ Analytical solutions ○ Polynomial functions, factor theorem ○ Solving trigonometric equations analytically ○ Symmetry properties ○ Continuous random variable probability density functions ○ Limits and first principles of differentiation
Significant content not in either DP mathematics subject *	
N/A	

*Significant content does not include topics which are typical to mathematical learning prior to upper secondary

Specialist Mathematics Units 1, 2, 3, and 4

For students completing SM Units 3 and 4, it is assumed that they will have at the very least, studied all MM units and the topics of 'Number systems and recursion' and 'Geometry in the plane and proof' found in SM Units 1 and 2.

Mathematics: analysis and approaches

Considering SM Units 3 and 4 with the assumed prior knowledge first, the content mapping shows that this content altogether strongly aligns with AA SL and AHL content. Nearly every sub-topic in 'Number and algebra', 'Functions', and 'Calculus' is present and most of the significant areas in the other main topics are covered either entirely or partially. Lesser alignment with 'Geometry and trigonometry' is due to differences in the AHL area of vectors, where similar content is covered, though SM then focuses on vector calculus rather than equations of planes and the vector product. There is good alignment with 'Statistics and probability' – the only significant area not covered being correlation and regression, which is instead covered in GM. As well as being strongly aligned with AA content, SM Units 3 and 4 include topics which are not covered by AA. These include a topic of 'Mechanics' and some different coverage of statistics (including hypothesis testing).

Further to this minimum assumed knowledge, it is important to note that most of the students taking SM Units 3 and 4 will have taken Units 1 and 2 in their entirety (not just the assumed knowledge). The inclusion of all SM Units 1 and 2 content does not have much impact on alignment, as most content relevant to AA is covered again and in greater depth in SM Units 3 and 4 – with the exception of sequences and counting principles. However, SM Units 1 and 2 offer further topics that are not present in AA. These include graph theory, a stand-alone topic of kinematics, more matrices, cartesian, polar and parametric forms, and graphs of lines, parabolas, circles, ellipses, and hyperbolas. However, it is important to note that some of these are optional content, thus not all may be studied.

Alternatively, in order to take SM Units 3 and 4, students can instead take GM with MM (though relevant knowledge from SM Units 1 and 2 is still assumed). In this case, the content from this combination of units brings further alignment with ‘Statistics and probability’ (for correlation and linear regression) and content beyond the scope of AA is graph theory and matrices – though these are optional topics.

In summary, SM Units 3 and 4 and the content assumed to have been learnt prior has strong alignment with AA SL and AHL content. Where SM Units 1 and 2 or GM Units 1 and 2 are also taken, there is opportunity for further topics which are not included in AA to be covered. Therefore, students taking SM are likely to cover greater content breadth than AA SL and HL. The depth of content in SM is comparable to AA HL.

Mathematics: applications and interpretation

The mapping of content shows that SM Units 3 and 4 – along with MM units and the assumed knowledge from SM Units 1 and 2 – is well-aligned with AI SL and AHL content for most topics and strongly aligned in two topics. SM is most strongly aligned with the topics of ‘Functions’ and ‘Calculus’, with nearly all SL and AHL sub-topics being present, though it can be noted that phase portraits are not covered.

SM is well-aligned with ‘Number and algebra’, however two significant areas are not covered in SM, which are eigenvalues and eigenvectors and sub-topics related to finance, such as interest and annuities. Furthermore, there is good alignment with ‘Geometry and trigonometry’ as AHL sub-topics such as vectors, radians, and trigonometric identities are included. SM would be further aligned with AHL if students take the topic of ‘Graph Theory’ in SM Units 1 and 2, which would satisfy two further AHL sub-topics. Though, regardless of this, the sub-topics of Voronoi diagrams and adjacency matrices would not be covered. The topic that SM has the least alignment with is ‘Statistics and probability’, as SM only covers hypothesis testing for a sample mean and does not include several significant areas such as chi-squared tests, linear and non-linear regression, the Poisson distribution, or transition matrices.

In the case where students take GM with, or instead of SM Units 1 and 2, this could bring more alignments with certain sub-topics, such as with financial applications, handling data and linear regression, graph theory, and decision maths.

Though SM units lack some areas of alignments with AI, especially in AHL ‘Statistics and probability’ content, they cover numerous other topics which are beyond the scope of AI. Some of these are in AA, such as proof, reciprocal trigonometric ratios and further identities, the absolute value function, probability density functions, and implicit differentiation (see table below for list). Further to these are also ‘Mechanics’ in SM Units 3 and 4, and (if taken) some of the additional topics in SM Units 1 and 2 such as counting principles, algebra and logic, kinematics as a stand-alone topic, and cartesian, polar and parametric forms and graphs.

In summary, SM has strong alignment with content with AI SL and AHL content. Alignment is somewhat affected by choice of optional content in SM Units 1 and 2 or GM. However, overall, there is good alignment with SL and HL content in all topics – except ‘Statistics and probability’ which is partially aligned. Furthermore, though not all SL and AHL AI sub-topics are covered, SM instead covers content found in AA or neither DP subject. If all SM and MM units are taken, then the content breadth is more than AI HL, though not with regard to the area of 'Statistics and probability'. Content depth of SM is comparable to AI HL.

Table 14: Specialist Mathematics content which is not covered by DP mathematics subjects

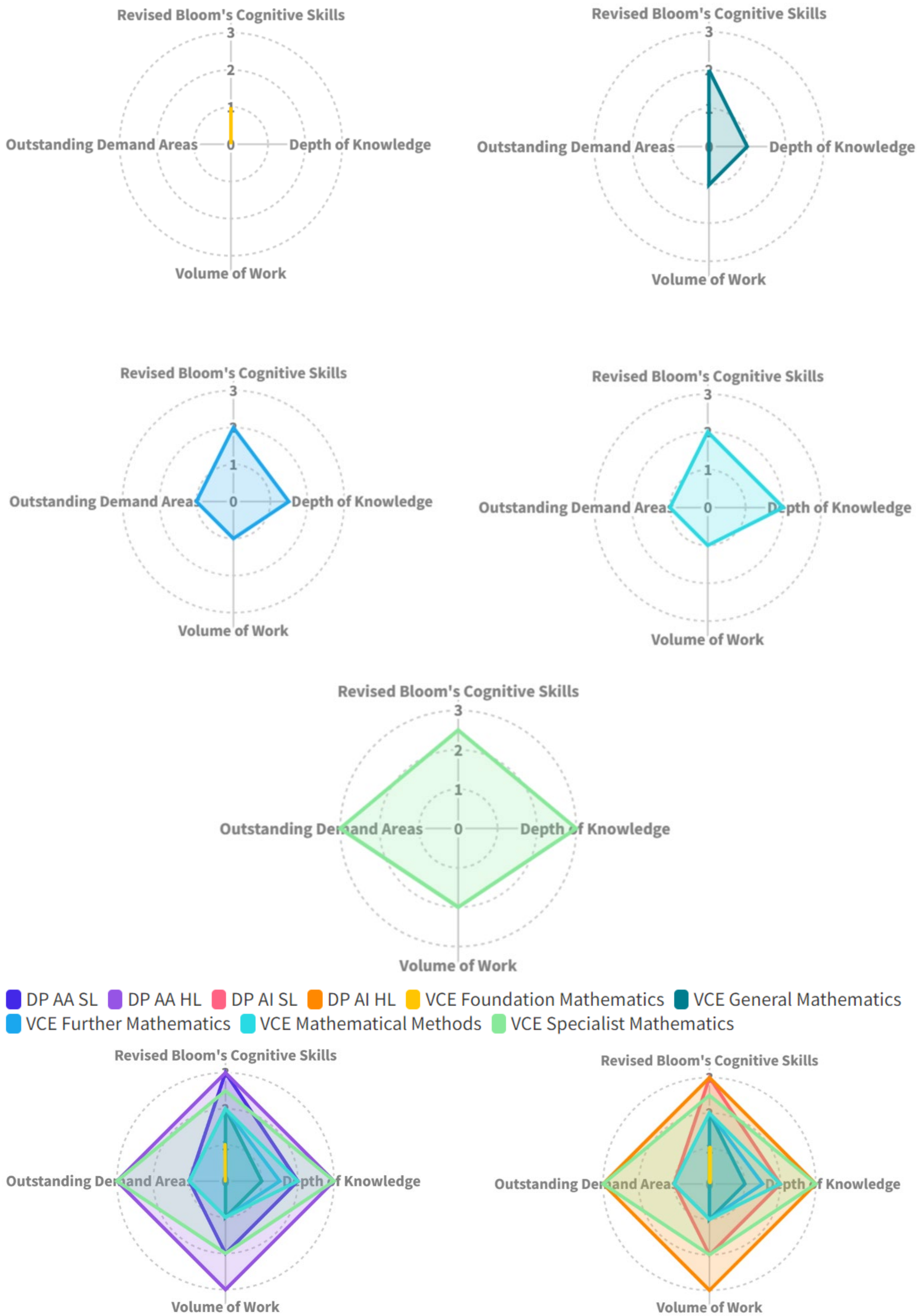
Significant SM content not in AA (only)*	Significant SM content not in AI (only)*
○ Definition and distribution of sample proportions, simulations, and confidence intervals ○ Hypothesis testing ○ Slope fields ○ Graphs and decision mathematics** ○ Matrices (transformations)**	○ Proof by induction and contradiction ○ Reciprocal trigonometric ratios and identities ○ Double and compound angle identities ○ Partial fractions ○ Rational functions ○ Absolute value function ○ Implicit differentiation ○ Counting principles**
Significant content not in either DP mathematics subject*	
○ Proof (circle theorems and geometric properties) ○ Locus definition and construction in the plane of lines, parabolas, circles, ellipses and hyperbolas ○ Cartesian, polar and parametric forms and graphs of lines, parabolas, circles, ellipses and hyperbolas ○ Polar form and graphs of other relations in the plane such as limaçons, cardioids, roses, lemniscates and spirals ○ Parametric form and graphs of other relations in the plane such as spirals, cycloids, lissajous figures and epicycles ○ Vector calculus Larger topics: ○ Mechanics ○ Logic and Algebra** ○ Kinematics**	

*Significant content does not include topics which are typical to mathematical learning prior to upper secondary
**Optional content

4.2.3 Demand

The DP and VCE curricula were analysed using the same demand tool in order to create a demand profile for mathematics: analysis and approaches (SL and HL), mathematics: applications and interpretation (SL and HL), VCE Foundation Mathematics, VCE General Mathematics, VCE Further Mathematics, VCE Mathematical Methods, and VCE Specialist Mathematics. The VCE demand profiles are presented below in the form of radar diagrams, with the last diagrams showing the DP subjects and VCE profiles superimposed in one place, enabling immediate visual comparison.

Figure 7: Visual representations of subject demand



DP AA SL · DP AA HL · DP AI SL · DP AI HL · VCE Foundation Mathematics · VCE General Mathematics · VCE Further Mathematics · VCE Mathematical Methods · VCE Specialist Mathematics

The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - DP subjects received a score of 3.
  - For the VCE, each subject is scored separately – since both the general 'Aims' and the unit-specific 'Outcomes' were used in demand decisions. Due to being more comprehensive, the unit-specific outcomes had precedence in the decision-making, however panel discussions noted that this specificity may lead to some VCE subjects receiving lower scores than others, which is not the case for subjects in curricula describing generalised outcomes only. Foundation Mathematics was given a score of 1 due to the overall emphasis on understanding and application of knowledge and limited evidence of higher order thinking skills. General Mathematics, Further Mathematics, and Mathematical Methods all received a score of 2 as their outcomes had more references to analysis and investigation but lacked complex reasoning and evaluation. Specialist Mathematics was given a score of 2.5 to reflect the higher order cognitive skills that its content material would demand, though evaluation and criticality were not present enough to justify a score of 3. Overall, the presence of higher order cognitive skills varied in the VCE's subject outcomes, however, none evidenced them as strongly as DP mathematics outcomes.

- Regarding the scores for **Depth of Knowledge**:
  - DP SL mathematics subjects received a score of 2 and HL subjects received a score of 3.
  - Looking at VCE subjects, Foundation Mathematics was given a score of 0 due to the basic nature of its content which was more typical of learning prior to upper-secondary mathematics. General Mathematics was awarded a score of 1, due to the inclusion of some sub-topics requiring study of new material and some pre-requisite knowledge, though generally these only contributed to main topics being studied in limited/some detail. A score of 1.5 was awarded for Further Mathematics as statistics and finance were studied in good, but not considerable detail. The subject went beyond a score of 1 for having sub-topics which were studied in greater detail than in General Mathematics, and thus required greater levels of pre-requisite knowledge. Mathematical Methods was deemed to cover the topics of 'Algebra', 'Functions', 'Statistics and probability', and 'Calculus' in considerable detail, similar to DP SL, hence a score of 2 was awarded. Finally, Specialist Mathematics covered topics in high levels of detail, extending each topic to include more advanced sub-topics which built upon the previous and thus increasing the level of complexity, hence it was awarded the same level 3 score as the DP HL subjects.

- Regarding the scores for **Volume of Work**:
  - The DP SL mathematics subjects received a score of 2 and HL subjects received a score of 3.
  - For Victoria, Foundation Mathematics had 100 hours to cover a very light amount of content and received a score of 0 for volume of work. General Mathematics and Further Mathematics each had 100 teaching hours and Mathematical Methods had 200 teaching hours, all of which were deemed standard time allocations to cover the topics within them and hence a score of 1 was given to each. Finally, Specialist Mathematics covered complex material in Units 3 & 4, with a time allocation of 100 hours, thus was deemed to lean towards a heavy workload. However, when considering the content learnt by the end of Specialist Mathematics (Mathematical Methods inclusive) the teaching hours total 400 – substantially more than the 240 hours for DP HL (though it can be noted that Specialist Mathematics covers several further topics). On average, given that some units had a standard volume of work, whilst others were more intensive, a score of 2 was deemed suitable for this subject.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - DP SL mathematics subjects received a score of 1 and HL subjects received a score of 3.
  - For Victoria, Foundational Mathematics and General Mathematics did not contain any areas of outstanding demand and were each awarded a score of 0. Further Mathematics had one or two areas of demand, identified as modelling time series data and transition matrices, and received a score of 1. Similarly, Mathematical Methods was also awarded a score of 1 for the inclusion of extended modelling and problem-solving tasks. Similar tasks were also present in Further Mathematics; however, these were not credited as another area of outstanding demand due to being described as ‘teacher-led’ in these units. Finally, Specialist Mathematics had further outstanding areas of demand identified, which were proof by induction, complex numbers (De Moivre’s theorem), slope fields, second order differential equations and vector calculus. Therefore, similarly to DP HL subjects, a score of 3 was awarded in this category.

4.3 Canada (Ontario)

The school system in the Canadian province of Ontario is overseen by the Ontario Ministry of Education. It is divided into primary school (grades 1-8) and secondary school (grades 9-12), with each grade being a year in duration.[34]

The Ontario Secondary School Diploma (OSSD) spans the last four years of Ontario's secondary school education, grades 9-12. Beginning in grade 9, students take courses to earn credits towards the OSSD, which is intended to grant access to higher education upon successful completion. There are several credit, and other, requirements, to gain the OSSD.

To earn a high school diploma in Ontario, students must:

- Earn a minimum of 30 credits, including 18 compulsory credits (see table below) and 12 optional credits selected from the courses listed as available in their school's programme and course calendar
- Meet the provincial secondary school literacy requirement (i.e. usually, this means passing the Ontario Secondary School Literacy Test[35])
- Complete a minimum of 40 hours of community involvement activities (a requirement aimed to provide students the opportunity to develop awareness and understanding about civic responsibility).[36]

Table 15: Compulsory credits for attaining the OSSD

Compulsory credits for attaining the OSSD (Total of 18)
• 4 credits in English (1 credit per grade) • 3 credits in mathematics (at least 1 credit in grade 11 or 12) • 3 credits for group 1, 2 and 3 courses (1 credit in each group) • 2 credits in science • 1 credit in Canadian history (grade 10) • 1 credit in Canadian geography (grade 9) • 1 credit in the arts • 1 credit in health and physical education • 1 credit in French as a second language • 0.5 credit in career studies • 0.5 credit in civics and citizenship

Courses are not only categorised by subject, but also by the stream within which they sit.

Types of courses available:

- Grades 9 and 10 – courses are either 'Academic', 'Applied', or 'Open'. More recently, some subjects that were previously streamed in grade 9 have now been de-streamed.[37]
- Grades 11 and 12 – courses are either 'University Preparation', 'College Preparation', 'University/College Preparation', 'Workplace', or 'Open'.

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[34] Government of Ontario, Ministry of Education. (2020). *Education and Training*. Available from: https://www.ontario.ca/page/educationand-training
[35] Education Quality and Accountability Office. (n.d.). *Ontario Secondary School Literacy Test (OSSLT)*. Available from: https://wwweqao.com/the-assessments/osslt/
[36] Government of Ontario, Ministry of Education. (2020). *High school graduation requirements*. Available from: https://wwwontario.ca/page/high-school-graduation-requirements#section-3
[37] Meaning that, all students take the same course.

The analysis for Ontario focuses on the mathematics courses offered in the University Preparation (UP) stream and their pre-requisite grade 9 and 10 courses. The exceptions to this are Functions and Applications, which is in the University/College stream, and its grade 10 pre-requisite course – Foundations of Mathematics. For content and demand, the analysis focuses on how grade 11 and grade 12 courses align with the DP, due to grades 9 and 10 being more typical of learning prior to upper secondary mathematics.

Grade 9 (De-streamed)[38]

Grade 9 is the first grade of high school in Ontario and is also the first grade in which a completed course can count as a credit towards the OSSD. Prior to 2021, grade 9 mathematics had two streams, 'Academic' and 'Applied', within which were the courses Principles of Mathematics and Foundations of Mathematics, respectively. From 2021, the grade 9 curriculum was de-streamed and now offers one course to be taken by all students and prepares them for all study options in grade 10.

Grade 10 Principles of Mathematics[39]

Principles of Mathematics is the course available in Ontario's grade 10 'Academic' stream. This course is suitable for students who are likely to enter the University Preparation stream and prepares students for grade 11 Functions.

Grade 10 Foundations of Mathematics[40]

Foundations of Mathematics is the course available in Ontario's grade 10 'Applied' stream. This course is suitable for those who are likely to enter the College Preparation stream and prepares students for grade 11 Functions and Applications.

Grade 11 Functions[41]

Grade 11 Functions is the first mathematics course to be taken in Ontario's 'University Preparation' stream. As such, many students who intend to go to university after high school will take this course. This course builds on learning developed in grade 10 Principles of Mathematics course and is designed to prepare students for grade 12 courses.

Grade 11 Functions and Applications[42]

Grade 11 Functions and Applications sits within Ontario's 'University/College Preparation' stream and provides preparation for those who intend to study technology-related programmes in college, whilst also leaving an option for students to study grade 12 Mathematics of Data Management (University Preparation). Functions and Applications build on grade 10 Foundations of Mathematics and introduces functions through an applied approach, with less emphasis on abstract concepts than grade 11 Functions.

[38] Government of Ontario, Ministry of Education. (2021). *Mathematics (2021).* Available from: https://www.dcp.edu.gov.on.ca/en/curriculum/secondary-mathematics/courses/mth1w

[39] Government of Ontario, Ministry of Education. (2005). *The Ontario Curriculum Grades 9 and 10 Mathematics.* Available from: math910curr.pdf (gov.on.ca)

[40] Ibid.

[41] Government of Ontario, Ministry of Education. (2007). *The Ontario Curriculum Grades 11 and 12 Mathematics.* Available from: The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007 (gov.on.ca)

[42] Ibid.

Grade 12 Advanced Functions[43]

Advanced Functions (AF) is one of three available courses in the Ontario's grade 12 University Preparation stream. Advanced Functions builds on and extends the concepts developed in its prerequisite course, grade 11 Functions. This course is intended to prepare students for university study in areas that include business, social science, and health science programs.

Grade 12 Calculus and Vectors[44]

Calculus and Vectors (CV) can only be taken following, or concurrently with, Advanced Functions and is designed to prepare students for university programmes, such as science, engineering, and economics, that include a calculus or linear algebra course in the first year.

Grade 12 Mathematics of Data Management[45]

Mathematics of Data Management (DM) can be accessed with either grade 11 Functions or Functions and Applications and is intended to prepare students for university study in programmes that may include statistical elements, such as those found in the social sciences and the humanities.

4.3.1 Learning Outcomes

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for AA and AI. Similarly, Ontario sets out a standard list of seven mathematical process expectations which are applicable to all courses in grades 9-12. In 2021, the grade 9 curriculum was updated to incorporate two further expectations: Strand AA (Social-Emotional Learning [SEL] Skills in Mathematics) and Strand A (Mathematical Thinking and Making Connections). The additional grade 9 learning outcomes are analysed and discussed separately in this section.

The following table demonstrates the learning outcome themes that were extracted from the DP mathematics curricula and indicates if and where they were judged to have presence within the learning outcomes of the OSSD mathematics curricula.

[43] Ibid.
[44] Ibid.
[45] Ibid.

Table 16: Presence of the DP mathematics subject group learning outcome themes in Ontario curricula (OSSD)

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the OSSD's mathematics learning outcomes	
1. Being aware of, and engaging with, mathematics in its wider context		Present in the Mathematical Process of 'Connecting' and in the grade 9 Strand A expectation.
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Present in the Mathematical Process of 'Reflecting' and in the grade 9 further expectations on 'Social-Emotional Learning Skills'
3. Using inquiry-based approaches		Present across three different Mathematical Process Expectations (Problem-solving, Reflecting, and Computational Strategies)
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Generally present in the Mathematical Process Expectations and in the grade 9 further expectations.
5. Making links and generalisations		Present in two expectations ('Connecting' and 'Representing') and in the grade 9 Strand A expectation
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Present across a wide range of the Process outcomes and the grade 9 further expectations.
7. Communicating mathematics clearly and in various forms		Present across three different Process outcomes.
8. Knowing how technology and mathematics influence each other and using technology to develop ideas and solve problems		Present in two Process expectations ('Selecting Tools and Computational Strategies' and 'Representing').

Key:

	This theme is well-evidenced in the learning outcomes of the OSSD.		*This theme is partially evidenced in the learning outcomes of the OSSD.*		*This theme is not evident in the learning outcomes of the OSSD.*

Presence of the DP's Learning Outcome Themes

As demonstrated in the table above, there is strong alignment with the DP mathematics learning outcome themes in Ontario's 'Mathematical Process Expectations' for grades 9-12, with the latter also reflecting an emphasis on the development of conceptual understanding and higher-level thinking skills.

Firstly, the theme of critical-thinking development is clearly present, as problem-solving and reasoning are identified separately as two of the seven processes and are embedded into others relating to technology, representations, and reflection. Also embedded in the OSSD learning outcomes is the theme of inquiry approaches, as the expectations make frequent references to working within the context of an investigation. Similarly, the OSSD Mathematical Process Expectations have a technology theme and expect that students use appropriate tools, not only for calculations and representations, but also to problem-solve and investigate ideas. There is further alignment with the DP themes, as the expectations include that students will be able to apply understanding to a range of contexts, communicate mathematics effectively in various ways, and reflect critically on their work.

Although there are many similarities, there are some differences within the themes. For example, though there is evidence that Ontario requires students to engage with mathematics in a wider context in ‘Connections’, this appears to mostly involve links to other disciplines and daily life, thus is not as extensive as the DP in considering global and historic perceptions and thinking critically about the implications of mathematics. Furthermore, although the Mathematical Process Expectations concentrate on developing some learning skills, such as critically reflective practice, they do not explicitly include outcomes directed at collaboration or students’ disposition towards the study of mathematics. However, although these elements are less present, they are mentioned in the extended description of ‘Problem-solving’, where ‘building confidence’ and ‘promoting collaborative sharing’ are listed as benefits.

Presence of DP Themes - Grade 9 Only

In 2021, Ontario de-streamed grade 9 (to no longer differentiate between academic and applied), which brought some changes to the curriculum. Here, two more expectations were added, in addition to the Mathematical Process Expectations also used in grades 10-12. Therefore, grade 9 expectations contain all the similarities mentioned above, with further considerations needed for the additional expectations introduced. The first addition is the expectation that students develop ‘Social-Emotional Learning (SEL) Skills’, which include developing healthy relationships (collaboration), identifying emotions that support mathematical learning (attitudes), and building perseverance. This expectation has strong alignment with the DP.

The other addition is that of ‘Mathematical Thinking and Making Connections’, which is split into two parts, the first referring to the development of conceptual understanding through the application of the Mathematical Processes and the second describing making connections to knowledge systems, lived experiences, and real-world applications. Hence, the inclusion of this expectation links to the DP’s theme of awareness and engagement with maths in its wider context. This makes the global perspectives theme more present in grade 9 in the OSSD compared to later grades, as it includes attention to knowledge systems. Therefore, with these additions, alignment with the DP is stronger in grade 9’s expectations than in the other grades (and some of the aforementioned differences are minimalised).

Other Themes in the OSSD

For Ontario grades 10-12, there is no significant theme in their learning outcomes that is not present in the DP, though a general theme of ‘Connecting’ is focused on more strongly, as it is one of the seven Processes and embedded throughout. Indeed, like the DP, the ‘Connecting’ outcome intends students to make links to other disciplines, but also goes further to expect that links are also made between different mathematical topics and amongst mathematical concepts and procedures. Furthermore, in ‘Representing’, students are expected to be able to create a variety of representations of mathematical ideas, in order to see the connections between them and develop their mathematical learning and understanding. Therefore, by having these outcomes, Ontario perhaps focuses more on connections made within mathematics and the development of flexible thinking about mathematical concepts, whereas the DP outcomes place more emphasis on thinking about mathematics in a wider interdisciplinary and global context.

Other Themes - Grade 9 Only

In grade 9 specifically, the outcome that is the most different to the DP's is that of expecting students to develop 'Social-Emotional Learning (SEL) Skills' during their mathematics studies. There is some overlap here with the DP's theme of developing learning skills, including a positive disposition, patience, collaboration, and critical and creative thinking. However, the SEL skills go beyond these to involve:

- 'recognizing and identifying emotions that support mathematical learning;
- recognizing sources of stress that present challenges to mathematical learning;
- identifying resources and supports that aid perseverance in mathematical learning;
- building healthy relationships and communicating effectively in mathematics;
- developing a healthy mathematical identity through building self-awareness;
- developing critical and creative mathematical thinking'.[46]

Therefore, the OSSD learning outcomes in grade 9 focus on developing students' self-awareness and self-regulation to a greater extent than the DP's learning outcomes overall. Moreover, these skills are more comprehensive with regard to expecting students to think about their mathematical learning and thus more obviously promote metacognition skills, which are present but not as strongly reflected in DP's learning outcomes for mathematics.

Summary

Overall, there is considerable alignment between the learning outcomes of the DP's mathematics subjects and Ontario's. Differences, where found, are generally at the level of emphasis rather than being substantial differences related to the absence and presence of important skills. The new grade 9 learning outcomes in Ontario bring the overall alignment closer than before these were updated, suggesting that the two systems may even further converge at the learning outcomes level when the OSSD curricula are further updated in the following grades.

4.3.2 Content

This section compares and contrasts the content of the DP and OSSD curricula falling within the category of mathematics. In order to support visual comparison at-a-glance, the OSSD mathematics content is presented in the below figure which shows the key topics and sub-topics included.

[46] Government of Ontario, Ministry of Education. (2021). *Grade 9 Maths Expectations by Strand*. Available from: Mathematics (2021) (gov.on.ca)

Figure 8: Ontario mathematics grades 9 and 10 content visualiser

Grade 9 De-streamed	**AA. Social-Emotional Learning (SEL) Skills in Mathematics**	AA1. Social-Emotional Learning Skills			
	A. Mathematical Thinking and Making Connections	A1. Mathematical Processes	A2. Making Connections		
	B. Number	B1. Development of Numbers and Number Sets	B2. Powers	B3. Number Sense and Operations	
	C. Algebra	C1. Algebraic Expressions and Equations	C2. Coding	C3. Application of Relations	C4. Characteristics of Relations
	D. Data	D1. Collection, Representation, and Analysis of Data	D2. Mathematical Modelling		
	E. Geometry and Measurement	E1. Geometric and Measurement Relationships			
	F. Financial Literacy	F1. Financial Decisions			
Grade 10 Foundations of Mathematics (Applied)	**1. Measurement and Trigonometry**	a. Solving Problems Involving Similar Triangles	b. Solving Problems Involving the Trigonometry of Right Triangles	c. Solving Problems Involving Surface Area and Volume, Using the Imperial and Metric Systems of Measurement	
	2. Modelling Linear Relations	a. Manipulating and Solving Algebraic Equations	b. Graphing and Writing Equations of Lines	c. Solving and Interpreting Systems of Linear Equations	
	3. Quadratic Relations of the Form $y = ax^2 + bx + c$	a. Manipulating Quadratic Expressions	b. Identifying Characteristics of Quadratic Relations	c. Solving Problems by Interpreting Graphs of Quadratic Relations	
Grade 10 Principles of Mathematics (Academic)	**1. Quadratic Relations of the Form $y = ax^2 + bx + c$**	a. Investigating the Basic Properties of Quadratic Relations	b. Relating the Graph of $y = x^2$ and Its Transformations	c. Solving Quadratic Equations	d. Solving Problems Involving Quadratic Relations
	2. Analytic Geometry	a. Using Linear Systems to Solve Problems	b. Solving Problems Involving Properties of Line Segments	c. Using Analytic Geometry to Verify Geometric Properties	
	3. Trigonometry	a. Investigating Similarity and Solving Problems Involving Similar Triangles	b. Solving Problems Involving the Trigonometry of Right Triangles	c. Solving Problems Involving the Trigonometry of Acute Triangles	

Figure 9: Ontario mathematics grades 11 and 12 content visualiser

Grade 11 Functions (University Preparation)	**A. CHARACTERISTICS OF FUNCTIONS** 1. Representing Functions 2. Solving Problems Involving Quadratic Functions 3. Determining Equivalent Algebraic Expressions*	**B. EXPONENTIAL FUNCTIONS** 1. Representing Exponential Functions 2. Connecting Graphs and Equations of Exponential Functions 3. Solving Problems Involving Exponential Functions	**C. DISCRETE FUNCTIONS** 1. Representing Sequences 2. Investigating Arithmetic and Geometric Sequences and Series 3. Solving Problems Involving Financial Applications	**D. TRIGONOMETRIC FUNCTIONS** 1. Determining and Applying Trigonometric Ratios 2. Connecting Graphs and Equations of Sinusoidal Functions 3. Solving Problems Involving Sinusoidal Functions	
Grade 11 Functions and Applications (University/ College Preparation)	**A. QUADRATIC FUNCTIONS** 1. Solving Quadratic Equations 2. Connecting Graphs and Equations of Quadratic Functions 3. Solving Problems Involving Quadratic Functions	**B. EXPONENTIAL FUNCTIONS** 1. Connecting Graphs and Equations of Exponential Functions 2. Solving Problems Involving Exponential Functions 3. Solving Financial Problems Involving Exponential Functions	**C. TRIGONOMETRIC** 1. Applying the Sine Law and the Cosine Law in Acute Triangles 2. Connecting Graphs and Equations of Sine Functions 3. Solving Problems Involving Sine Functions		
Grade 12 Advanced Functions (University Preparation)	**A. EXPONENTIAL AND LOGARITHMIC FUNCTIONS** 1. Evaluating Logarithmic Expressions 2. Connecting Graphs and Equations of Logarithmic Functions 3. Solving Exponential and Logarithmic Equations	**B. TRIGONOMETRIC FUNCTIONS** 1. Understanding and Applying Radian Measure 2. Connecting Graphs and Equations of Trigonometric Functions 3. Solving Trigonometric Equations	**C. POLYNOMIAL AND RATIONAL FUNCTIONS** 1. Connecting Graphs and Equations of Polynomial Functions 2. Connecting Graphs and Equations of Rational Functions 3. Solving Polynomial and Rational Equations 4. Solving Inequalities	**D. CHARACTERISTICS OF FUNCTIONS** 1. Understanding Rates of Change 2. Combining Functions 3. Using Function Models to Solve Problems	
Grade 12 Calculus and Vectors (University Preparation)	**A. RATE OF CHANGE** 1. Investigating Instantaneous Rate of Change at a Point 2. Investigating the Concept of the Derivative Function 3. Investigating the Properties of Derivatives	**B. DERIVATIVES AND THEIR APPLICATIONS** 1. Connecting Graphs and Equations of Functions and Their Derivatives 2. Solving Problems Using Mathematical Models and Derivatives	**C. GEOMETRY AND ALGEBRA OF VECTORS** 1. Representing Vectors Geometrically and Algebraically 2. Operating with Vectors 3. Describing Lines and Planes Using Linear Equations 4. Describing Lines and Planes Using Scalar, Vector, and Parametric Equations		
Grade 12 Mathematics of Data Management (University Preparation)	**A. COUNTING AND PROBABILITY** 1. Solving Probability Problems Involving Discrete Sample Spaces 2. Solving Problems Using Counting Principles	**B. PROBABILITY DISTRIBUTIONS** 1. Understanding Probability Distributions for Discrete Random Variables 2. Understanding Probability Distributions for Continuous Random Variables	**C. ORGANIZATION OF DATA FOR ANALYSIS** 1. Understanding Data Concepts 2. Collecting and Organizing Data	**D. STATISTICAL ANALYSIS** 1. Analysing One-Variable Data 2. Analysing Two-Variable Data 3. Evaluating Validity	**E. CULMINATING DATA MANAGEMENT INVESTIGATION**1. Designing and Carrying Out a Culminating Investigation 2. Presenting and Critiquing the Culminating Investigation

4.3.2.1 Structure

For students to graduate secondary school with an OSSD, they must achieve a minimum of three mathematics credits, with at least one credit in grade 11 or 12. The mathematics programme for the OSSD spans three-four years, compared to the DP's two years. Similarly, both programmes require students to continue studying mathematics during the last two years of secondary education, for the OSSD this is a minimum of 110 hours in either grade 11 or 12 (for one credit) and for DP students this is a minimum of 150 hours (for SL).

Furthermore, like the DP, Ontario offers courses designed for different students' future education and careers. Indeed, mathematics content in both programmes is differentiated with regard to where emphasis is placed (pure versus applied). For the DP, this is in the form of AA and AI subjects and for the OSSD this is the offering of Academic or Applied courses in grade 10 and University or College Preparation courses from grade 11. Applied and College Preparation courses' content explore maths with a stronger focus on practical applications. However, where both DP subjects' content prepares students for university programmes, in the OSSD students need to take University Preparation courses. All students in this stream take grade 11 Functions and then have a choice of grade 12 courses.

From grade 11 onwards, there are differences in the structure of content between the mathematics subjects in the DP and the OSSD University Preparation (UP) stream. The UP courses available do not separate topic content into levels, such as the DP's SL and HL, nor do they integrate a range of main topics, instead each course tends to focus on a specific area of mathematics. For instance, calculus is studied in all DP mathematics subjects, whereas in the UP stream it is studied only within the grade 12 Calculus and Vectors course. Therefore, the variable in the UP courses' mathematics content is the number of main topics which are studied, whereas for the DP it is the amount of content within each main topic.

4.3.2.2 Content Alignment

From here onwards, the analysis focuses on grades 11 and 12. Although grades 9 and 10 are part of the programme, the content and demand of these grades is typical of learning *prior* to upper secondary mathematics. Therefore, for meaningful comparison to the upper secondary DP, it is logical to direct attention to the last two grades of secondary education in Ontario.

To complement the analysis, the figures below represent a simplified summary of the OSSD's content alignment, at topic-level, with AA (SL and HL) and AI (SL and HL).[47]

[47] The content of grades 9 and 10 was also mapped but is not represented in the table. Very little alignment was found with these grades due to their content being more typical of prior learning to upper secondary mathematics.

Table 17: Summary of the content alignment OSSD has with the main topics in AA

Mathematics: analysis and approaches topics		Presence in OSSD					
		Grade 11 Functions and Applications	Grade 11 Functions	Grade 12 Advanced Functions	Grade 12 Calculus and Vectors	Grade 12 Data Management	University Preparation (combined)
SL	1. Number and algebra						
	2. Functions						
	3. Geometry and trigonometry						
	4. Statistics and probability						
	5. Calculus						
AHL	1. Number and algebra						
	2. Functions						
	3. Geometry and trigonometry						
	4. Statistics and probability						
	5. Calculus						

<u>Key:</u>

	Strong presence of this topic in the OSSD		*Partial presence of this topic in the OSSD*		*Little or no presence of this topic in the OSSD*

NB: Where applicable, content alignments found in pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

Table 18: Summary of the content alignment the OSSD has with the main topics in AI

Mathematics: applications and interpretation topics		Presence in OSSD					
		Grade 11 Functions and Applications	Grade 11 Functions	Grade 12 Advanced Functions	Grade 12 Calculus and Vectors	Grade 12 Data Management	University Preparation
SL	1. Number and algebra						
	2. Functions						
	3. Geometry and trigonometry						
	4. Statistics and probability						
	5. Calculus						
AHL	1. Number and algebra						
	2. Functions						
	3. Geometry and trigonometry						
	4. Statistics and probability						
	5. Calculus						

Key:

	Strong presence of this topic in the OSSD		*Partial presence of this topic in the OSSD*		*Little or no presence of this topic in the OSSD*

NB: Where applicable, content alignments found in pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

Grade 11

Mathematics: analysis and approaches

Courses offered in grade 11 are ‘Functions’ (University Preparation) and ‘Functions and Applications’ (College Preparation). There are more instances of alignment with AA in ‘Functions’ content than in ‘Functions and Applications’, though not a significant amount in most topics.

In grade 11, functions are introduced in both courses and each cover quadratic, exponential, and trigonometric functions. Therefore, there are strong and partial alignments with ‘Functions’ sub-topics in AA SL. Indeed, the topic of ‘Functions’ is the topic that the OSSD has the most alignment with for grade 11 content. However, OSSD Functions and OSSD Functions and Applications further include sub-topics which involve the modelling of these functions, which is not a focus for AA. Both courses also have a similar number of alignments with AA SL at sub-topic level in the topic ‘Geometry and trigonometry’ – though fewer than those found with ‘Functions’. No ‘Geometry and trigonometry’ AHL content areas are covered in OSSD grade 11, nor are some SL sub-topics such as solving trigonometric equations and introducing radians.

The alignment difference to AA between ‘Functions’ and ‘Functions and Applications’ is most obvious in ‘Number and algebra’, which is due to grade 11 Functions containing the topic of ‘Discrete Functions’, which is not in Functions and Applications. This area explores sequences and the binomial theorem, hence there are alignments with these sub-topics in AA, along with evidence of simple proof (SL) from other topics. Hence, grade 11 Functions covers most SL sub-topics in AA ‘Number and algebra’, but no AHL sub-topics. With regard to other topics, no ‘Calculus’ is covered by the end of OSSD grade 11, and no further statistics or probability content is covered.

Overall, grade 11 content aligns with a good amount of AA SL content in ‘Number and algebra’, ‘Functions’, and ‘Geometry and trigonometry’, minimal amounts of SL content in ‘Statistics and probability’, and no content from ‘Calculus’. OSSD grade 11 Functions and Applications is similar but has less content shared with ‘Number and algebra’. There is no alignment with AHL content. There are few areas which are covered in grade 11 that are not covered by AA. Where these do exist, they relate to aspects of modelling and some further financial applications, such as annuities. Students taking only grade 11 courses will study a lesser breadth and depth of content than AA SL and HL.

Mathematics: applications and interpretation

As with AA, the AI topic that grade 11 content has the most sub-topic alignments with is ‘Functions’. Like AI, OSSD Functions and OSSD Functions and Applications place importance on modelling with functions and using modelling processes, thus there is strong alignment in this topic – however, these courses do not model with as many functions as AI. There are less instances of alignment in AI ‘Number and algebra’, though grade 11 Functions has more alignment than Functions and Applications due to including sequences and their financial applications as part of its ‘Discrete Functions’ section. Grade 11 Functions and Applications also considers financial applications, though within the context of exponential functions rather than sequences. There are limited examples of alignment with ‘Geometry and trigonometry’, with most coming from the more basic sub-topics in this area. There is no further ‘Statistics and probability' content covered in OSSD grade 11 to extend content covered in grade 9, and calculus is not studied until grade 12.

Overall, grade 11 Functions has alignment with most SL content in 'Functions', plus parts of AHL; a good amount of SL content in 'Number and algebra'; some SL content in 'Geometry and trigonometry'; minimal SL content in 'Statistics and probability'; and no content in 'Calculus'. The same applies for Functions and Applications, with the difference being coverage of fewer 'Number and algebra' areas and some small aspects of functions and trigonometry not being included either. There is no alignment with AHL content. Students taking only taking a grade 11 course will study a lesser breadth and depth of content than DP SL.

Table 19: OSSD grade 11 content which is not covered by DP mathematics subjects

Significant grade 11 content not in AA (only)	Significant grade 11 content not in AI (only)
• A focus on modelling with functions, e.g. sinusoidal models • Some financial applications (annuities)	
Significant content not in either DP mathematics subject *	
N/A	

*Significant content does not include topics which are typical to mathematical learning *prior to* upper secondary

Grade 12

This section considers OSSD content alignment with AA and AI if all University Preparation courses are taken in grade 12 (i.e. Advanced Functions, Calculus and Vectors, and Data Management). However, it is important to note that, to achieve the OSSD, it is not necessary that students take all (or in fact any) grade 12 University Preparation courses. That said, it is likely that those going to university will need Calculus and Vectors (and, by default, Advanced Functions), with some also recommending that Data Management be taken. The relevant alignments from pre-requisites (such as grade 11 Functions) will also be taken into account, to create a picture of all the content covered from grades 9-12. Following this subsection, there will be consideration of how alignment is affected in the cases where certain grade 12 courses are not taken.

Mathematics: analysis and approaches

The University Preparation courses' content has sub-topic alignments with all topics in AA, to varying degrees. The AA topic that UP has the strongest alignment with is that of 'Functions', due to the OSSD offering two courses focusing on this area of mathematics – grade 11 Functions and grade 12 Advanced Functions. Grade 12 builds on grade 11 to include rational, polynomial, logarithmic, and composite functions. However, not all AHL 'Functions' content is covered, Advanced Functions instead focuses on modelling and solving real-world problems with some of these functions.

After 'Functions', the next AA topic UP courses have the most alignments with is 'Geometry and trigonometry', as they have partial or strong alignment with nearly every SL and AHL sub-topic. Similarly, UP courses combined include 'Geometry and trigonometry' concepts such as solving trigonometric equations, vectors, trigonometric identities, reciprocal trigonometric functions, and radians.

For ‘Number and algebra’, the UP courses have very strong alignment with SL content, though only minimal instances of alignments are with AHL content; indeed, none of the UP courses include complex numbers or proof by contradiction/induction. Many of the sub-topics in SL ‘Statistics and probability’ are present in UP courses, with Data Management covering some SL sub-topics such as correlation and regression, introducing probability and set notation, and looking at several probability distributions – including different ones such as hypergeometric. Similar to both DP subjects, an investigation is conducted in Data Management which involves students choosing an area, carrying out an investigation, reporting and presenting their findings, and critically reviewing theirs and others’ work.

For AA ‘Calculus’, UP courses have good alignment with SL content, except for sub-topics related to the area of integration – UP courses only cover differentiation. Furthermore, apart from understanding concepts of limits and first principles, no other AHL content is covered from ‘Calculus’; indeed, the UP courses do not cover integrals, methods of integration, further derivatives, implicit differentiation, first order differential equations or Maclaurin series. However, it can be noted that a large amount of their calculus content is focused on investigating the graphs of functions and their derivatives, verifying differentiation rules, and using first principles, thus UP courses do give conceptual depth to the parts of calculus which they do include.

In summary, UP courses combined have strong alignment with all SL topics – except ‘Calculus’ – strong alignment with AHL ‘Geometry and trigonometry’, and partial alignment with ‘Functions’ AHL content. Overall, the UP courses’ content is less broad than that of AA AHL, though is larger than SL to some extent. The only significant UP area less present in AA is that of modelling with functions.

Mathematics: applications and interpretation

The UP courses’ content has sub-topic alignments with all topics in AI – to varying degrees. As with AA, the topic that the UP courses are the most strongly aligned with is ‘Functions’. Courses which involve functions in the OSSD specify modelling with a range of functions, identifying key features of graphs, transformations of a variety of functions, and finding composite and inverse functions. Thus, all SL topics are present, with also two out of four AHL topics (some harder models are not covered). However, it can be noted that Advanced Functions include exploration of functions beyond the scope of AI, such as rational and polynomial, and include a heavier focus on analytical solutions.

For AI ‘Geometry and trigonometry’, UP courses have a mixture of SL and AHL alignments. Most significantly, vectors are covered, as are radians and trigonometric ratios and identities. Advanced Functions goes beyond the scope of AI in this topic by including equations of vector planes, reciprocal trigonometric functions, further identities, and analytically solving trigonometric equations. However, no sub-topics related to Voronoi diagrams, matrices, graph theory, or decision mathematics are present in the UP courses.

For AI ‘Statistics and probability’, UP courses have strong alignment with most SL sub-topics – presenting data, correlation, probability, and probability distributions – though there are no clear alignments with AHL content such as hypothesis testing, non-linear regression, or Markov chains. For AI ‘Calculus’, UP courses have a mixture of SL and AHL sub-topic alignments, including second derivatives, optimisation, and kinematic problems. However, the area of integration is not covered by any UP courses, hence a considerable number of sub-topics have no alignment. Furthermore, harder topics such as differential equations and phase portraits are also not included. That said, the Calculus and Vectors course gives conceptual depth by exploring graphical representation of derivatives of functions, verifying rules graphically, and looking in detail into first principles.

For AI ‘Number and algebra’, UP courses have strong alignment with most SL sub-topics and the two AHL sub-topics involving the laws of logarithms and exponentials, however UP courses do not cover complex numbers, matrices, or eigenvalues and eigenvectors.

Overall, content in UP courses is not strongly aligned with AI AHL content – though some topics have a few AHL sub-topics present – but is better aligned with SL topics. Thus, the subject content covered in OSSD is not as broad as HL, but is broader than SL.

Table 20: OSSD University Preparation content which is not covered by DP mathematics subjects

Significant UP content not in AA (only)	Significant UP content not in AI (only)
• A focus on modelling with some functions • Some financial applications, e.g. annuities	• Certain functions, e.g. rational functions and polynomials • Reciprocal ratios • Further trigonometric identities • Differentiation from First Principles • Solving inequalities • Solving equations, both graphically and analytically • Counting principles • Vectors – equations of planes
Significant content not in either DP mathematics subject *	
• Hypergeometric probability distribution	

*Significant content does not include topics which are typical to mathematical learning prior to upper secondary

The section below will look at how alignment is affected in different scenarios where a certain grade 12 University Preparation course is not studied. Since the only mathematics requirement is one credit in either grade 11 or 12, it is likely that pathways such as these will be taken.

(i) Mathematics of Data Management (DM)

Generally, DM is not a pre-requisite of many university programmes outside of those with a strong statistical element, though there is evidence that it is recommended for STEM (Science, technology, engineering, and mathematics) courses. If DM is not taken, then students in the University Preparation stream do not encounter any statistics and probability topics after grade 9. Thus, without this course content, there is very minimal alignment with the topic of ‘Statistics and probability’ in both AA and AI. The alignment with other topics remains the same.

(ii) Calculus and Vectors

Calculus and Vectors can only be taken with or after Advanced Functions and is considered to be the highest-level course that can be taken in OSSD mathematics. Often this course is a requirement of university programmes, especially if the course has a STEM element. If the course is not taken, the level of alignment is significantly affected in two main topics for both AA and AI. In ‘Geometry and trigonometry’, only the alignment with AHL would be affected, due to vectors being no longer covered. In ‘Calculus’, there would be no alignment with either SL or AHL content for both AA and AI, as this topic is not studied in any other course.

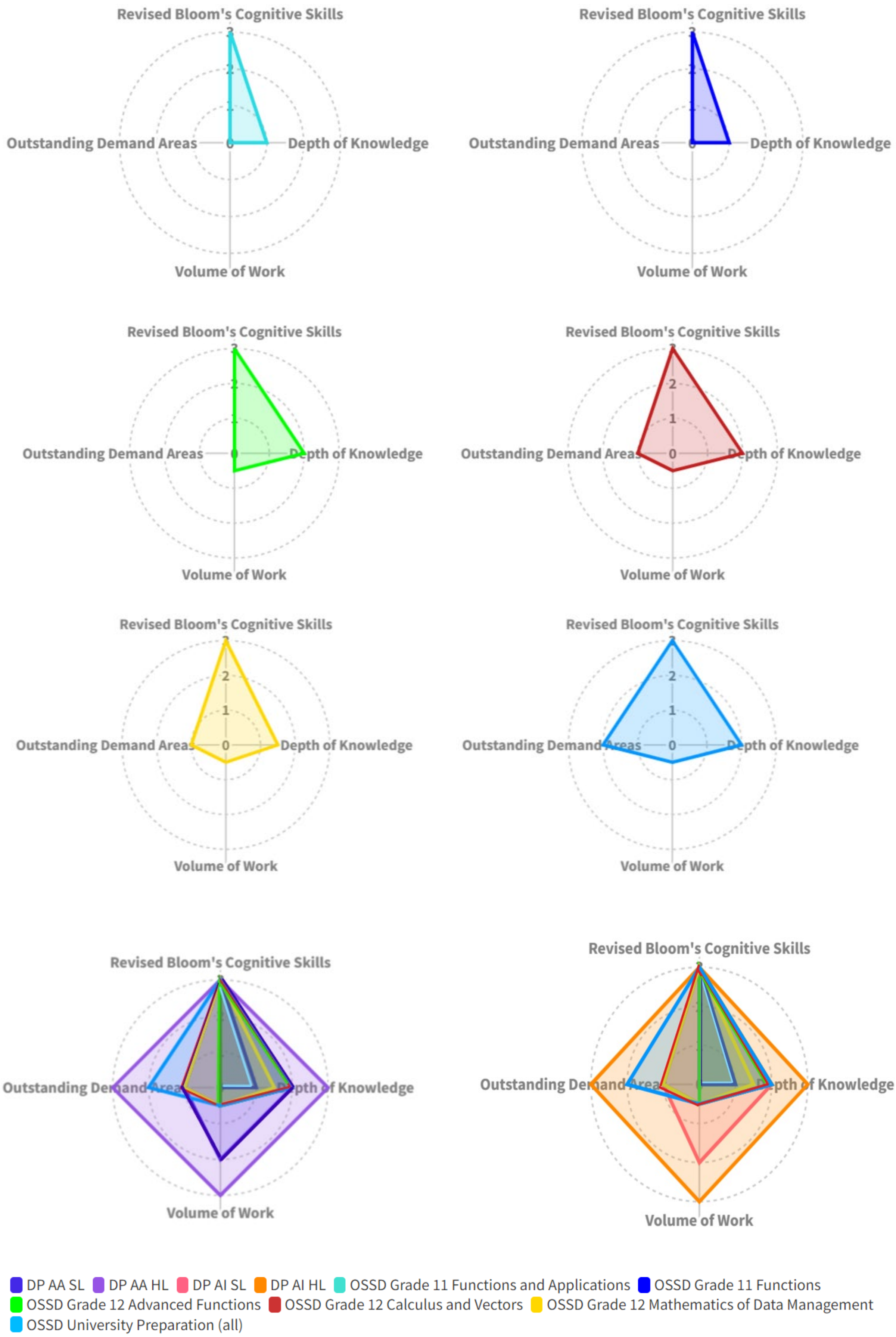
(iii) Advanced Functions

If a student does not study Advanced Functions, they cannot study Calculus and Vectors. Thus, the effects on alignment mentioned in the section above would also apply again in this case. Without Advanced Functions, alignment with both AA and AI topics of ‘Functions’ and ‘Geometry and trigonometry’ would decrease further – though more significantly for AA. Advanced Functions includes logarithmic, rational, and polynomial functions, meaning that these alignments with AA would be lost. Composite functions are also introduced in Advanced Functions; thus, not studying the course would affect alignment with both AA and AI. For ‘Geometry and trigonometry’, there would be fewer alignments with AA due to Advanced Functions including reciprocal trigonometric ratios, radians, further identities and solving trigonometric equations. Some alignment would also be lost with the AI AHL sub-topic of radians.

4.3.3 Demand

The DP and OSSD curricula were analysed using the same demand tool in order to create a demand profile for AA (SL and HL), AI (SL and HL), OSSD grade 11 Functions and Applications, OSSD grade 11 Functions, OSSD grade 12 Advanced Functions, OSSD grade 12 Calculus and Vectors, OSSD grade 12 Mathematics of Data Management, and OSSD University Preparation (all mathematics University Preparation courses combined). The OSSD demand profiles are presented below in the form of radar diagrams, with the last two diagrams showing the DP subjects and OSSD profiles superimposed in one place, enabling immediate visual comparison.

Figure 10: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - DP subjects received a score of 3.
  - Similarly, the OSSD learning outcomes applied to all courses and thus each has the same score. Like the DP, the OSSD learning outcomes were given a score of 3, which was on the basis that elements of evaluation, creation and analysis were woven into most of their outcomes through references to posing problems, reflecting, investigating, generalising, assessing, and justifying.

- Regarding the scores for **Depth of Knowledge**:
  - DP SL subjects received a score of 2 and HL subjects received score of 3.
  - For the OSSD, grade 11 courses had some detail in the topics of functions and trigonometry, however the restricted range of functions introduced, and the generally small number of sub-topics covered, meant complexity was limited and was not on par with DP SL, hence a score of 1 was deemed appropriate for both courses. Grade 12 Advanced Functions extended the knowledge from grade 11 Functions to include more complex functions and concepts, hence the higher level of complexity and requirement of considerable pre-requisite knowledge warranted a score of 2. Furthermore, grade 12 Calculus and Vectors required considerable pre-requisite knowledge and covered the topic of vectors in high detail – though grasping the foundations of differentiation was given greater precedence than including integration and building complexity, hence an overall score of 2 was awarded. For grade 12 Data Management, a limited amount of pre-requisite information was needed, though the course contained a good amount of analysis and level of detail, thus an overall score of 1.5 was deemed appropriate for depth of knowledge. Finally, the depth of knowledge score was considered in the case where all grade 12 University Preparation courses were studied, for which it judged that most topics were typically studied in considerable, rather than a high, level of detail, thus a score of 2 was awarded.

- Regarding the scores for **Volume of Work**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For the OSSD, courses in grade 11 had a generous amount of time to cover mostly basic concepts, hence the volume of work was low and a score of 0 was given to all. A good comparison which demonstrated this was grade 11 Functions, which had 110 teaching hours to cover a small range of functions and a few sub-topics from other areas, whereas the DP allocated considerably fewer hours (21-31) for 'Functions' at SL. Furthermore, grade 12 Advanced Functions, Calculus and Vectors, and Data Management also each had 110 hours to cover a small number of topics. A score of 0.5 was deemed suitable here as the content was more complex than lower grades but was still too generous to be considered a standard amount of time to justify a score of 1. As another comparison, to study a broad range of topics in considerable detail in the OSSD would require 440 teaching hours (for grade 11 Functions and all grade 12 courses), whereas the DP has 150

hours to cover a similar number of themes and concepts, or 240 hours to cover all topics in further detail than the OSSD. Overall, the volume of work in mathematics was a significant difference between the OSSD and the DP.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - DP SL subjects received a score of 1 and HL subjects received a score of 3.
  - For Ontario, no areas of outstanding demand were found in grade 11, nor were any identified in grade 12 Advanced Functions. These courses included content that was either typical of upper secondary mathematics or below, thus each was awarded a score of 0 for this category. Calculus and Vectors was agreed to merit a score of 1 due to identified areas of outstanding demand within vector sub-topics (C3 Describing lines and planes using linear equations and C4 Describing lines and planes using scalar, vector, and parametric equations). Data Management was also awarded a score of 1 for its 'Culminating Investigation' which required students to collect, model and analyse data, apply ethical practices, consider bias, produce a report, present findings to peers, respond to questioning, and offer own critiques. Finally, in the scenario where all grade 12 courses are taken, the scores for outstanding areas would be combined from Calculus and Vectors and Data Management to give a total score of 2.

4.4 Finland

The school system in Finland is overseen by the Finnish National Agency for Education (EDUFI).[48] Compulsory education is divided into Basic Education (years 1-9) and General Upper Secondary Education (GUSE). In Finland, students must study until they graduate from secondary education or reach the age of eighteen.[49] GUSE is typically three years in duration, yet students may complete the certificate in two or four years; as a result, the qualification is very fluid and there are no grade-specific classes.

An extensive reform to GUSE was launched in 2017, with changes to the Act on GUSE and the Government Decree on GUSE.[50] The aforementioned documents form the basis of the Finnish National Core Curriculum (FNCC), produced by EDUFI, with input from teaching staff, students, students' guardians, and relevant authorities. The FNCC underpins all local curriculum offerings.

In the FNCC/GUSE, each subject area is composed of different *subjects*, some of which contain different choices of *syllabi* – e.g. biology has only one syllabus, while mathematics offers two syllabi for student to choose from. Subjects/syllabi, in turn, contain *studies* (also referred to as 'modules'). There are *compulsory studies* and *national optional studies* – each study carrying different credit values.

In order to complete general upper secondary education, students must have 'passed the subject syllabi and completed the minimum scope of general upper secondary education studies, or 150 credits'.[51] Of these 150 credits, a minimum of 20 must have been obtained from the completion of national optional studies. For reference, each credit is equivalent to fourteen hours and fifteen minutes of teaching, in addition to independent study.[52]

Students must study the *compulsory credits* from all of the subjects listed in the table below.

Table 21: List of subjects and compulsory credits in the FNCC[53]

Subjects and their compulsory credits in the FNCC
• mother tongue and literature (12 credits) • second national language, either Finnish or Swedish. (10 or 12 credits)* • foreign languages (12 credits) • mathematics (12 or 20 credits)* • biology (4 credits) • physics (2 credits) • chemistry (2 credits)

[48] Finnish National Agency for Education. (2022). Available at: https://www.oph.fi/en
[49] Ministry of Education and Culture, Finland. (2020). *Act on Compulsory Education (1214/2020).* Available from: https://www.finlex.fi/fi/laki/ajantasa/2020/20201214
[50] Finnish National Agency for Education. (2019). *National Core Curriculum for General Upper Secondary Education*. p. 10.
[51] Ibid. p. 62.
[52] Ministry of Education and Culture, Finland. (2018). *Government Decree on General Upper Secondary Education (810/2018).* Available from: https://www.finlex.fi/fi/laki/alkup/2018/20180810
[53] Finnish National Agency for Education. (2019). *National Core Curriculum (NCC) for General Upper Secondary Education*.

Subjects and their compulsory credits in the FNCC
• geography (2 credits) • philosophy (4 credits) • psychology (2 credits) • history (6 credits) • social studies (6 credits) • religion or worldview studies (4 credits) • health education (2 or 4 credits)* • physical education (4 credits) • music (4 credits) • visual arts (2 or 4 credits)* • guidance counselling (4 credits). * Depending on the syllabus chosen.

As can be seen from the table above, students are required to study mathematics as part of the FNCC/GUSE. The Finnish curriculum offers two mathematics syllabi for students to choose from, as described below.

Basic syllabus[54]
The basic syllabus is one of two options offered in the FNCC for general upper-secondary mathematics education. The syllabus is comprised of seven compulsory modules, worth 12 credits, and two optional modules, worth four credits. Through focusing on applications of mathematics, such as modelling, finance, and statistics, this syllabus equips students with the knowledge and skills for everyday life and will support further studies for which a grounding in basic mathematical concepts will be beneficial.

Advanced syllabus[55]
The advanced syllabus is one of two options offered in the FNCC for general upper-secondary mathematics education. The syllabus is comprised of nine compulsory modules, worth 20 credits, and three optional modules, worth six credits. This syllabus focuses more on pure mathematics than the basic syllabus and goes into further depth and complexity in its topics. Therefore, this option is suitable for students with good mathematical ability, who may require more advanced mathematical knowledge in their future studies and careers.

4.4.1 Learning Outcomes

The DP's learning outcomes are the same for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics applications and interpretation. Similarly, Finland's curriculum sets out the same outcomes for both its basic and advanced syllabi. Finland's mathematics-specific learning outcomes take the form of 'general objectives', 'transversal competencies' (contextualised for mathematics), and a section describing the 'task of the subject'.

The following summary table demonstrates the learning outcome themes that were extracted from the DP mathematics and indicates if and where they were judged to have presence within the learning outcomes of the FNCC mathematics curricula.

[54] Finnish National Agency for Education. (2019). *NCC for General Upper Secondary Education.*
[55] Ibid.

Table 22: Presence of the DP mathematics subject group learning outcome themes in the FNCC

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the FNCC	
1. Being aware of, and engaging with, mathematics in its wider context		Present in the general objectives, transversal competencies, and task of the subject. Specifically, the 'societal, ethical, and environmental', and 'global and cultural' competencies strongly demonstrate this theme.
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Present in the general objectives and transversal competencies – specifically the 'well-being' and 'interaction' competencies.
3. Using inquiry-based approaches		Present in the general objectives, where use of experimental and investigative actions are described.
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present mainly in the task of the subject, which describes understanding and application. Also implied in other outcomes, such as those relating to modelling and environmental contexts.
5. Making links and generalisations		Present in the general objectives, transversal competencies, and the task of the subject. Specifically, the 'multidisciplinary and creative competency' strongly demonstrates this theme.
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Present in the general objectives, transversal competencies, and task of the subject. Problem-solving, reasoning, evaluation, and modelling, are frequently referenced.
7. Communicating mathematics clearly and in various forms		Present in the general objectives and the task of the subject. Students are expected to be able to understand and communicate mathematics taking different forms.
8. Knowing how technology and mathematics influence each other and using technology to develop ideas and solve problems		Present in the task of the subject. Students are expected to use technology, as well as to evaluate it.

Key:

	This theme is well-evidenced in the learning outcomes of the FNCC.		*This theme is partially evidenced in the learning outcomes of the FNCC.*		*This theme is not evident in the learning outcomes of the FNCC.*

Presence of the DP's Learning Outcome Themes

As demonstrated in the table above, all of the DP's themes are present in Finland's mathematics learning outcomes. Similarly, to the DP, Finland expects students to understand and apply mathematics in wider contexts, including societal, ethical, environmental, global, and cultural. For example, the 'global and cultural' transversal competence expects students to 'appreciate the significance of mathematics for different cultures and in the development of history, and to understand its nature as a universal language'.[56] Moreover, it is expected that students will reflect on how they can use mathematical skills to solve problems relating to

[56] Ibid. *6.6 Mathematics.*

humankind and the environment, thus demonstrating the ‘societal, ethical, and environmental’ transversal competency. Overall, these expectations of appreciating different perspectives, gaining awareness of ethical, environmental, and societal issues, and exploring mathematical solutions to these strongly echo the aims of DP mathematics.

Furthermore, the transversal competencies also demonstrate other DP themes. The ‘multidisciplinary and creative’ competence speaks to students understanding how mathematical concepts are connected to each other and with other subjects, thus this reflects the DP’s theme of a similar nature regarding making links to other disciplines. In addition, the transversal competencies of interaction and well-being correlate with the DP’s theme of the development of learning skills. Indeed, these competencies describe building students’ confidence, independence, and perseverance, as well as developing decision-making, collaboration, and interaction skills.

Another strongly demonstrated DP theme is that of critical and creative thinking skills. All the elements of Finland’s learning outcomes reference this theme, specifically they describe problem-solving, evaluating mathematical information and the soundness of results, constructing and substantiating mathematical arguments, creative thinking, and modelling.

Together, the general objectives and task of the subject encompass the rest of the DP’s themes. Like the DP, the FNCC aims to encourage students to use inquiry-based approaches, specifically it states that students should ‘become confident in using experimental and investigative actions, finding solutions, and presenting them clearly’.[57] Furthermore, communication skills are also demonstrated, as the task of the subject describes students being able to communicate mathematics, using the language of mathematics in spoken, written and other forms. The task of the subject section also describes the use of technology, which, similarly to the DP, expects students to utilise technology in research, exploration, problem-solving, and to evaluate its reasonableness and limitations. Finally, understanding and application of mathematics is implied in various other themes and explicitly referenced in the task of the subject.

Other Themes in the FNCC

In general, there are no significant themes that are not present in the DP. However, it can be noted that Finland’s curriculum learning outcomes made more explicit reference to modelling and making connections within the subject of mathematics.

Summary

Overall, there is strong alignment between the learning outcomes of the DP’s mathematics subjects and Finland’s. Both the DP and FNCC take a holistic approach and similarly emphasise critical and creative thinking, use of inquiry-based approaches, building transferable learning skills, and developing awareness of the wider contexts of mathematics.

[57] Ibid.

4.4.2 Content

This section compares and contrasts the content of the DP and FNCC falling within the category of mathematics. In order to support visual comparison at-a-glance, the FNCC mathematics syllabi are presented below in a diagram showing the key topics included in each.

Figure 11: FNCC mathematics content visualiser

Basic syllabus	Compulsory	MAY1 Numbers and equations (2 credits)	MAB2 Expressions and equations (2 credits)	MAB3 Geometry (2 credits)	MAB4 Mathematical models (2 credits)	MAB5 Statistics and probability (2 credits)	MAB6 Elements of mathematical economics (1 credit)	MAB7 Mathematical economics (1 credit)		
	Optional	MAB8 Mathematical analysis (2 credits)	MAB9 Statistical and probability distributions (2 credits)							
Advanced syllabus	Compulsory	MAY1 Numbers and equations (2 credits)	MAA2 Functions and equations 1 (3 credits)	MAA3 Geometry (2 credits)	MAA4 Analytical geometry and vectors (3 credits)	MAA5 Functions and equations 2 (2 credits)	MAA6 Derivative (3 credits)	MAA7 Integral calculus (2 credits)	MAA8 Statistics and probability (2 credits)	MAA9 Mathematical economics (1 credit)
	Optional	MAA10 3D geometry (2 credits)	MAA11 Algorithm and number theory (2 credits)	MAA12 Analysis and continuous distribution (2 credits)						

4.4.2.1 Structure

In a similar way to SL and HL in DP mathematics, Finland's curriculum offers two syllabi of different levels – basic and advanced – from which one is chosen. The basic syllabus aims to give a good grounding in mathematics, whilst the advanced offers more in-depth study for students who are able and may need mathematics in further education. However, the important difference is that the advanced syllabus does not follow on from the basic syllabus in the way that AHL content follows on from SL content. Indeed, the basic and advanced syllabi are separate, with students usually completing one of them – though it is possible for students to change from one to the other. Furthermore, unlike the DP, Finland's national curriculum does not offer syllabi of similar size which differ in thematic focus, as AA and AI do in the DP.

Whilst DP mathematics subjects are broken down into five main topics, which remain the same for both SL and HL, Finland's syllabi are broken down into smaller topics, called modules, and are different for basic and advanced – except for one common topic, MAY1 Numbers and equations. Furthermore, each topic is worth a number of credits, with some being compulsory and others optional. The basic syllabus has 12 compulsory credits and 4 optional credits, the advanced syllabus has 20 compulsory credits and 6 optional credits. There is no minimum requirement for optional credits specific to mathematics, however, students must complete a minimum of 20 optional credits in general upper-secondary education. This is different from the DP, which offers no optional topics; hence, Finland's curriculum offers an element of flexibility which the DP does not.

4.4.2.2 Content Alignment

The figures below show a simplified summary of the extent to which FNCC mathematics aligns with the main topics of the DP's subjects.

Table 23: Summary of the content alignment FNCC has with the main topics in AA

Mathematics: analysis and approaches topics		Presence in FNCC	
		Basic	Advanced
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		Majority of vector content is part of an optional credit
	4. Statistics and probability		Optional credit
	5. Calculus		

Key:

	Strong presence of this topic in the FNCC.		*Partial presence of this topic in the FNCC.*		*Little or no presence of this topic in the FNCC.*

* Where applicable, content alignments found in pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

Table 24: Summary of the content alignment the FNCC has with the main topics in AI

Mathematics: applications and interpretation topics		Presence in FNCC	
		Basic	Advanced
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus	Optional credit	
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in the FNCC.		*Partial presence of this topic in the FNCC.*		*Little or no presence of this topic in the FNCC.*

* Where applicable, content alignments found in pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

Basic syllabus

Mathematics: analysis and approaches

The mapping of content shows that the basic syllabus has some significant alignment with the SL content of AA, though not in every main topic. The SL topic that the basic syllabus has the most alignment with is that of 'Number and algebra'. The basic syllabus covers arithmetic and geometric sequences and series in similar detail – including financial applications. However, binomial theorem, simple proof, and laws of logarithms are not included.

The basic syllabus is partially aligned with SL 'Functions' content due to its inclusion of quadratic functions and solutions, exponential functions, and graphing of functions. However, rational functions, logarithmic functions, inverse functions, composite functions, and transformations are not present in the syllabus – though it can be noted that polynomials, which is an AHL sub-topic in AA, is present. Furthermore, the basic syllabus frequently references modelling of functions, similar to AI.

Another topic with which the basic syllabus partially aligns is that of SL 'Statistics and probability', due to the basic syllabus including similar coverage of correlation, linear regression, and probability. More sub-topic alignments are found with the binomial and normal distributions; however, these are only covered in the national optional credit MAB9. There are a few sub-topic alignments with SL 'Calculus', however, none of this content is compulsory in the basic syllabus – it is offered as national optional credit MAB8. This credit only includes basic calculus content, such as derivatives of polynomials and finding maximum/minimum points. Additionally, geometry and trigonometry content is compulsory, though only aligns with a couple of the more basic sub-topics in AA SL.

Most of the content in the basic syllabus is present in AA, however, compulsory credits MAB6 and MAB7 may offer economics content which is not present in AA, though there will be some overlap with AA's financial sub-topics. Furthermore, the basic syllabus contains some areas which are more typical of AI than AA, such as modelling, confidence intervals, and margins of error – though the latter two only appear in the national optional credit MAB9.

In summary, there is partial alignment with AA SL content and no alignment with AA AHL content. Where there is alignment with SL content, this is most significant in 'Number and algebra', 'Functions', and 'Statistics and probability'. Only a few sub-topic alignments are found for 'Geometry and trigonometry' and 'Calculus' – the latter originating from an optional credit. Overall, AA SL content exceeds the basic syllabus in depth and has greater breadth in its compulsory content.

Mathematics: applications and interpretation

The mapping of content reveals that the basic syllabus generally has partial alignment with the main topics of AI SL content. The basic syllabus is partially aligned with 'Number and algebra' SL content by covering arithmetic and geometric sequences and series in similar detail, but not including amortization and annuities or approximation. For SL 'Functions', the basic syllabus does include a focus on modelling with polynomial and exponential functions, however, this is a smaller range of functions than covered by AI. That said, the content references similar modelling skills such as using technology, evaluating usability of models, making comparisons, and making predictions.

As with AA, there is good alignment with 'Statistics and probability' SL content, with the basic syllabus also including concepts of correlation and regression, probability, goodness-of fit, and probability distributions – though study of the binomial and normal distributions is only part of an optional credit. Additionally, there is partial alignment with SL 'Calculus', due to the inclusion of finding derivatives of polynomials and maximum/minimum points; however, it can be noted that these are only covered in the national optional credit MAB8 and do not include integration or optimisation. The basic syllabus has limited alignment with 'Geometry and trigonometry' – only covering some of the simpler sub-topics.

Mostly, the basic syllabus does not include content which is not present in AI. The only notable additional content is that found in the two compulsory economic credits, however, much of the content of these overlaps with AI financial sub-topics.

Overall, there is partial alignment with AI SL content and no alignment with AHL content. All main topics at SL, except 'Geometry and trigonometry', had partial alignment – though all 'Calculus' content is optional. Therefore, the basic syllabus content has less depth than AI SL and HL, and less breadth in its compulsory content.

The table below summarises the basic syllabus content that is not present in either DP mathematics subjects, or only one of them.

*Table 25: FNCC Basic mathematics content which is not covered in the DP**

Significant content not in AA (only)	Significant content not in AI (only)
• Mathematical models (linear, exponential models, and understanding goodness-of-fit) • Further financial applications • Concepts of confidence intervals and margin of error	
Significant content not in either DP mathematics subject	
• Stronger focus on economic applications	

* Significant content does not include topics which are typically studied *prior* to upper secondary

Advanced syllabus

Mathematics: analysis and approaches

The mapping of content shows that the advanced syllabus has strong alignment with AA SL content and some limited alignment with AHL content. Firstly, alignment with ‘Number and algebra’ SL content is strong, as the advanced syllabus includes exponentials, logarithms, binomial theorem, and sequences and series. Furthermore, the syllabus also includes combinatorics, thus this aligns with the similar sub-topic in AHL, however, this is the only alignment with AHL in this topic.

The advanced syllabus aligns strongly with ‘Functions’ SL content due to its coverage of quadratic, exponential, logarithmic, and rational functions. Whilst most SL ‘Functions’ content is present, transformations and composite functions are not included and finding the inverse of functions only appears within the optional credit MAA11.

Again, most ‘Geometry and trigonometry’ SL content is present in the advanced syllabus, including volume and surface area, basic trigonometry and Pythagoras, sine and cosine functions, radians, and solving trigonometric equations. However, transformations and composite functions are again not present in this topic. Regarding AHL content, the advanced syllabus covers an introduction to vectors in its compulsory credits and offers further in-depth study in the optional credit MAA10 – which includes the scalar and cross products and finding vector equations of lines and planes. However, the advanced syllabus coverage of vectors does not appear to be as extensive as AA and also does not extend to include reciprocal trigonometric functions and identities other than the Pythagorean identity.

Similarly, there is strong alignment with SL ‘Statistics and probability’ content, as the advanced syllabus includes central tendency and dispersion, correlation and linear regression, probability, and discrete random variables. The optional credit MA11 also offers study of the normal distribution and continuous random variables, hence this credit creates partial alignment with AA’s ‘Statistics and probability’ AHL content.

Unlike the basic syllabus, studies of calculus are compulsory in the advanced syllabus. These include sub-topics such as derivatives of polynomials and other functions, product and quotient rules, finding maximum and minimum points, definite integrals, and using integration to find area and volume. There is also an optional credit, MAA12, which explores limits and the continuity and differentiability of functions – hence some AHL sub-topics are present. However, a few SL sub-topics are not included, such as the second derivative, kinematic applications, and indefinite integrals. A larger amount of AHL content is also not included, such as differential equations, Maclaurin series, integration methods, and implicit differentiation.

Most advanced syllabus content can be found in AA, with one or two exceptions. The advanced syllabus offers an optional credit, MAA11 ‘Algorithm and number theory’, which includes programming algorithms, concepts of logic, and number theory. This may include some similar concepts to the proof sub-topics in AA, but proof is not stated as a requirement of this optional credit. Furthermore, as with the basic syllabus, there is a stronger focus on modelling and economics in the advanced syllabus than in AA.

In summary, there is strong alignment with AA SL content in all main topics, with the advanced syllabus covering most, though sometimes not all, significant SL sub-topics. Alignment with AA AHL content is considerably weaker, with the most significant alignment originating from the optional module covering vectors. Therefore, the advanced syllabus slightly exceeds AA SL in some areas, though has lesser depth and breadth than AA HL content overall.

Mathematics: applications and interpretation
The mapping of content shows that the advanced syllabus has mostly strong alignment with AI SL content, and partial alignment with AHL content in one main topic. Firstly, some ‘Number and algebra’ SL sub-topics are present in the advanced syllabus, including arithmetic and geometric sequences and series and financial applications, though it is unclear whether annuities and amortization are included. A few AHL sub-topics are also present, specifically these are laws of exponentials and logarithms.

The advanced syllabus includes the modelling of polynomial, exponential, logarithmic and trigonometric functions, hence there is strong alignment with AI SL content in the topic of ‘Functions’. However, there is no alignment with AHL content due to composite functions, transformations, and further complex functions not being covered.

Most SL sub-topics from AI ‘Geometry and trigonometry’ are present in the advanced syllabus, except Voronoi diagrams. There is also a considerable amount of AHL content which includes vectors, radians, trigonometric functions, and identities. However, it can be noted that some vector content is optional and other AHL sub-topics such as matrix transformations, graph theory, adjacency matrices, and decision mathematics are not included. However, it can be noted that the optional credit MA11 ‘Algorithms and number theory’ may develop some similar concepts involving algorithms.

Furthermore, there is strong alignment with SL ‘Statistics and probability’, with the advanced syllabus covering central tendency and dispersion, correlation and linear regression, probability, discrete random variables, the binomial distribution, and, as part of an optional credit, the normal distribution. However, Spearman’s rank, hypothesis testing, chi-squared tests and goodness of fit do not appear to be included in the syllabus. Also, the advanced syllabus does not align with the AHL content in this topic, including nonlinear regression, further hypothesis testing, Markov chains, and Poisson distribution. Similarly, the advanced syllabus has strong alignment with SL ‘Calculus’ content, though does not cover optimisation and most AHL sub-topics, such as differential equations, Euler’s method, slope fields, the second derivative, applications to kinematics, and phase portraits.

There are a few areas which are present in the advanced syllabus and not present in AI. Some of these are sub-topics that can be found in AA, such as polynomial and rational functions, counting principles, continuity and differentiability, and continuous random variables – the last two being part of optional credit MAA12. Furthermore, as previously mentioned, the advanced syllabus also includes economics and an optional credit of MA11 'Algorithm and number theory' which has different content from AI.

In summary, the advanced syllabus is well-aligned with SL content, though usually includes most, rather than all, SL sub-topics. Alignment with AHL is mostly limited to some partial alignment in the topic of 'Geometry and trigonometry'. Therefore, the advanced syllabus in some places slightly exceeds the content of SL, though has considerably less depth and breadth than AI HL content.

The table below summarises the advanced syllabus content that is not present in either DP mathematics subjects, or only one of them.

*Table 26: FNCC Advanced mathematics content which is not covered in the DP**

<table>
<tr><th>Significant content not in AA (only)</th><th>Significant content not in AI (only)</th></tr>
<tr><td>Mathematical modelling using software
Further financial applications</td><td>Rational functions
Absolute value functions and equations
Continuity of functions
Combinatorics
Limits of functions
Differentiability of functions</td></tr>
<tr><th colspan="2">Significant content not in either DP mathematics subject</th></tr>
<tr><td colspan="2">Stronger focus on economic applications
Algorithm and number theory – may contain elements which are in AA or AI (such as properties of prime numbers, divisibility, and use of algorithms)</td></tr>
</table>

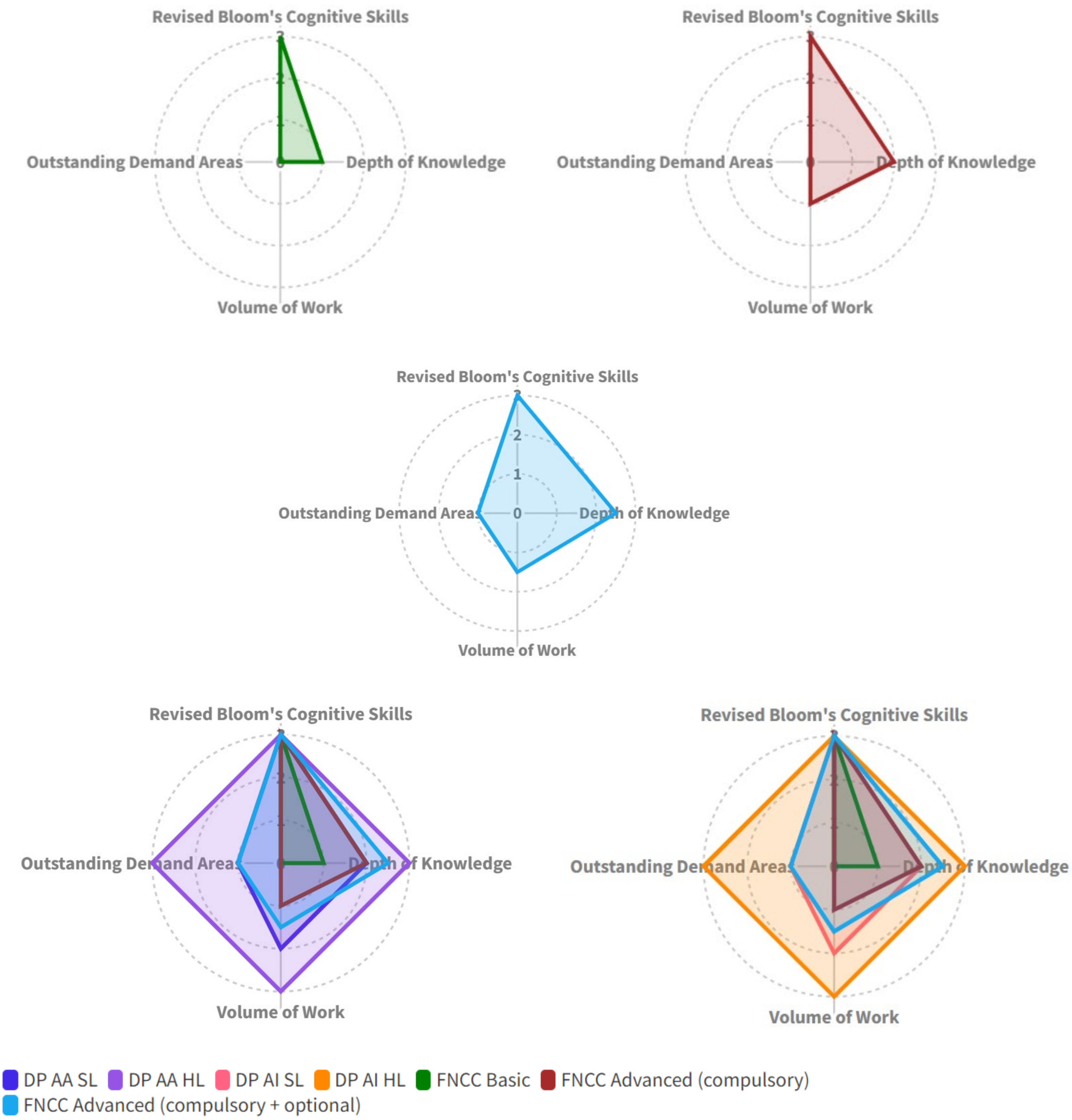
* Significant content does not include topics which are typically studied *prior* to upper secondary

4.4.3 Demand

This section considers the alignment between the DP and FNCC mathematics curricula in terms of demand.

The DP and FNCC curricula were analysed using the same demand tool in order to create a demand profile for AA (SL and HL), AI (SL and HL), FNCC Basic, FNCC Advanced (compulsory) and FNCC Advanced (compulsory and optional). The scores for the advanced syllabus were split into two profiles as the panellists agreed that the scores would slightly differ depending on whether only the compulsory modules were taken, as opposed to all the modules, both compulsory and optional. For the basic syllabus, this split was not deemed necessary and the scores account for all compulsory and optional modules combined. The FNCC demand profiles are presented below in the form of radar diagrams, with the last two diagrams showing both DP mathematics subjects and the FNCC profiles superimposed, enabling immediate visual comparison.

Figure 12: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - DP subjects received a score of 3.
  - Similarly, the FNCC learning outcomes are the same for both the basic and advanced syllabus and were given a score of 3. The general objectives, task of the subject, and transversal competencies together had a strong presence of higher order thinking skills through their focus on students' ability to discuss mathematics,

substantiate arguments, use investigative actions, solve real-world problems, and evaluate mathematical information.

- Regarding the scores for **Depth of Knowledge**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For FNCC, the basic syllabus was given a score of 1 for depth. Generally, the coverage of topics was shallow and often focused on basic concepts. Furthermore, the optional studies did not offer a considerable amount of further depth, as they included the binomial and normal distributions and an introduction to calculus. Hence, complexity was limited and was not on par with DP SL, thus a score of 1 was deemed appropriate. For the advanced syllabus, it was logical to consider two scenarios, (i) compulsory modules only and (ii) compulsory and optional modules combined. For the compulsory modules alone, a score of 2 was given due to topics being covered in considerable detail, which was comparable to that of SL. When considering the compulsory and optional modules together, there is some evidence of further depth, as the optional modules extend learning in vectors and statics and probability, however, this was not deemed enough to warrant a 3, and thus a score of 2.5 was given.

Regarding the scores for **Volume of Work**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For the FNCC, each credit has 14.25 hours of teaching hours allocated. The total teaching time for the basic syllabus (for all modules) is 228 hours, which was deemed a generous amount of time to cover the amount of topics in the syllabus, thus it received a score of 0. For the advanced syllabus, two scenarios were considered again. For the compulsory modules only, a teaching time of 285 hours was deemed a standard amount of time for the breadth and depth of the content described, thus was given a score of 1. For compulsory and optional modules combined, panellists decided that the allocated time of approximately 28 hours for each optional module was slightly more demanding, given the increase in complexity of these modules. To acknowledge this, whilst also considering that most modules had a standard amount of time allocated, a score of 1.5 was deemed appropriate for the combination of compulsory and optional modules.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - DP SL subjects received a score of 1 and HL subjects received a score of 3.
  - For FNCC, no outstanding areas were found in the basic syllabus, which covered content that was typical of upper secondary mathematics, thus was awarded a score of 0 for this category. For the advanced syllabus, the compulsory modules alone did not contain any areas of outstanding demand, thus were given a score of 0. When considering the optional modules combined with the compulsory modules, some areas of outstanding demand emerged in the module of 3D geometry, and thus a score of 1 was deemed appropriate.

4.5 Singapore

The school system in Singapore is overseen by the Ministry of Education (MOE). It is divided into primary school (six years/grades), secondary school (four to five years/grades), and post-secondary/pre-university education (two to three years/grades).[58] This report focuses on the Singaporean GCE A Level (SGA) curriculum taught at junior colleges (JCs) and the Millenia Institute (MI), leading to the GCE A Level examination and qualification[59] and represents the most common route to higher education.

The SGA curriculum is taught over two to three years (two years in JCs and three in MI) and organises courses into three main groups:

- Life skills: co-curriculum activities, character and citizenship education, physical education, and values in action
- Knowledge skills: general paper, knowledge and inquiry, and project work
- Content-based subjects: various courses offered within the subject areas of:
    - languages
    - humanities and the arts
    - mathematics and sciences.[60]

Life skills are not assessed in the GCE A Level examinations.

Each content-based and ‘knowledge skills’ subject will be described by a level. The three possible levels of study are the following:[61]

- Higher 1 (H1) level
- Higher 2 (H2) level
- Higher 3 (H3) level.

H2 is double the size of H1 in terms of curriculum time and thus enables a greater breadth and depth of content to be studied.[62] The syllabi are designed such that students study **either** H1 or H2, they do not study both. In contrast, H2 is a requirement to study H3, as H3 is designed to build on the content of H2 and offers diverse learning opportunities for in-depth study. However, not all subjects are offered at H1, H2 and H3 levels – e.g. the General Paper is only offered as H1.

In order to sit their GCE A Level examinations, students must complete 10 to 12 academic units (AU) of study, where:

- each H1 subject carries 1 AU
- each H2 subject carries 2 AU

[58] MOE, Singapore. (2022). *Post-secondary Education Booklet*. Available from: post-secondary-education-booklet-2022.ashx (moe.gov.sg)
[59] SEAB. (2022). *GCE A Level*. Available from: www.seab.gov.sg/home/examinations/gce-A Level
[60] MOE, Singapore. (2022). *A Level curriculum and subject syllabuses.* Available from: www.moe.gov.sg/post-secondary/A Level-curriculum-and-subject-syllabuses
[61] Ibid.
[62] MOE, Singapore. (2015). *The ‘A’ Level Experience. Levels of Study.* Available from: Ministry of Education Singapore: New 'A' Level Curriculum 2006 (archive.org)

- each H3 subject carries 1 AU (as it is taken in addition to the H2 subject); and
- the total number of AU should not exceed 12.[63]

In practice, this typically means that students take:

- at least four content-based subjects, usually:
  - three H2 content-based subjects
  - one H1 content-based subject
- H1 Mother Tongue Language (MTL) (or MT syllabus B in some circumstances[64])
- H1 General Paper, or H2 Knowledge and Inquiry in lieu – the latter counting as a fourth H2 subject.[65]
- H1 Project Work.

It is required that at least one of the four content-based subjects is from a contrasting discipline.

Hence, the SGA does not require students to study mathematics, but for those wishing to do so, the following courses are offered and will be used in the analysis:

SGA H1 mathematics[66]

H1 mathematics is designed to provide students with a foundation in mathematics and statistics that will support their business or social sciences studies at university. It is particularly suitable for students without an Additional Mathematics[67] background because it offers an opportunity for them to learn important mathematical concepts and skills in algebra and calculus that are covered in Additional Mathematics.

SGA H2 mathematics[68]

H2 mathematics is designed to prepare students for a range of university courses, including mathematics, sciences and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through the applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world. For this level, it is assumed that students have knowledge from Additional Mathematics.

SGA H2 further mathematics[69]

H2 further (H2F) mathematics is designed for students who are mathematically-inclined and who intend to specialise in mathematics, sciences, engineering or disciplines with higher demand on mathematical skills. It extends and expands on the range of mathematics and

[63] SEAB. (2022). *2022 Singapore-Cambridge GCE A Level Examinations - Registration Information for School Candidates.* Available from: [2022 Instructions For School Candidates (seab.gov.sg)]

[64] Students should sit the Year-End mother tongue syllabus B examination if they did not obtain at least D7 in GCE O Level MTL or take MTL Syllabus B at GCE O Level.

[65] SEAB. (2022). *2022 Singapore-Cambridge GCE A Level Examinations - Registration Information for School Candidates.*

[66] MOE, Singapore. (2019). *Mathematics Syllabus. Pre-University. Higher 1. Syllabus 8865.* Available from: 2020-pre-university-h1-mathematics.ashx (moe.gov.sg)

[67] Additional Mathematics is an O Level which students can take prior to upper-secondary.

[68] MOE, Singapore. (2019). *Mathematics Syllabus. Pre-University. Higher 2. Syllabus 9758.* Available from: 2020-pre-university-h2-mathematics.ashx (moe.gov.sg).

[69] MOE, Singapore. (2019). *Mathematics Syllabus. Pre-University. Higher 2. Syllabus 9649.* Available from: 2020-pre-university-h2-further-mathematics.ashx (moe.gov.sg)

statistics topics in H2 mathematics and provides these students with a head start in learning a wider range of mathematical methods and tools that are useful for solving more complex problems in mathematics and statistics. H2F is to be offered with H2 as ‘double mathematics’.

SGA H3 mathematics[70]
H3 mathematics provides students who intend to pursue mathematics at university with an insight into the practices of a mathematician. It equips students with the knowledge and skills to understand and write mathematical statements, proofs and solutions, and the techniques and results helpful in their work. Students will develop these competencies through proving mathematical results and solving non-routine mathematical problems during their learning. H3 must be offered with H2, from which knowledge of the content is assumed.

4.5.1 Learning Outcomes

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics applications and interpretation. The learning outcomes for the SGA are represented by the ‘mathematics curriculum framework’ (MCF), the ‘21st century competencies’ (21CC), and the ‘syllabus aims’. The MCF and the 21CC are applicable to all subjects in the curriculum (H1, H2, H2F, and H3), whereas the syllabus aims are unique to each subject. 21CC are to be developed across the curriculum but are adapted specifically for mathematics in the subject syllabi. This section will first review the learning outcomes relevant to all subjects, followed by the learning outcomes specific to each subject.

The following summary table demonstrates the learning outcome themes that were extracted from DP mathematics and indicates if and where they were judged to have presence within the learning outcomes of the SGA mathematics curricula.

[70] MOE, Singapore. (2019). *Mathematics Syllabus. Pre-University. Higher 3. Syllabus 9820.* Available from: 2020-pre-university-h3-mathematics.ashx (moe.gov.sg)

Table 27: Presence of the DP mathematics subject group learning outcome themes in SGA curricula

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the SGA	
1. Being aware of, and engaging with, mathematics in its wider context		Present in the 21CC.
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Present in the 'metacognition' and 'attitudes' components of the MCF.
3. Using inquiry-based approaches		Not present.
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present in the syllabus aims of H1, H2, H2F, and H3. Also present in the 'concepts', 'attitudes', and 'processes' components of the MCF.
5. Making links and generalisations		Present most strongly in the 'processes' component of the MCF, which encompasses connections and thinking skills. Making connections is also evident across the syllabus aims.
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Present in the MCF, of which problem-solving is the center. Also present in the syllabus aims, especially H3.
7. Communicating mathematics clearly and in various forms		Present in the MCF, 21CC, and syllabus aims.
8. Knowing how technology and math influence each other and using technology to develop ideas and solve problems		Present in the 'skills' component of the MCF and the 21CC.

Key:

	This theme is well-evidenced in the learning outcomes of the SGA.		*This theme is partially evidenced in the learning outcomes of the SGA.*		*This theme is not evident in the learning outcomes of the SGA.*

Presence of the DP's Learning Outcome Themes

All subjects:

There is a strong presence of the DP's mathematics learning outcome themes in the syllabi of the SGA. Like the DP, Singapore has a strong emphasis on problem-solving, which is at the core of their MCF. Indeed, developing problem-solving skills is the central focus of the SGA's mathematics curriculum, and is supported by five inter-related components, namely 'concepts', 'skills', 'processes', 'metacognition' and 'attitudes'. These supporting components are also coherent with DP learning outcome themes. For example, 'attitudes and metacognition' contain very similar concepts to the DP, such as the development of interest, enthusiasm, reflective working, perseverance, and confidence in students. Furthermore, 'processes' includes 'reasoning, communication, and connections', which correlate with the respective DP themes of critical thinking skills, communication, and making links and generalisations. More specifically, the MCF expects that students will be able to analyse and construct mathematical arguments; express ideas and arguments logically, accurately, and precisely; and make connections between mathematical concepts, to other subjects, and to real life. Critical thinking skills and making links are further represented by the two other parts of the 'processes' component, which are 'applications and modelling' and 'thinking skills and heuristics'.

In addition, the DP's theme of understanding mathematics is demonstrated through the components of 'concepts' and 'attitudes', whereby it is expected that students will learn a broad range of mathematical concepts and have appreciation of the beauty and power of mathematics. Another theme accounted for is that of technology, as the 'skills' component describes the use of tools, spreadsheets, and software. The last DP theme identified is that of wider contexts, which is found in the 21CC. Indeed, this theme expects that students will have an awareness of, and ability to engage with, the local and global issues around them. The 21CC also reinforce some of the previously mentioned themes by referring to thinking critically, reasoning logically, communicating effectively, using ICT tools, and working individually as well as collaboratively.

Finally, the only DP learning outcome theme not explicitly identified in the MCF or 21CC is that of inquiry-based approaches. SGA syllabi do not specify that students are expected to investigate, make conjectures, or test for validity. However, inquiry-based learning is referred to in the 'Teaching Processes'.

H1 and H2/H2F:
The 'syllabus aims' for H1, H2, and H2F are very similar, with the main difference being the associated outcomes of each – tertiary study into the fields of business and social science for H1 and mathematics, sciences, engineering for H2/H2F. The aims for each syllabus correlate to DP themes through demonstration of some of the previously mentioned expectations such as understanding mathematical concepts, critical thinking skills, making connections, and appreciating the value of mathematics.

H3 only:
The 'syllabus aims' for H3 differ to those for H1, H2, and H2F, though refer to some of the same themes. H3 aims focuses on advanced skills, such as complex problem-solving in non-routine contexts, learning methods of proof, and developing mathematical rigour and precision. Therefore, these aims reinforce the presence of the DP themes of problem-solving, reasoning, making generalisations, and communicating mathematics.

Other Themes in the SGA
Though the learning outcomes taken from the MCF and the 21CC often relate to a theme extracted from the DP, some have unique elements which are not replicated. These differences are present in the MCF, in the 'metacognition' and 'skills' sections, and two sub-areas of 'processes', specifically 'thinking skills and heuristics', and 'applications and modelling'. For instance, though the DP places emphasis on students reflecting on their work, this differs from metacognition – which goes further to involve the ability to monitor thinking processes and self-regulate learning. Furthermore, 'Thinking skills and heuristics' from the 'processes' component includes different skills, such as identifying patterns, visualization, and heuristics, which are not explicitly mentioned in the DP. Similarly, the 'skills' component includes other considerations to those in the DP, such as spatial visualization. Finally, 'Applications and modelling' from the 'processes' component places a greater focus on modelling than the DP does in its learning outcomes and expects that students will learn to deal with complexity and ambiguity through mathematical modelling.

H3 only:

Though the learning outcomes represented by H3's syllabus aims correlate with the DP's themes of critical thinking skills and communication, they place higher emphasis on the advanced skills within these themes. This can be partially attributed to SGA giving level-specific outcomes, which the DP does not do. That said, H3 has a higher focus on readiness for university-level mathematics than the DP, meaning that skills such as mathematical rigour and methods of proof will have a stronger emphasis.

Summary

Overall, there is a high degree of alignment between the learning outcomes of the DP's mathematics subjects and the SGA's. Nearly all the DP's themes are well-evidenced in the SGA learning outcomes, with the exception of inquiry-based approaches – though these can be found in the 'Teaching Processes'. Generally, the outcomes in the SGA have a higher level of detail than the DP's, especially with the inclusion of course-specific 'syllabus aims'. Partially due to the detailed nature of SGAs outcomes, a slightly greater number of skills are included. However, metacognition skills and some of H3's advanced skills, such as mathematical rigour, do have more emphasis than in the SGA than the DP.

4.5.2 Content

This section compares and contrasts the content of the DP and SGA curricula falling within the category of mathematics. In order to support visual comparison at-a-glance, the SGA mathematics subjects are presented below in a diagram which shows the key topics and sub-topics included in each.

Figure 13: SGA mathematics content visualiser for H1, H2, H2F, and H3

H1 mathematics	**Section A: Pure mathematics**	**1 Functions and graphs**	1.1 Exponential and logarithmic functions and graphing techniques	1.2 Equations and inequalities				
		2 Calculus	2.1 Differentiation	2.2 Integration				
	Section B: Probability and statistics	**3 Probability and statistics**	3.1 Probability	3.2 Binomial distribution	3.3 Normal distribution	3.4 Sampling	3.5 Hypothesis testing	3.6 Correlation and linear regression
H2 mathematics	**Section A: Pure mathematics**	**1 Functions and graphs**	1.1 Functions	1.2 Graphs and transformations	1.3 Equations and inequalities			
		2 Sequences and series	2.1 Sequences and series					
		3 Vectors	3.1 Basics properties of vectors in two-and three-dimensions	3.2 Scalar and vector products in vectors	3.3 Three-dimensional vector geometry			
		4 Introduction to complex numbers	4.1 Complex numbers in cartesian form	4.2 Complex numbers and expressed in polar form				
		5 Calculus	5.1 Differentiation	5.2 Maclaurin series	5.3 Integration techniques	5.4 Definite integrals	5.5 Differential equations	
	Section B: Probability and statistics	**6 Probability and statistics**	6.1 Probability	6.2 Discrete random variables	6.3 Normal distribution	6.4 Sampling	6.5 Hypothesis testing	6.6 Correlation and linear regression
H2 further mathematics	**Section A: Pure mathematics**	**1 Algebra and calculus**	1.1 Mathematical inductions	1.2 Complex numbers	1.3 Polar coordinates	1.4 Conic sections	1.5 Applications and definite integrals	1.6 Differential equations
		2 Discrete mathematics, matrices and numerical methods	2.1 Recurrence relations	2.2 Matrices and linear spaces	2.3 Numerical methods			
	Section B: Probability and statistics	**3 Probability and statistics**	3.1 Discrete random variables	3.2 Continuous random variables	3.3 Hypothesis testing and confidence intervals	3.4 Non-parametric tests		

H3 mathematics	*Students learn to prove properties and results, and solve non-routine problems involving:* **(1) H2 mathematics content areas** **(a) Functions,** e.g. graphs, symmetries, derivatives, integrals, differential equations, limiting behaviours, bounds **(b) Sequences and series,** e.g. general terms, sum, limiting behaviours, bounds *The examples in (a) and (b) illustrate some types of problems that are based on content in H2 mathematics.* **(2) Additional content areas** **(a) Inequalities:** AM-GM inequality, Cauchy-Schwarz inequality, triangle inequality **(b) Numbers:** primes, coprimes, divisibility, greatest common divisor, division algorithm, congruences and modular arithmetic **(c) Counting:** distribution problems, Stirling numbers of the second kind, recurrence equations, bijection principle, principle of inclusion and exclusion. *The above define the expected scope of content knowledge that may be assessed.* *Notwithstanding the content areas defined above, students will also prove results and solve problems outside these defined areas or at the intersection of two or more such areas using their ability to understand and apply given definitions or results.*

4.5.2.1 Structure

Like the DP, Singapore offers different levels to students studying mathematics – H1, H2, H2F, and H3. The content in each increases in complexity, with H2, H2F, and H3 all aiming towards the higher-end of upper-secondary mathematics and H1 offering a more basic, though still substantial, coverage of upper-secondary topics. Therefore, for students who are able to study mathematics at a higher level, it can be noted that SGA offers slightly more options (H2, H2F, and H3) than the DP, which offers AA HL or AI HL. SGA students can also study a larger number of mathematics subjects, as they can take H2, H2F, and H3 (or H2 and one other), whereas DP students may only study one HL subject, as they cannot take both AA and AI. Moreover, Singapore offers a subject (H3) which is dedicated to developing skills and methods for students who are specifically intending to study mathematics at university. Though the DP aims to prepare students for university study, H3 more closely resembles some of the material and skills students will use in higher education mathematics. In terms of subjects that take an applied focus, the SGA somewhat offers this in the form of H1. H1 assessment contains 40% pure mathematics and 60% applied mathematics (statistics and probability), thus being similar to AI in that statistics and probability is the largest topic area. However, unlike the DP, an 'applied-focus' subject is not offered at a higher level; instead, the higher levels incorporate some applied content – except for H3, which focuses primarily on abstract concepts and methods of proof.

With regard to main topics, there are strong similarities between the DP and Singapore's subjects. Just as 'Number and algebra', 'Functions', 'Geometry and trigonometry', 'Statistics and probability', and 'Calculus' will be taken by all students studying the DP, the topics of 'Functions', 'Calculus', and 'Statistics and probability' will be covered by all students taking SGA mathematics – regardless of the level chosen. Interestingly, however, geometry and trigonometry content does not feature as a main topic in the SGA, as most of this content is covered in prior learning (GCE Mathematics O Level and GCE Additional Mathematics O Level) – instead, 'Vectors' is a main topic in H2. Also, unlike the DP, though all students will study the main topics described above, there are other topics which vary depending on the level; they are not the same for all subjects as they are in the DP.

Another key difference is the way in which content follows on from previous studies. Whereas additional higher level (AHL) content follows on from SL in the DP HL, H2 does not follow on from H1 in the SGA. Instead, H1 content directly follows on from GCE O Level Mathematics and H2 content directly follows on from GCE O Level Additional Mathematics; therefore, either H1 *or* H2 is studied, not both. However, students must study H2 to study H2F and H3.

Finally, whereas H1, H2, and H2F break content down into main topics and sub-topics akin to the DP structure, H3 takes a different approach. The content detail for H3 is less specific and focuses on the skills of mathematical communication, advanced problem-solving, and methods of proof, rather than specific topics. Nevertheless, the syllabus does give examples of the areas to which mathematical proofs, results, and advanced problem-solving will relate.

4.5.2.2 Content Alignment

See the table below for a simplified summary of the extent to which SGA mathematics aligns with the main topics of the DP's subjects. As indicated in the content structure section above, various topics are not present in H1, H2, H2F, and H3 themselves, but are featured in the content of O Levels, which, although not strict pre-requisites, contain content that is regarded as assumed knowledge by the SGA subjects. H1 and H2 assume knowledge from O Level Mathematics and H2 also assumes knowledge from O Level Additional Mathematics. The A Level syllabi are then designed to follow on from the content of O Level. The analysis has taken this into account and carries forward the O Level alignments to represent the cumulative content learnt by students studying the SGA. As Additional Mathematics contains a significant amount of overlapping content with the DP, it has been included in the table. The topics where prior learning has impacted alignment significantly will be noted in the analysis.

Table 28: Summary of the content alignment SGA has with the main topics in AA

Mathematics: analysis and approaches topics		Prior learning	Presence in SGA mathematics			
		Additional Mathematics	H1	H2	H2F	H3
SL	1. Number and algebra					
	2. Functions					
	3. Geometry and trigonometry					
	4. Statistics and probability					
	5. Calculus					
AHL	1. Number and algebra					
	2. Functions					
	3. Geometry and trigonometry					
	4. Statistics and probability					
	5. Calculus					

Table 29: Summary of the content alignment the SGA has with the main topics in AI

Mathematics: applications and interpretation topics		Prior learning	Presence in SGA mathematics			
		Additional Mathematics	H1	H2	H2F	H3
SL	1. Number and algebra					
	2. Functions					
	3. Geometry and trigonometry					
	4. Statistics and probability					
	5. Calculus					
AHL	1. Number and algebra					
	2. Functions					
	3. Geometry and trigonometry					
	4. Statistics and probability					
	5. Calculus					

Key:

	Strong presence of this topic in the SGA.		*Partial presence of this topic in the SGA.*		*Little or no presence of this topic in the SGA.*

* Where applicable, content alignments found in assumed knowledge or pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

H1 Mathematics

Mathematics: analysis and approaches

The mapping of content shows that sub-topic alignments for H1 are usually located in the SL content of AA, to varying degrees depending on the topic. The main topic with the most alignment is that of SL 'Statistics and probability', as H1 covers many similar subtopics, including presenting data, regression and correlation, probabilities, and probability distributions. Following this, H1 has a considerable number of sub-topic alignments with

'Functions' SL, due to inclusion of function notation, graphing of functions, and a range of functions, such as exponential and logarithmic. However, the terms 'domain' and 'range' are excluded, as are transformations and rational, reciprocal, composite, and inverse functions.

H1 also has partial alignment with 'Geometry and trigonometry' SL, due to its inclusion of some basic geometry and trigonometry sub-topics, and the exclusion of trigonometric functions, identities, and equations. Moreover, it can be noted that the sub-topic alignments identified are not in H1 but are carried over from prior learning in O Level Mathematics. There is substantial alignment with 'Calculus' SL, as H1 introduces differentiation and integration. However, sub-topics regarding applications and the second derivative are not included, and other sub-topics have partial alignment due to less coverage (for example, only the chain rule is included).

The SL topic that H1 has the fewest sub-topic alignments with is 'Number and algebra'. Though H1 includes a basic coverage of exponents and logarithms, it does not cover sequences and series. However, there are a few statistical areas covered by H1 which are not included in AA. These are sampling and hypothesis testing, which are topics that appear in AI.

Overall, H1 mostly aligns with AA SL content, particularly in the topics of 'Statistics and probability', 'Calculus', and 'Functions'. Therefore, H1 has similar depth to SL in some topics; however, it has less breadth and is overall smaller in content size than SL. There are no significant alignments with AHL content.

Mathematics: applications and interpretation

The mapping of content shows that the sub-topic alignments for H1 are usually located in the SL content of AI, to varying degrees depending on the topic. A main topic which H1 has strong alignment with is that of 'Calculus' SL, as H1 introduces differentiation and integration, including definite integrals, finding maximum and minimum points, and tangents and normals. Similarly, H1 has strong alignment with 'Statistics and probability' SL due to its inclusion of presenting data, correlation, linear regression, probabilities, probability distributions, and hypothesis testing.

H1 has reasonable alignment with 'Geometry and trigonometry' SL sub-topics; however, these come from prior learning (GCE O Level) and do not include Voronoi diagrams. There is partial alignment with 'Functions' SL due to the graphing of functions. Nevertheless, modelling of functions is not a focus of H1, thus this significantly impacts alignment with this topic. The topic with the least alignment is 'Number and algebra', as sequences and series are not covered in H1.

Overall, H1 mostly aligns with AI SL content, particularly with the topics of 'Calculus' and 'Statistics and probability'. H1 has comparable depth to SL in certain topics, though sometimes differs in its coverage of the topic, and generally is smaller in breadth and overall size. There are occasional sub-topic alignments with AHL content (such as laws of logarithms), though these are generally insignificant.

*Table 30: SGA H1 mathematics content which is not covered in the DP**

Significant content not in AA (only)	Significant content not in AI (only)
• Sampling (sample mean as a random variable, use of the Central Limit Theorem, unbiased estimates) • Hypothesis testing (formulation of hypotheses and testing for a population mean)	• Combinations and permutations
Significant content not in either DP mathematics subject	
N/A	

* Significant content does not include topics which are typically studied *prior* to upper secondary.

H2 Mathematics

Mathematics: analysis and approaches

The mapping of content shows that H2 has a considerable number of SL and AHL sub-topic alignments with AA. Firstly, there is strong alignment within ‘Geometry and trigonometry’ content, as vectors are covered in similar depth and detail in H2. Furthermore, there are sub-topic alignments with trigonometric functions (including reciprocal), trigonometric equations, radians, and trigonometric identities – though it can be noted that these are not in the syllabus of H2 but are covered prior in the O Level Additional Mathematics. There is also strong alignment with the SL and AHL content of ‘Functions’, as H2 has an equally wide range of functions and includes composite and inverse functions, transformations, analytical solutions, absolute value, and rational functions.

There is also strong alignment with SL, and most AHL, content of ‘Calculus’, as H2 includes the second derivative, methods of integration, differential equations, and Maclaurin series. Though, again, it can be noted that several of the SL sub-topics alignments arise from prior learning in the Additional Mathematics. Furthermore, the concept of a limit is not introduced in H2, therefore there are some missing sub-topic alignments regarding this, such as evaluation of limits and differentiation from first principles. Again, for ‘Number and algebra’ there are strong alignments with SL and some AHL content, as H2 covers sequences and series in a similar level of detail, as well as laws of logarithms and rational exponents. For AHL content, counting principles, systems of equations, and complex numbers are present in H2; however, complex numbers is covered in lesser detail than AA and proof by contradiction and induction is not included.

For ‘Statistics and probability’, there is strong alignment with SL content, as discrete random variables, linear regression, and probability distributions are all included in H2, though there is no AHL content covered, such as continuous random variables. However, there are a few statistical areas covered by H2 which are not included in AA. These are ‘Sampling’ and ‘Hypothesis Testing’, which respectively correlate to AHL and SL sub-topics in AI.

Overall, H2 has strong alignment with SL and AHL content, particularly in the topics of ‘Geometry and trigonometry’ and ‘Functions’. Therefore, H2 content exceeds AA SL in depth, and is more comparable to that of AA HL.

Mathematics: applications and interpretation

The mapping of content shows that H2 has strong alignment with AI SL content and generally partial alignment with AHL content. For 'Functions', there is partial alignment with AHL content, as H2 includes transformations and composite and inverse functions but does not include modelling with AHL functions. There is considerable alignment with SL 'Functions' due to modelling with linear, quadratic, exponential, logarithmic, and trigonometric functions being covered in prior learning (GCE O Level Additional Maths).

For 'Number and algebra', H2 has considerable alignment with SL, as sequences and series are covered in similar depth and detail, and financial applications are listed in the 'Applications and Contexts' section of the syllabus. Furthermore, logarithms and exponentials are covered, as are complex numbers (for the most part); however, matrices are only covered in basic detail (from O Level Mathematics) and eigenvalues and eigenvectors are not included. Similarly, there is strong alignment with SL 'Geometry and trigonometry' content and partial alignment with AHL content. In this area, vectors, trigonometric functions, and radians are included, though matrix transformations and graph theory and decision mathematics sub-topics are not. Additionally, H2 has strong alignment with SL 'Calculus' and some AHL content. Indeed, H2 includes the second derivative, area of a region and volumes of revolution, applications to kinematics, and some differential equations. However, slope fields, Euler's Method, and phase portraits are not included. For 'Statistics and probability', there is strong alignment with SL content as correlations and linear regression, probabilities, discrete random variables, probability distributions, and hypothesis testing are all included. Furthermore, there are a few sub-topic alignments with AHL content as H2 covers 'Sampling', though not enough to be considered significant – H2 does not cover non-linear regression, transition matrices, confidence intervals, and has lesser detail in hypothesis testing.

However, there are a few topics which H2 covers in more or different detail to AI – these are vectors, functions, and trigonometric functions, which more closely resemble AA content. Overall, there is strong alignment with SL content and partial-strong alignment with AHL (due to the exclusion of some significant topic areas). In terms of depth, H2 content exceeds AI SL and is comparable to AI HL, albeit sometimes covering different advanced content.

*Table 31: SGA H2 mathematics content which is not covered in the DP**

Significant content not in AA (only)	**Significant content not in AI (only)**
• Sampling (sample mean as a random variable, use of the Central Limit Theorem, unbiased estimates) • Hypothesis testing (formulation of hypotheses and testing for a population mean)	• Solving inequalities • Absolute value • Further three-dimensional vector geometry content e.g. equations of planes • Maclaurin series • Further integrals and integration techniques • Permutations and combinations
Significant content not in either DP mathematics subject	
N/A	

* Significant content does not include topics which are typically studied *prior* to upper secondary

H2 Further Mathematics

This subject is taken with H2 mathematics, combining to create double mathematics. All of the alignments found within H2 still apply (see above); therefore, this section will focus on where H2F content strengthens alignments, or covers topics not included in the DP.

Mathematics: analysis and approaches

Topics in H2F which are relevant to AA and strengthen alignment with AHL content are 'Mathematical Induction', 'Complex Numbers', 'Differential Equations', 'Numerical Methods', and 'Continuous Random Variables', which strengthen alignment with 'Number and algebra', 'Calculus', and 'Statistics and probability'. Overall, almost the entire content of AA can be identified in the SGA when combining the content of H2 and H2F.

However, there is a considerable number of H2F topics which are of an advanced level, but not included in AA. These are 'Polar Coordinates', 'Conic Sections', 'Recurrence Relations', 'Matrices and Linear Spaces', 'Discrete Random Variables (Poisson and Geometric)', 'Hypothesis Testing and Confidence Intervals', and 'Non-parametric Tests'. Furthermore, some topics have similarities with AA but are covered in slightly more or different detail, such as 'Numerical Methods', 'Differential Equations', and 'Applications of Definite Integrals'.

Therefore, students taking H2F are likely to encounter a greater breadth and depth of content than those taking AA HL.

Mathematics: applications and interpretation

Topics in H2F which are relevant to AI and strengthen alignment with AHL content are 'Complex Numbers', 'Differential Equations', 'Matrices and Linear Spaces', 'Numerical Methods', 'Discrete Random Variables (Poisson and Geometric)', and 'Hypothesis Testing and Confidence Intervals'. Like AI content, 'Matrices and Linear Spaces' covers eigenvalues/ eigenvectors and matrix transformations, whilst also including the concepts of nullity, rank, basis, independence, and dimension. Overall, H2F strengthens alignment with 'Number and algebra', 'Statistics and probability', and 'Calculus' – enabling the majority of AI content to be identified in SGA. However, some sub-topics, such as non-linear regression, graph theory, decision mathematics, Markov chains, and phase portraits, are still not present in H2F.

There is a considerable number of H2F topics which are of an advanced level but are not present in AI. These are 'Mathematical Induction', 'Polar Coordinates', 'Conic Sections', 'Recurrence Relations', 'Continuous Random Variables', and 'Non-parametric Tests'. Furthermore, 'Matrices and Linear Spaces', 'Complex Numbers', 'Applications of Definite Integrals', and 'Numerical Methods' all share similarities to AI, though include some additional details and concepts which are not present in the latter.

Overall, students taking H2F are likely to encounter a greater breadth and depth of content than those taking AI HL.

*Table 32: SGA H2F mathematics content which is not covered in the DP**

Significant content not in AA (only)	Significant content not in AI (only)
• Second order differential equations • Matrices • Eigenvalues and eigenvectors • Approximation of solutions of first order differential equations using Euler method • Poisson distribution • Hypothesis testing and confidence intervals (formulation of hypotheses, testing for a population mean using t-test, difference of population means, contingency tables and χ^2 tests of independence and goodness of fit, and confidence intervals for population mean and proportion)	• Mathematical induction • De Moivre's theorem • Continuous random variables (probability density function, mean, and variance)
Significant content not in either DP mathematics subject	
• Complex numbers – small amount of additional content • Polar coordinates • Conic sections • Surface area of revolution • Recurrence relations • Linear spaces – additional content • Numerical methods – small amount of additional content • Geometric distribution • Non-parametric tests (formulation of hypotheses and testing for: a population median using Sign test and identical probability distributions for two sampled populations in a paired difference design using Wilcoxon matched-pair signed rank test)	

*Significant content does not include topics which are typically studied *prior* to upper secondary. Furthermore, content listed in this table is specific to H2F. As H2 is offered alongside H2F, all topics in the H2 table should be considered in addition to the topics in this table.

H3 Mathematics

H3 is offered with H2 and gives students who intend to study mathematics at university the opportunity to develop and practice the skills needed. Essentially, this subject aims to introduce students to the practices of a mathematician, including thinking mathematically, applying mathematical rigour to work, advanced problem-solving, and being able to prove statements and results. This way of working is usually not seen until university; thus, H3 is likely to support students in their first year of higher education.

H3 states that these skills are applied in some areas taken from H2 ('Functions' and 'Sequences and Series') and additional areas ('Inequalities', 'Numbers', and 'Counting'). As part of this, students learn new language (such as converse, theorem, definition, and conditional statements) and new notation (such as set notation, logical connectives, and existential and universal qualifiers). In terms of proof, students will cover a broad range of techniques, such as proof by induction, contradiction, counterexample, and construction. For this reason, and the abstract nature of it, this subject is more akin to AA than to AI. However, H3 goes significantly beyond the AA content regarding proof – as can be seen by the size of the subject and the wider range of areas with which mathematical rigour and proof are applied.

*Table 33: SGA H3 mathematics content which is not covered in the DP**

<table>
<tr><th>Significant content not in AA (only)</th><th>Significant content not in AI (only)</th></tr>
<tr><td></td><td>Methods of proof (induction, counterexample, and contradiction)
Further mathematical language</td></tr>
<tr><th colspan="2">Significant content not in either DP mathematics subject</th></tr>
<tr><td colspan="2">Further mathematical properties, results and proofs in the areas below
<u>(1) H2 mathematics content areas</u>
(a) Functions: graphs, symmetries, derivatives, integrals, differential equations, limiting behaviours, bounds.
(b) Sequences and series: general terms, sum, limiting behaviours, bounds.
<u>(2) Additional content areas</u>
(a) Inequalities: AM-GM inequality, Cauchy-Schwarz inequality, triangle inequality
(b) Numbers: primes, coprimes, divisibility, greatest common divisor, division algorithm, congruences and modular arithmetic
(c) Counting: distribution problems, Stirling numbers of the second kind, recurrence, equations, bijection principle, principle of inclusion and exclusion, pigeonhole principle.

The syllabus also states ‘Notwithstanding the content areas defined above, students will also prove results and solve problems outside these defined areas or at the intersection of two or more such areas using their ability to understand and apply given definitions or results’.</td></tr>
</table>

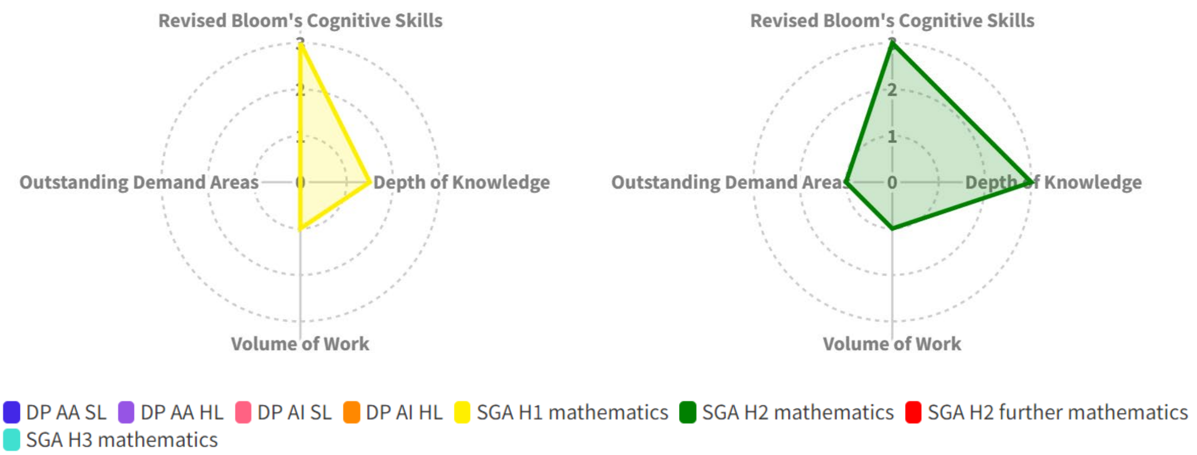
*Significant content does not include topics which are typically studied *prior* to upper secondary. Furthermore, content listed in this table is specific to H3. As H2 is offered alongside H3, all topics in the H2 table should be considered in addition to the topics in this table.

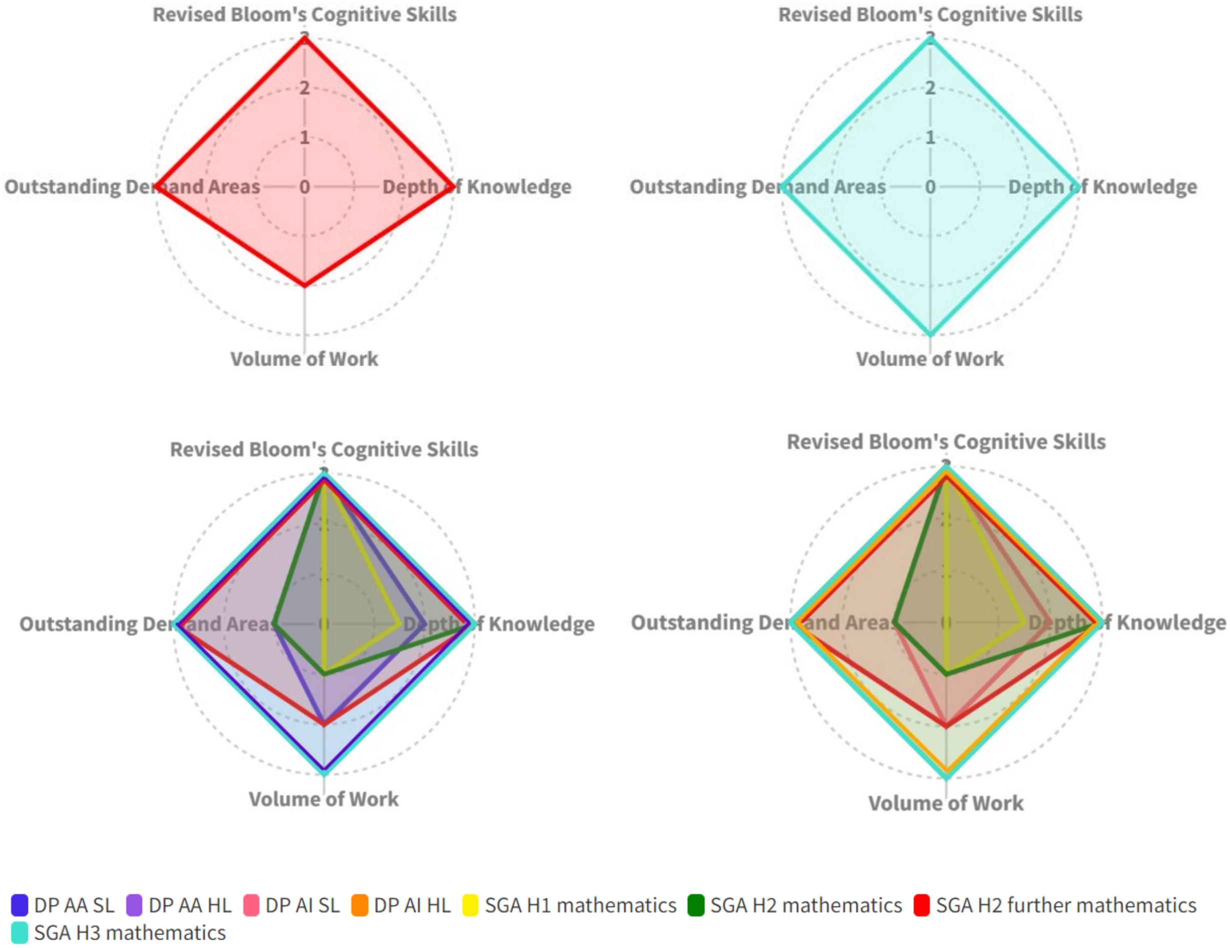
4.5.3 Demand

This section considers the alignment between the DP and SGA mathematics curricula in terms of demand.

The DP and SGA curricula were analysed using the same demand tool in order to create a demand profile for AA (SL and HL), AI (SL and HL), H1, H2, H2F, and H3. The SGA demand profiles are presented below in the form of radar diagrams, with the last two diagrams showing DP subjects and SGA subjects superimposed, enabling immediate visual comparison.

Figure 14: Visual representations of subject demand





The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - DP subjects received a score of 3.
  - The learning outcomes of the SGA were drawn from three sources: the syllabus aims, the MCF, and the 21CC. The panel judged that, although the syllabi aims increased in evidence of higher-cognitive skills from H1 to H3, that the skills listed in the MCF and 21CC, which apply to all subjects, were sufficient to justify a score of 3 for all SGA subjects. In these components, skills of reasoning, analysis, evaluation, critical thinking, generalisation, and metacognition were present. H2F further evidenced this level with references in its syllabus aims to appreciating rigour and abstraction, as did H3 with references to advanced problem-solving skills, rigour in mathematical argument, and writing and evaluation of proofs.

- Regarding the scores for **Depth of Knowledge**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For SGA, H1 received a score of 1.5 as some topics are studied in considerable depth, in cases comparable to that of SL; however, there was not enough evidence

of depth to warrant a score of 2. For the rest of the SGA subjects, a score of 3 was given for depth. H2 required a large amount of pre-requisite knowledge from Additional Mathematics O Level and covered only new content, i.e. prior learning was not re-visited. Furthermore, all topics were studied in a high level of detail – comparable to the DP's HL subjects. Also scoring a 3, H2F focuses on advanced topics, either extending on topics in H2 or introducing new areas which are also explored in depth. Finally, H3 also received a 3 due to the nature of the subject being generally akin to first-year university mathematics, developing practices of a mathematician through focusing on rigor, proof, and advanced problem-solving.

- Regarding the scores for **Volume of Work**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For the SGA, there are difficulties in judging the volume as it varies based on where students will take the qualification. For students attending junior colleges, the programme has a length of two years, whereas in the millennia institute it is three years. Therefore, students in the millennia institute will have a longer period to study their subjects, which impacts the demand of volume. Likely as a result of these differences, the MOE do not publish guidelines for the teaching hours of each subject, therefore scores can only be approximations. For the following scores, it was assumed that the subjects are studied as part of a junior college two-year programme. H1 can be studied over one year, which the panel deemed to be a standard amount of time for the content and requirements of the course. Similarly, H2 was given a score of 1, as two years was deemed a standard amount of time. H2F was raised to a score of 2, due to the majority of time being spent on new and complex topics. Finally, H3 subjects are not required to be taken, therefore they are studied on top of the regular number of subjects, hence the panel speculated that this would likely increase the volume of work considerably enough to warrant a score of 3 – also considering the advanced nature of a H3 subject.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - DP SL subjects received a score of 1 and HL subjects received a score of 3.
  - For SGA subjects, a score of 0 was given to H1, as all content was typical of upper secondary mathematics. For H2, a score of 1 was given, with 3D geometry and calculus containing sub-topics which demonstrated opportunity for stretch. H2F was given a score of 3 as it contained a large number of topics that were beyond the usual scope of upper secondary mathematics, such as matrices and linear spaces, polar coordinates, conic sections, second-order differential equations, and complex numbers (De Moivre's theorem). Finally, H3 was also given a 3 due to requiring students to prove properties and results and solve advanced problems in areas such as sequences and series, functions, numbers, inequalities, and combinatorics.

4.6 South Korea

The school system in South Korea is overseen by the Ministry of Education (MOE – 교육부).[71] It is divided into primary school (grades 1-6, typically from ages 6 to 12), middle school or lower secondary school (grades 7-9, typically from ages 12 to 15), and high school or upper secondary education (grades 10-12, typically from ages 15-18).[72]

High school education is offered by several different types of schools, including:

- General high schools
- Vocational high schools
- Special purpose high schools
- Autonomous high schools.[73]

The focus of this report is on the curriculum offered by general high schools and leading to the High School Certificate of Graduation (KHSCG); other routes will not be discussed in depth.

The KHSCG curriculum spans the three years of South Korea's upper secondary education. Beginning in grade 10, students select courses which earn them units towards the KHSCG, which is intended to grant access to higher education upon successful completion. The high school curriculum is comprised of common courses, electives, and what is referred to as 'creative experiential activities' (CEA), consisting of 'discretionary activities, club activities, community services, and career-related activities'.[74]

Common courses are mandatory courses studied by all students. All students completing a KHSCG must study the common courses in: Korean Language, Mathematics, English, Korean History, Integrated Social Studies, Integrated Science, Scientific Investigation. Students must also study courses in Physical Education, Arts, and Life/Liberal Arts (i.e. Technology, Home Economics, Second Foreign Language, Classical Chinese or Liberal Arts).

Electives are non-compulsory courses selected by students according to their aptitudes and career plans. They are divided into general electives and career-related electives.

To successfully complete the KHSCG, students must study a minimum of 204 units (i.e. 2,890 hours) over the course of three years, of which:

- 180 units (i.e. 2,550 hours) are typically earned through successful completion of common courses and elective courses and/or specialised courses combined. Notably, all units studied from the 'Foundation' subject group may not exceed the 50% of the total number of units in the programme.[75]
- At least 24 units (i.e. 340 hours) must be earned through the completion of CEA.

[71] MOE, Republic of Korea. (2018). *Education in Korea*. Available from: http://english.moe.go.kr/boardCnts/viewRenewal.do?boardID=282&boardSeq=80324&lev=0&searchType=null&statusYN=W&page=1&s=english&m=0502&opType=N
[72] Ibid.
[73] MOE, Republic of Korea. (2022). *Secondary Education.* Available from: english.moe.go.kr/sub/infoRenewal.do?m=0303&page=0303&s=english
[74] Ibid.
[75] Ibid.

Each unit in the KHSCG is equivalent to 17 lessons of 50 minutes each.[76] The following table provides a summary of the unit allocation per subject cluster:

Table 34: Unit requirements of the KHSCG

Subject areas	Subject clusters	Minimum required units per subject cluster (units from the common courses)	Electives and/or specialised courses
Foundation	Korean Language	**10 units** Korean Language (8 units)	Selected subjects from general electives, career-related electives, or specialised subjects*
	Mathematics	**10 units** Mathematics (8 units)	
	English	**10 units** English (8 units)	
	Korean History	**6 units** Korean history (6 units)	
Inquiry	Social Studies (including History/ Moral Education)	**10 units** Integrated social studies (8 units)	
	Science	**12 units** Integrated science (8 units) Scientific investigation (2 units)	
Physical Education/ Arts	Physical Education	**10 units**	
	Arts	**10 units**	
Life/Liberal Arts	Life/Liberal Arts (i.e. Technology/ Home Economics/ Second Foreign Language/ Classical Chinese/Liberal Arts)	**16 units**	
Sub-total		**94 units**	**86 units**
Creative experiential activities (CEA)			
Discretionary activities, club activities, community services, and career-related activities			**24 units**
Total		**204**	
NB: Units assigned to common courses can be reduced up to 2 units, except for Korean History, where the full 6 units are required.[77]			

As seen in the table above, all students are required to study mathematics as part of the high school curriculum. The minimum requirement is 10 units, with the common course making up eight of these. See the below descriptions for detail on the courses and selected pathway which will be used in the analysis.

Common course: Mathematics[78]

Mathematics is a common subject in the South Korean high school curriculum and thus has a common course required to be taken by all students. Worth eight units, this course is taken in the first grade of high school and directly follows the mathematical learning of the last grade

[76] MOE, Republic of Korea. (2015). *The National Curriculum for the Primary and Secondary Schools*. p. 20
[77] Ibid. p.19.
[78] MOE, Republic of Korea. (2020). *Mathematics Curriculum.*

in middle school. The aim of the common course is to provide a foundation for further study of general electives, career-related electives, specialised mathematics subjects, and other disciplines. Furthermore, the course aims to develop skills and competences which will enable students to effectively participate in society. The minimum requirement of mathematics units to complete over three years of high school is ten, therefore students will need to take at least one elective in addition to the common course. Electives are separated into general and career-related.

General electives[79]
For students wishing to build on the content of the common course and study mathematics at a higher level, the following general electives may be chosen:

- **Mathematics I** – focuses on functions and sequences.
- **Probability and Statistics** – focuses on number of cases, probability, and statistics.
- **Mathematics II** – introduces calculus and concepts of limits and continuity of functions.
- **Calculus** – this elective can be chosen following study of Mathematics I and Mathematics II. It focuses on limits of a sequence, differentiation methods, and integration methods.

Career-related electives[80]
For students who want to build on content learnt in the common course and study mathematics that may be relevant to a future career path, the career-related electives offer the following options:

- **Geometry** – focuses on the quadratic curve, plane vectors, and spatial figure and spatial coordinates.
- **Economic Mathematics** – focuses on number and economy, sequences and finance, functions and economics, and differentiation and economics.
- **Artificial Intelligence Mathematics** – focuses on how mathematics is used in artificial intelligence and learning processes of classifying data and using it to make predictions.
- **Mathematical Inquiry Task** – focuses on developing inquiry skills and gives opportunity for students to independently explore an area of mathematics or problem and present their findings.

KHSCG mathematics pathway[81]
For content and demand, the analysis focuses on comparing the DP subjects to a pathway in the KHSCG mathematics curriculum – consisting of Mathematics, Mathematics I, Mathematics II, Probability and Statistics, Calculus, and Geometry. The electives have been selected from the South Korean curriculum on the basis that they are the topic areas assessed in the 'Swuri-Ka' section of the CSAT[82] and also because of their similarity to the main topics in the DP.

[79] Ibid.
[80] Ibid.
[81] Kim.J, Lee. J, Park. M, Han.I. (2012). *Mathematics Education In Korea - Vol. 1: Curricular And Teaching And Learning Practices*. p. 290-294.
[82] There are two options offered to students in the mathematics section of the CSAT, 'Swuri-Na' and 'Swuri-Ka'. The latter is more advanced and assesses a broader range of mathematical areas.

4.6.1 Learning Outcomes

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics: applications and interpretation. Similarly, the South Korean curriculum sets out its learning outcomes in the form of objectives, which are mostly identical for the common course and electives, only slightly differing in their references to the content that the subject/elective focuses on. The objectives include six mathematical competencies, which are problem-solving, reasoning, creativity and convergent thinking, communication, information processing, and attitudes and practices. These competencies are described in the 'character' section of each syllabus; thus these sections were also used to assess the presence of DP themes.

The following summary table demonstrates the learning outcome themes that were extracted from the DP mathematics and indicates if and where they were judged to have presence within the learning outcomes of the KHSCG mathematics curricula.

Table 35: Presence of the DP mathematics subject group learning outcome themes in KHSCG curricula

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in KHSCG	
1. Being aware of, and engaging with, mathematics in its wider context		Somewhat present. The desired result of the competencies and objectives is that students will be effective members of society.
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Present in the third objective and the 'attitudes and practice' competency.
3. Using inquiry-based approaches		Somewhat present in the 'information processing' competency.
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present in the first objective.
5. Making links and generalisations		Present in the 'creative and convergent' competency.
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Present in 'problem-solving', 'creative and convergent thinking', and 'reasoning', competencies.
7. Communicating mathematics clearly and in various forms		Present in the second objective and the 'communication' competency.
8. Knowing how technology and math influence each other and using technology to develop ideas and solve problems		Somewhat present in the 'information processing' competency.

Key:

	This theme is well-evidenced in the learning outcomes of the KHSCG.		*This theme is partially evidenced in the learning outcomes of the KHSCG.*		*This theme is not evident in the learning outcomes of the KHSCG.*

Presence of the DP's Learning Outcome Themes

All of the extracted themes from the DP's learning outcomes are identifiable in the South Korean curriculum, with five of them being strongly present. Most strongly evidenced is the DP's theme of critical and creative thinking. Indeed, three out of the six mathematical competencies are dedicated to this theme, namely the problem-solving, reasoning, and creative and convergent thinking competencies. These describe skills such as exploring solution strategies, analysing facts, and justifying ideas. Furthermore, the creative and convergent thinking competency also aligns with another of the DP's themes – making links and generalisations. This competency states that students should be able to create new knowledge and solve problems by linking together mathematical concepts and skills and drawing upon information from experiences and other subjects.

Also well-evidenced is the DP's theme of learning skills, as the competency of attitudes and practices describes students having confidence and curiosity, as well as being independent learners. Although collaboration is not explicitly mentioned, there are references to 'understanding others` ideas' in the competency of communication. Communication as a theme generally is present, as this competency details expressing results, ideas, and problem-processes in various formats, such as words, text, pictures, and mathematical notation.

Turning to themes which are present in a weaker sense, some inquiry-based approaches are detailed, such as collecting, organising, and analysing data, however, these are within a different context of information processing, rather than inquiry. Therefore, elements of inquiry, such as making conjectures and testing the reliability of results, are not present in South Korean outcomes, though it can be noted that inquiry-based learning is mentioned in the teaching and learning section of the curriculum. In addition, 'information processing' only describes 'selecting and using appropriate tools', hence the theme of technology is present, but to a lesser extent than in the DP, which describes using technology to solve problems and explore new ideas.

Finally, there are elements in the South Korean outcomes which speak to the wider contexts of maths, as the curriculum intends that students will be able to use their mathematical knowledge and competencies to engage effectively in society. However, the learning outcomes do not explicitly detail that students will be able to understand global issues and historic perspectives or think critically about the implications of mathematics.

Other Themes in the KHSCG

There are no significant themes in the South Korean mathematics syllabi which are not present in the DP. Differences are usually in wording, rather than intended outcomes. For example, as previously mentioned, South Korea describes a competency regarding information processing, which, whilst not explicitly described in the DP, shares similarities with the DP's outcome of using inquiry-based approaches. Another example is South Korea's reference to convergent thinking, which, whilst not a term used in the DP, is strongly present in its outcomes involving problem-solving, reasoning, critical reflection, and consideration of mathematical implications. Lastly, South Korea perhaps has more explicit detail around making mathematical connections and creating new ideas.

Summary

Overall, there is considerable alignment between the learning outcomes of the DP's mathematics subjects and South Korea's. Most of the DP's themes are present and similarly emphasised, especially critical and creative thinking, understanding and application, communication, and making links and generalisations. However, it can be noted that wider contexts, inquiry-based approaches, and technology, though present, had less emphasis in the KHSCG curriculum. There are no significant themes which are unique to the KHSCG.

4.6.2 Content

This section compares and contrasts the content of the DP and KHSCG curricula falling within the category of mathematics. In order to support visual comparison at-a-glance, the KHSCG mathematics curriculum is presented below in a diagram showing the common course and electives and the key topics and sub-topics included in each.

Figure 15: KHSCG mathematics content visualiser

Common Subject	Mathematics	Algebra • Polynomials • Equations and inequalities	Geometry • Equations of figures	Numbers and arithmetic operations • Sets and propositions	Functions • Functions and graphs	Probability and statistics • Number of cases	
General electives	Mathematics I	Exponential and logarithmic functions	Trigonometric functions	Sequences			
	Mathematics II	Limits and continuity of functions	Differentiation	Integral calculus			
	Calculus	Limit of a sequence	Differentiation	Integration			
	Probability and statistics	Number of cases	Probability	Statistics			
Career-related electives	Geometry	Quadratic curves	Plane vectors	Spatial figures and coordinates			
	Practical Mathematics	Geometry: rules	Geometry: spaces	Statistics: Data			
	Economic Mathematics	Numbers and daily economics	Sequences and finance	Functions and economics	Differentiation and economics		
	Mathematical Inquiry Task	Understanding of inquiry task	Inquiry task and evaluation				
	Basic Mathematics	Number of cases	Polynomials	Equations and inequalities	Sets	Functions and graphs	Equation of figure
	Artificial Intelligence Mathematics	Artificial intelligence and mathematics	Representation of materials	Classification and prediction	Optimisation		

4.6.2.1 Structure

Mathematics content in the DP is separated by focus, AA and AI, and level, SL and HL. DP HL includes the content of SL, as well as additional higher level (AHL) content. For high school, the mathematics content in the South Korean curriculum is structured into the common course, general electives, and career-related electives. General electives are Mathematics I, Mathematics II, Probability and Statistics, and Calculus. Career-related electives are Geometry, Practical Mathematics, Economic Mathematics, Mathematical Task Inquiry, Basic Mathematics, and Artificial Intelligence Mathematics. Each elective usually focuses on a few areas of one main topic – unlike DP mathematics subjects which integrate content from five main topic areas. Students can choose the electives that they take, therefore there is a greater degree of flexibility than there is in the DP. Furthermore, content is not separated into levels like SL and HL in the DP; however, a few of the electives require more pre-requisite knowledge than others.

Similarly to AA and AI in the DP, the electives offer opportunities to focus on pure mathematics and applied mathematics. For example, Calculus offers more pure-focused content, whereas Economic Mathematics offers more applied-focused content. Furthermore, the Mathematical Inquiry Task elective is similar to the mathematical exploration offered in the DP, though it is not compulsory. However, the South Korean content structure has a stronger focus on careers than the DP and offers electives which have a more specific focus on certain fields, such as Economic Mathematics and Artificial Intelligence Mathematics. Furthermore, the electives also cater for those who need further time with basic concepts (Basic Mathematics) and for those who wish to focus on solving real-world problems from everyday life (Practical Mathematics). Altogether, the South Korean curriculum caters to a wider range of mathematical abilities than the DP and allows students to choose a more specific focus in their studies, based on their interests and future plans for education and careers.

4.6.2.2 Content Alignment

The figures below show a simplified summary of the extent to which KHSCG mathematics aligns with the main topics of the DP's subjects. The following analysis in this section will first be split into AA and AI. For both AA and AI, the analysis reviews how the content of each KHSCG common course/elective affects the alignment with DP subjects, before combining this information to compare a pathway to DP SL/HL mathematics. Following this, several other electives which are offered in the KHSCG and not included in the 'pathway', will be reviewed for their similarities and differences in content to the DP curriculum.

Table 36: Summary of the content alignment KHSCG has with the main topics in AA

Mathematics: analysis and approaches topics		Presence in KHSCG						
		Maths	Maths I	Maths II	Calculus	Prob & Stats	Geometry	Pathway
SL	1. Number and algebra							
	2. Functions							
	3. Geometry and trigonometry							
	4. Statistics and probability							
	5. Calculus							
AHL	1. Number and algebra							
	2. Functions							
	3. Geometry and trigonometry							
	4. Statistics and probability							
	5. Calculus							

Table 37: Summary of the content alignment the KHSCG has with the main topics in AI

Mathematics: applications and interpretation topics		Presence in KHSCG						
		Maths	Maths I	Maths II	Calculus	Prob & Stats	Geometry	Pathway
SL	1. Number and algebra							
	2. Functions							
	3. Geometry and trigonometry							
	4. Statistics and probability							
	5. Calculus							
AHL	1. Number and algebra							
	2. Functions							
	3. Geometry and trigonometry							
	4. Statistics and probability							
	5. Calculus							

Key:

	Strong presence of this topic in the KHSCG.		*Partial presence of this topic in the KHSCG.*		*Little or no presence of this topic in the KHSCG.*

NB: Where applicable, content alignments found in pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

The table below summaries KHSCG content that is in neither DP mathematics subject or only one of them.

*Table 38: KHSCG mathematics content which is not covered in the DP**

Significant content not in AA (only)	Significant content not in AI (only)
• Some financial applications (e.g. annuities). *(Economic Mathematics).*	• Limits of functions. *(Mathematics II).* • Continuity and differentiability *(Mathematics II).* • Combinatorics. *(Probability and Statistics).*
Significant content not in either DP mathematics subject	
• Additional content regarding economic applications of mathematics. *(Economic Mathematics).* • Applications to artificial intelligence. *(Artificial Intelligence Mathematics).* • Further detail regarding convergence and divergence of functions and limits of sequences. *(Calculus).* • Parabolas, ellipses, and hyperbolas. *(Geometry).*	

* Significant content does not include topics which are typically studied *prior* to upper secondary

Mathematics: analysis and approaches

Mathematics

The mapping of content shows that the common subject Mathematics has partial alignment with a few of the main topic areas in AA. Namely, these are SL 'Functions' and AHL 'Number and algebra'. Mathematics partially aligns with SL 'Functions' content as it covers concepts of functions, quadratic and rational functions, and composite and inverse functions. However, exponential and logarithmic functions, transformations, and some analytical solutions are not included in this subject. Although Mathematics does not align with SL 'Number and algebra' content, partial alignment was found with the AHL content in this topic. Indeed, Mathematics covers counting principles, proof by induction, and some coverage of complex numbers. Overall, Mathematics on its own has very limited alignment with SL or AHL content of AA.

Mathematics I

Mathematics I follows Mathematics, therefore previously mentioned alignments still apply. The mapping of content shows that Mathematics I has alignment with some, but not all, of the main topics of AA SL. Topics which Mathematics I has the strongest alignment with are, at SL, 'Number and algebra' and 'Functions'. Of 'Number and algebra' content, Mathematics I covers sequences and series, simple proof, and laws of exponents and logarithms. Of 'Functions' content, Mathematics I builds on content from Mathematics to include exponential and logarithmic functions and some reference to analytical solutions, thus strengthening the alignment with SL content in this topic. Furthermore, Mathematics I incorporates trigonometric functions and radians, which gives partial alignment with 'Geometry and trigonometry' SL content. However, there is limited detail for this topic, such as for solving trigonometric equations, and no reference to trigonometric identities or transformations. There are a few sub-topic alignments with 'Statistics and probability' from prior learning, however, not enough to be significant. No 'Calculus' content is covered in Mathematics I.

In summary, Mathematics I has good alignment with 'Number and algebra', and SL 'Functions'. There is also partial alignment with SL 'Geometry and trigonometry' content. Overall, Mathematics I has less breadth and depth than the content of AA SL.

Probability and Statistics

Probability and Statistics can be taken following the common subject Mathematics, therefore the alignments mentioned in relation to the common subject still apply. As expected, Probability and Statistics mostly impacts alignment with the AA topic on the same theme. Probability and Statistics includes probability, discrete random variables, and the binomial and normal distributions. Other SL sub-topics regarding presenting data and measures of central tendency and dispersion are covered in middle school. Therefore, there is good alignment with 'Statistics and probability' SL content, though it can be noted that linear regression is not covered. For students who are also taking Mathematics II, Probability and Statistics includes some coverage of continuous random variables, therefore there is partial alignment with AHL content in this case. Finally, this elective contains the binomial theorem and counting principles, hence, when taken with Mathematics I, there is strong alignment with 'Number and algebra' SL content.

Geometry

Geometry can be taken following the common subject Mathematics, therefore alignments mentioned in relation to the common subject still apply. In general, Geometry has little impact on the extent of alignment with AA SL content, though it does slightly increase the number of AHL sub-topics alignments in 'Geometry and trigonometry'. This is due to some coverage of vectors, including an introduction to vectors, the scalar product, and vector equations of lines. However, other AA AHL sub-topics are not covered, such as the cross product, equations of planes, reciprocal trigonometric ratios, and further trigonometric identities. Therefore, there is only weak alignment with AHL content in this topic overall. Aside from vectors, the Geometry career-elective focuses on other topics which are less, or not, present in AA. For example, Geometry focuses on parabolas, ellipses, and hyperbolas, as well going into more depth in coordinate geometry in its section named 'Spatial figures and coordinates'.

Mathematics II

Mathematics II would usually be taken following Mathematics I, therefore previous alignments mentioned for the latter still apply. Mathematics II focuses on limits and continuity of functions, differentiation, and integration, therefore alignment with AA only changes with regard to the topic of Calculus and the alignment with other topics remain the same as for Mathematics I.

The mapping of content shows that Mathematics II has considerable alignment with the SL content of AA Calculus. Indeed, the elective includes concepts of limits, increasing and decreasing functions, graphs, differentiation of polynomials, equations of tangents, maxima and minima problems, and integration. Furthermore, continuity and differentiability of functions is also covered, which is an AHL sub-topic in AA. Within this are some concepts not explicitly mention in the AA syllabus, such as the mean value theorem, Rolle's theorem, and the interval theorem. Overall, Mathematics II has similar depth in the topic of calculus as AA SL, but not AA HL.

Calculus

Calculus can be studied following Mathematics I and Mathematics II, therefore previous alignments with those electives also apply. The Calculus elective requires the most pre-requisite study and therefore can be considered the most advanced elective to be chosen. As expected, the Calculus elective only significantly impacts the alignment of the main topic of the same name in AA. Calculus builds on the learning in Mathematics II and includes several AHL sub-topics and further SL topics. These are the second derivative, further derivatives, indefinite integrals, optimisation problems, evaluation of limits, implicit differentiation, integration methods, and finding area and volume. However, the elective does not include first order differential equations or Maclaurin series.

The Calculus elective also explores convergence and divergence of sequences and series and finding the limits of sequences. Overall, the elective has greater depth in the topic of calculus than AA SL, and similar depth to AA AHL.

Pathway
This section will consider the alignment between a particular pathway in KHSCG mathematics, consisting of the subject/electives above, and DP AA content.

In summary, for AA SL content, the pathway has strong alignment with 'Number and algebra', 'Functions', 'Statistics and probability', and 'Calculus'. There is also partial alignment with SL 'Geometry and trigonometry'. For AHL content, there is strong alignment with 'Calculus' and partial alignment with 'Number and algebra' and 'Statistics and probability'. There are also some sub-topic alignments within the area of vectors. Most of the content in the electives is present in AA, with the exception of some different geometry coverage and some additional coverage of continuity, differentiability, limits and sequences.

Altogether, these electives have comparable breadth to DP subjects and greater depth in content than AA SL, though slightly less depth than HL. Specifically, there is less depth in vectors, complex numbers, and trigonometry.

Mathematics: applications and interpretation

Mathematics
The mapping of content shows that Mathematics has some limited alignment with a few of the main topic areas in AI. Namely, these are SL 'Functions' and 'Geometry and trigonometry'. Mathematics partially aligns with SL 'Functions' content as it covers concepts of functions and graphing of functions, however, modelling with functions is not a focus in this subject. Furthermore, sub-topic alignments in SL 'Geometry and trigonometry' include volume and area, Pythagoras, and circles, however, these are studied in middle school, prior to Mathematics. It can be noted that there are some sub-topics in 'Statistics and probability' which are also covered in middle school. Mathematics also includes solving equations with complex roots, however, there are no other alignments with 'Number and algebra'. Mathematics covers some sub-topics which are not present in AI but are present in AA. These include proof, rational and polynomial functions, and counting principles. Overall, Mathematics on its own has very limited alignment with SL or AHL content of AI.

Mathematics I
Mathematics I can be taken following Mathematics, therefore previously mentioned alignments still apply. The mapping of content shows that Mathematics I has alignment with some, but not all, of the main topics of AI SL. Topics which Mathematics I has alignment with, at SL, are 'Number and algebra', 'Functions', and 'Geometry and trigonometry'. Of 'Number and algebra' content, Mathematics I covers sequences and series, exponents and logarithms, and some of complex numbers – thus there is partial alignment with AHL content also in this topic. Regarding 'Functions' content, Mathematics I builds on content from Mathematics to

include exponential and logarithmic functions – modelling of which is referenced. However, there is limited detail with regard to modelling and fewer functions are used to model than in AI.

Furthermore, Mathematics I incorporates trigonometric functions and radians, which creates some sub-topic alignment with ‘Geometry and trigonometry’ AHL content, though not a significant amount. Alignment with SL content remains the same, as Voronoi diagrams are not present. Moreover, there are a few sub-topic alignments with ‘Statistics and probability’, however, not enough to be significant. No Calculus content is covered in Mathematics I.

In summary, Mathematics I has some alignment with ‘Number and algebra’, SL ‘Functions’ and ‘Geometry and trigonometry’. Overall, Mathematics I has less breadth and depth than the content of AI SL.

Probability and Statistics
Probability and Statistics can be taken following the common subject Mathematics, therefore the alignments mentioned in relation to the common subject still apply.

As expected, Probability and Statistics mostly impacts alignment with the AI topic on the same theme. Probability and Statistics includes probability, discrete random variables, and the binomial and normal distributions. Other SL sub-topics regarding presenting data and measures of central tendency and dispersion are covered in middle school. Therefore, there is partial alignment with ‘Statistics and probability’ SL content, noting that linear regression, Spearman’s rank, and hypothesis testing are not covered. Regarding AHL content, there are a couple of sub-topic alignments with the central limit theorem and confidence intervals. For students who are also taking Mathematics II, it includes some coverage of continuous random variables, which is not present in AI. In addition, this elective contains the binomial theorem and counting principles, which are also not sub-topics that are present within AI.

In summary, this elective has reasonable alignment with AI SL ‘Statistics and probability’ but is missing some of the key sub-topics such as regression and hypothesis testing.

Geometry
Geometry can be taken following the common subject Mathematics, therefore alignments mentioned in relation to the common subject still apply. It is likely that students taking Geometry will also at least take Mathematics I. In general, Geometry has little impact on the alignment of AI SL content, though it does slightly increase the number of AHL sub-topics alignments in ‘Geometry and trigonometry’. This is due to some coverage of vectors, including an introduction to vectors, the scalar product, and vector equations of lines. Therefore, when taken with Mathematics I, there is partial alignment with AHL content in this topic. However, other AI AHL sub-topics are not covered, such as the cross product, matrix transformations, graph theory, and decision mathematics. Aside from vectors, the Geometry career-elective focuses on other topics which are less, or not, present in AI. For example, Geometry focuses on parabolas, ellipses, and hyperbolas, as well going into more depth in coordinate geometry through its section named ‘Spatial figures and coordinates’.

Mathematics II

Mathematics II is usually taken following Mathematics I, therefore previous alignments mentioned for the latter still apply. Mathematics II focuses on limits and continuity of functions, differentiation, and integration, therefore alignment with AA only changes with regard to the topic of ‘Calculus’ and the alignment with other topics remain the same as for Mathematics I. The mapping of content shows that Mathematics II has considerable alignment with the SL content of AI ‘Calculus’. Indeed, the elective includes concepts of limits, increasing and decreasing functions, graphs, differentiation of polynomials, equations of tangents, maxima and minima problems, and integration. Furthermore, continuity and differentiability of functions is also covered, which is an AHL sub-topic in AA. Within this are some concepts not included in the AI syllabus, such as the concepts of continuity and differentiability, the mean value theorem, Rolle’s theorem, and the interval theorem.

Overall, Mathematics II has similar depth and detail in the topic of calculus as AI SL, but not AI HL.

Calculus

Calculus can be studied following Mathematics I and Mathematics II, therefore previous alignments with those electives also apply. This elective requires the most pre-requisite study and therefore can be considered the most advanced elective to be chosen. As expected, the Calculus elective only significantly impacts the alignment of the main topic of the same name in AI. Calculus builds on the learning in Mathematics II and includes further SL sub-topics and several AHL sub-topics from the DP. These are differentiation methods, the second derivative, further derivatives, optimisation problems, integration methods, and finding area and volume. However, the elective does not include first order differential equations, slope fields, Euler’s method and numerical solutions, phase portraits, or second order differential equations.

The Calculus elective also explores convergence and divergence of sequences and series and finding the limits of sequences. It also contains other areas which are not part of AI, such as implicit integration and further derivatives.

Overall, the Calculus elective has greater depth in the topic of calculus than AI SL, and similar depth to AI AHL, though with different coverage of the topic.

Pathway

This section will consider the alignment between a particular pathway in KHSCG mathematics, consisting of the subject/electives above, and DP AI content.

In summary, for AI SL content this pathway has strong alignment with ‘Calculus’ and ‘Geometry and trigonometry’. There is also partial alignment with ‘Number and algebra’, ‘Functions’, and ‘Statistics and probability’. For AHL content, there is partial alignment with ‘Number and algebra’, ‘Geometry and trigonometry’, and ‘Calculus’. There are several areas covered in this pathway which are not present in AI, these included certain functions content, proof, continuous random variables, continuity and differentiability, limits of sequences, and different geometry coverage. Altogether, these electives have greater depth in content than AI SL, but have slightly less depth than HL. Furthermore, it can be noted that this pathway is slightly more aligned with AA than AI.

Other Electives

Mathematical Inquiry Task
This is a career elective which can be taken following the study of the common subject, Mathematics. This elective has strong similarities with the exploration component of DP mathematics, as it aims to develop skills of inquiry. Like with the DP, this elective allocates time to both learning and understanding inquiry techniques and then executing an independent exploration. The content describes how students will, supported by examples, learn the necessity of exploring maths, the procedures involved with inquiry, and recognise the importance of research ethics. The exploration itself has many similar components to the DP's. Indeed, students taking this elective will select an area based on their interests, plan their inquiry, collaborate, use research methods (e.g. literature review, case study, and data collection), organise and present findings, use correct referencing where appropriate, and conduct self-evaluation and peer-evaluation. Furthermore, teachers are also expected to support students throughout the process, offering frequent guidance and feedback. However, there are some differences to the DP which are worth noting. Firstly, this elective is not compulsory, thus not all students will be assessed on their ability to carry out and present the findings of an independent exploration of a mathematical area. Assessment is also different, as teachers' evaluation is not based solely on the final product, but also the process, with teachers encouraged to conduct various types of evaluation throughout the inquiry such as interview and observation. Furthermore, the South Korean curriculum does not state at what level the mathematics within the inquiry needs to be, whereas the DP states that the mathematics used should be commensurate with the level of the course, SL or HL. Since this elective can be taken following the common subject Mathematics, this may mean that students taking this elective are able to use mathematics that is below the level of DP SL/HL.

The career electives also contain other options that are of a similar level but focus on areas of mathematics that are not prominent in the DP curriculum. Namely, these are the Economic Mathematics and Artificial Intelligence Mathematics. Below are summaries of these electives and how they compare to the DP.

Economic Mathematics
This elective contains four areas 'Numbers and daily economics', 'Sequences and finance', 'Functions and economics', and 'Differentiation and economics'. As can be seen by these areas, Economic Mathematics incorporates topics that can be found within the DP, such as number, sequences, functions, and calculus, but differs by having a stronger focus on their application to economics. Regardless, there are some sub-topic alignments with the DP, such as with arithmetic and geometric sequences and their financial applications, some functions and graphs, concept of a limit, differentiation, and optimisation. Due to its applied nature, Economic Mathematics has more instances of overlap with AI than AA, indeed it especially aligns with AI SL Calculus – with the exception that integration is not covered. Overall, many of the fundamental concepts of Economic Mathematics can be identified within the DP curriculum, though it has additional economic-specific content, such as further economic indicators and use of functions in economics (demand and supply curve, utility functions, and equilibrium price).

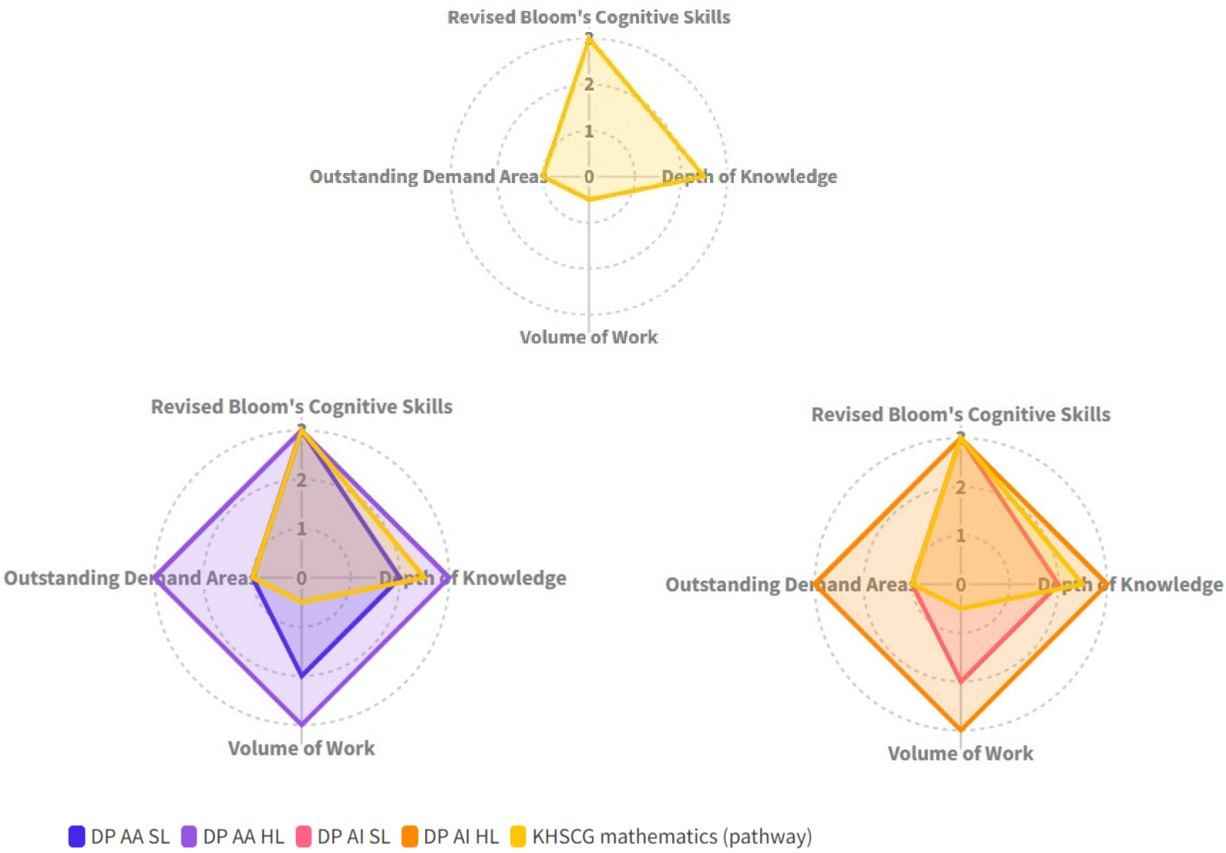
Artificial Intelligence Mathematics

This elective is divided into four main concepts – 'Artificial Intelligence and Mathematics', 'Data Representation', 'Classification and Prediction', and 'Optimisation'. This elective diverges from the DP curriculum, though does contain similar concepts such as organising and analysing data, identifying trends, conditional probability, limits, differentiating quadratic functions, and optimisation. Differently to the DP, this elective has a strong focus on artificial intelligence, with each of the four main topics looking at a different area of this. 'Artificial Intelligence and Mathematics' explores the history of mathematics in artificial intelligence and deals with logic, truth tables and flowcharts; 'Data Representation' looks at representation of text and image data; 'Classification and Prediction' deals with classifying image data and using data for prediction; and 'Optimisation' deals with finding the maximum and minimum of functions and using optimisation methods to solve problems and decision making. Like with AI, this elective has a strong focus on technology, with reference to the use of engineering tools in most topics.

4.6.3 Demand

The DP and KHSCG curricula were analysed using the same demand tool in order to create a demand profile for AA (SL and HL), AI (SL and HL), and a pathway from the South Korean curriculum, comprising Mathematics, Mathematics I, Mathematics II, Calculus, Probability and Statistics, and Geometry. These pathway profiles are presented below in the form of radar diagrams, with the last two diagrams showing the two DP mathematics subjects superimposed with the KHSCG pathway, enabling immediate visual comparison.

Figure 16: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - DP subjects received a score of 3.
  - Similarly, the KHSCG's learning outcomes are mostly the same for the common course and the electives, hence all received the same score of 3. The KHSCG learning outcomes evidenced creativity through their 'creative and convergent thinking' competency, which described producing and refining new ideas. Analysis, evaluation and reflection were also evidence by their 'reasoning' and 'information processing' competencies.

- Regarding the scores for **Depth of Knowledge**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For the KHSCG, the pathway was given a score of 2.5 for depth. All electives require pre-requisite knowledge from the common course and Calculus requires substantial pre-requisite knowledge from Mathematics I and Mathematics II. Furthermore, most topics were covered in a high level of detail – though slightly less than what can be observed in the DP HL, hence a 2.5 rather than a 3 was awarded.

- Regarding the scores for **Volume of Work**:
  - DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - For the KHSCG, each unit is equivalent to approximately 14 hours of teaching time. The mathematics common subject is eight units and each elective is five units, therefore this pathway has a total of 33 units and a combined teaching time of 462 hours. This amount of time was deemed a generous amount of time to cover the number of topics in the syllabus, thus it received a score of 0.

Regarding the scores for **Outstanding Areas of Subject Demand**:

  - DP SL subjects received a score of 1 and HL subjects received a score of 3.
  - For the KHSCG, the low level of detail in the documented content meant that areas of outstanding subject demand were somewhat difficult to identify. However, a limited number of outstanding areas were found in the Calculus elective, with regard to limits of sequences and applications, and in the Mathematics I elective, with regard to mathematical induction, thus a score of 1 was deemed appropriate for this category.

4.7 United States (CCSS)

The Common Core State Standards (CCSS) are a set of college and career ready standards designed to ensure that students of high-school graduation age are prepared for their progression either into higher education or the workforce. The standards outline basic skills and knowledge that students should have gained by each grade level, from kindergarten through to grade 12, in English language, arts/literacy, and mathematics. 43 states have currently adopted these standards.[83]

The standards were developed by the National Governors Association (NGA) Centre for Best Practices and the Council of Chief State School Officers (CCSSO), with input from educators, students, administrators, parents and experts from across the US. The standards intend to promote equity and collaboration among states to ensure consistency in education. As such, the CCSS promote standardisation and provision of:

- Teaching materials such as textbooks and media.
- Comprehensive assessment system implementation and development to measure student performance.
- Support for institutions and students in meeting the standards.[84]

Notably, the standards do not dictate teaching methods or curriculum design, but rather set out goals upon which teachers are to devise their curriculum and lesson plans, allowing for flexibility and tailored delivery. The standards impact teachers by:

- Providing consistent goals and benchmarks for teachers to base their curriculum and lessons on, ensuring students' progress with the skills needed for the next steps in their academic or professional careers.
- Providing consistent expectations for students who move between states or districts, allowing for an easier transition in the education system.
- Providing the opportunity to collaborate with teachers across the country in the development of curricula, materials, and assessments.
- Helping with teacher education in preparation for beginning a career.[85]

The implementation of the standards is decided at state and local levels, allowing for flexibility and different approaches of their implementation alongside any other state-specific goals.[86]

Where the CCSS have been adopted, students are required to study mathematics in high school (grades 9-12). See below for the description of standards used in this analysis.

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[83] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards. Frequently Asked Questions.* Available from: http://www.corestandards.org/wp-content/uploads/FAQs.pdf
[84] Ibid.
[85] Ibid.
[86] Ibid.

Common Core State Standards for Mathematics[87]

The Common Core State Standards for Mathematics (CCSSM) include standards for mathematical practice and standards for mathematical content. These standards describe what students should be able to understand and do in their mathematics study, with this analysis focusing on high school standards (grades 9-12). However, the standards do not comprehensively detail all mathematics content that may be studied in high school, especially for advanced courses usually taken in grade 11/12, known as 'fourth courses'[88]. Furthermore, when comparing to the DP, it is important to consider that the CCSSM are not a curriculum, therefore there will be variation on how these are applied in practice.

4.7.1 Learning Outcomes

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same AA and AI. Similarly, CCSSM set out the Standards for Mathematical Practice, which are the same from kindergarten to grade 12. The mathematical practices comprise of seven standards, with detailed descriptions for each. The table below shows the comparison of these with the DP's learning outcome themes.

Table 39: Presence of the DP mathematics subject group learning outcome themes in the CCSSM

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the CCSSM's learning outcomes	
1. Being aware of, and engaging with, mathematics in its wider context		Not present.
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Present in CCSS are students' learning skills, including critical reflection on work, perseverance in problem solving, and building a positive attitude to mathematics.
3. Using inquiry-based approaches		Not present.
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present in CCSS, as they promote conceptual understanding and the development of skills.
5. Making links and generalisations		Present in the CCSS. The reasoning standard demonstrates the theme by incorporating abstraction skills, such as the ability to 'de-contextualise'. Furthermore, standards 7 and 8 expect students to 'look for and make use of structure' and 'regularity in repeated reasoning'
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Present in the CCSS, as critical thinking skills such as problem-solving and reasoning are two of the seven standards and present in others.

[87] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Available from: Mathematics Standards | Common Core State Standards Initiative (corestandards.org)

[88] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core Standards for Mathematics. Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards.* p. 8. Available from: Mathematics Appendix A_Teal1.indd (corestandards.org)

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the CCSSM's learning outcomes	
7. Communicating mathematics clearly and in various forms		Partially present in the CCSS as, for example, 'attending to precision' is one of the seven standards.
8. Knowing how technology and mathematics influence each other and using technology to develop ideas and solve problems		Present in the CCSS, as students are expected to use a range of technology to solve problems and deepen understanding.

Key:

	This theme is well-evidenced in the learning outcomes of the CCSS.		*This theme is partially evidenced in the learning outcomes of the CCSS.*		*This theme is not evident in the learning outcomes of the CCSS.*

Presence of the DP's Learning Outcome Themes:

There is strong alignment with some of the DP's learning outcome themes in the mathematical practices, with the CCSS also promoting conceptual understanding and the development of higher-order skills. Like the DP, critical thinking skills such as problem-solving and reasoning are both key outcomes as they make up two of the seven standards and are also highly present in others. Also embedded into the standards are other critical processes such as analysis and evaluation.

The DP's theme of making links and generalisations is present in several practices. The 'reasoning' standard demonstrates the theme by incorporating abstraction skills, such as the ability to 'de-contextualise', and two other standards demonstrate the theme by describing students' ability to look for 'patterns' and 'regularity in repeated reasoning' (with the aim to be able to abstract and/or make generalisations).

As in the DP, the standards in the mathematical practices also expect students to be able to use a range of technology and to be able to communicate mathematical ideas concisely and accurately. Furthermore, the standards outline the development of similar learning skills to the DP, such as being able to critically reflect on work, persevere in problem solving, and build a positive attitude to mathematics.

Despite the many similarities between the learning outcome themes of the DP and the CCSSM, some themes had less alignment. The least present theme is that of engaging with maths in its wider context, as real-world relevance is only considered with respect to modelling. Unlike the DP, the CCSSM do not include global and historical perspectives, links to other disciplines, or questioning of the social, moral, and ethical implications of mathematics. Also notably, though the CCSSM practices have elements of inquiry-based approaches, taking this approach in general is not a focus, and working within the context of an investigation is not a feature in the standards. Furthermore, the standards do not explicitly describe the development of collaboration skills or the ability to work independently.

Other Themes in the CCSSM

The mathematical practices do not contain any significant theme or learning outcome that is not present in the DP. However, a difference of note is that the CCSSM expectations include more specific outcomes of how students will employ and refine their powers of generalisation and abstraction, which they state will be through 'looking for and expressing regularity in repeated reasoning'. They also state that students should be able to 'look closely and discern a pattern or structure'. Hence, the CCSSM are more specific in their outcomes about the types of deeper learning they want their students to be engaging with and exploring in their mathematical studies. However, though these are not explicitly described in DP's learning outcomes, similar ideas (and others) are reflected in the 'conceptual understandings' and 'connections' sections of the DP content descriptions.

Another outcome which is given more attention in the CCSSM is that of modelling, which is one of the seven standards in the mathematical practices. This standard describes students being able to solve problems, analyse relationships, use assumptions and approximations in complex situations, interpret results, and improve models. Although modelling is a not a focus in the learning outcomes of the DP, it is present in the curriculum documents which describe modelling processes.

Summary

There is moderate alignment between the learning outcome themes of the DP and CCSS. Most of the DP's themes can be identified in the CCSS, however the themes of wider contexts and inquiry-based approaches are not present in the CCSS. Furthermore, the CCSS place a higher emphasis on modelling skills in its learning outcomes, though this is focused on in other sections of the DP's subject documentation.

4.7.2 Content

This section compares and contrasts the content of the DP mathematics subjects and the CCSSM. In order to support visual comparison at-a-glance, the mathematics content in the CCSSM are presented below in diagrams which show the key topics and sub-topics included.

Figure 17: CCSS for mathematics content visualiser

Number and quantity	The real number system	Quantities	The complex number system	Vector & matrix quantities		
Algebra	Seeing structure in expressions	Arithmetic with polynomials and rational functions	Creating equations	Reasoning with equations and inequalities		
Functions	Interpreting functions	Building functions	Linear, quadratic, and exponential models	Trigonometric functions		
Modelling	*'Modelling is best interpreted not as a collection of isolated topics but rather in relation to other standards'*					
Geometry	Congruence	Similarity, right triangles, and trigonometry	Circles	Expressing geometric properties with equations	Geometric measurement and dimension	Modelling with geometry
Statistics and probability	Interpreting categorical and quantitative data	Making inferences and justifying conclusions	Conditional probability and the rules of probability	Using probability to make decisions		

4.7.2.1 Structure

The CCSSM are learning goals that are expected to be achieved by students completing grades 9-12. Unlike how the DP sets out all the content for the whole mathematics subject group, the standards are not intended to be seen as the curriculum for all mathematics study. For high school, the CCSSM are structured into ‘Conceptual Categories’, namely ‘Number and quantities’, ‘Algebra’, ‘Functions’, ‘Geometry’, ‘Modelling’, and ‘Statistics and probability’. ‘Modelling’ is not intended as an isolated topic, but as an area to be developed in relation to other standards. Taking this away, the number of main topics is the same as the DP, which has ‘Number and algebra’, ‘Functions’, ‘Geometry and trigonometry’, ‘Statistics and probability’, and ‘Calculus’. As can be seen, there are clear similarities in the types of main topics covered.

For the CCSSM, trigonometry is found within ‘Functions’ and ‘Geometry’, rather than as a separately named topic area. Like the DP, each main area in the CCSSM is broken down into smaller areas, with these being the ’domains’, within which are the standards. Similarly, in this breakdown, both have content that is marked as higher level and is intended for more difficult courses, in the DP this is the additional higher level (AHL) content and for the CCSSM this is content studied in a ‘fourth course’. In the US, schools often teach Algebra I, Geometry, and Algebra II to all students, followed by a choice of fourth courses ranging in difficulty, such as Pre-calculus, Calculus, and Advanced Statistics. However, where the DP details all the higher level content required to be taught, the CCSSM only point to a few new areas or further extensions, which are not themselves reflective of all content that can be studied in a fourth course. Many students take fourth courses, with choices depending on what students will do after high school.

Another main difference in the structure of mathematical content between the DP and the CCSSM is the number of years with which they relate to, for the DP this is two years, where for the US it can be 3-4 years, with also options for fast tracking and starting content earlier in grades 7 or 8. Finally, the CCSSM do not separate content with regard to focus, such as pure and applied, as the DP does to an extent with AA and AI.

4.7.2.2 Content Alignment

To complement the analysis, the figures below represent a simplified summary of the CCSSM content alignment, at topic-level, with mathematics: analysis and approaches (SL and HL) and mathematics: applications and interpretation (SL and HL).

Table 40: Summary of the content alignment between the topics in AA and the CCSSM content.

Mathematics: analysis and approaches topics		Presence in the CCSSM
SL	1. Number and algebra	
	2. Functions	
	3. Geometry and trigonometry	
	4. Statistics and probability	
	5. Calculus	
AHL	1. Number and algebra	
	2. Functions	
	3. Geometry and trigonometry	
	4. Statistics and probability	
	5. Calculus	

Table 41: Summary of the content alignment between the topics in AI and the CCSSM content.

Mathematics: applications and interpretation topics		Presence in the CCSSM
SL	1. Number and algebra	
	2. Functions	
	3. Geometry and trigonometry	
	4. Statistics and probability	
	5. Calculus	
AHL	1. Number and algebra	
	2. Functions	
	3. Geometry and trigonometry	
	4. Statistics and probability	
	5. Calculus	

Key:

	There is strong presence of this topic in the CCSS		*There is partial presence of this topic in the CCSS*		*There is little or no presence of this topics in the CCSS*

<u>*Mathematics: analysis and approaches*</u>

Mathematics: analysis and approaches (AA) content is present in the CCSSM for all topics, except 'Calculus'. Calculus is usually taken in grade 12 by those who are mathematically able and may intend to pursue a STEM-related course in higher education. The mapping of content shows that CCSSM have the most alignments with the AA topics of 'Number and algebra' and 'Functions'. The standards which relate to studied content in the first three courses of mathematics study involve nearly all of the key areas in both of these main topics, including sequences, rational exponents, binomial theorem, complex numbers, and several functions and graphs. In these topics, most significant SL content is covered by the CCSSM, along with some partial AHL alignments. If the CCSSM relating to fourth courses are also considered, then alignment with AHL is further strengthened due to more functions being explored and the inclusion of more difficult concepts in complex numbers. Although proof is a feature in the CCSSM, it can be noted that methods of proof by induction, comparison, or counterexample were not required.

Some SL content from 'Geometry and trigonometry' is present in the CCSSM, with fourth course standards also including the study of trigonometric functions and equations. With regard to AHL content, the CCSSM indicate that an introduction to vectors would be studied in a fourth course, with further depth of study likely to depend on the course. In contrast to AA, the CCSSM focus on areas such as similarity, congruence, constructions, circle theorems, and conic sections.

For 'Statistics and probability', the CCSSM have strong alignment with the sub-topics of presenting data, measures of central tendency and spread, correlation and linear regression, and probability formulae, with random variables being an area of focus in fourth course study. The CCSSM also include the development of concepts involving populations and sampling, which are not in the scope of AA.

Further, in contrast to AA, the standards place a greater emphasis on modelling and also indicate that matrices will be covered in a fourth course. Overall, the CCSSM strongly align with AA SL content in 'Number and algebra' and 'Functions', and also align well with 'Geometry and trigonometry' and 'Statistics and probability'. Of these topics, for AHL content, the CCSSM have some alignment with 'Number and algebra' and 'Functions', limited alignment with 'Geometry and trigonometry', and none with 'Statistics and probability'. Lastly, the CCSSM do not include standards related to calculus, thus there is no alignment with the topic of 'Calculus' for both SL and AHL content. Therefore, the content depth in AA HL exceeds that of the CCSSM in this area.

Mathematics: applications and interpretation
Mathematics: applications and interpretation (AI) content is present in the CCSSM for all topics, except 'Calculus'. The mapping of content shows that the CCSSM strongly align with SL content in the topics of 'Functions' and 'Geometry and trigonometry' – Voronoi diagrams excluded. Strong alignment with 'Functions' comes from the CCSSM focus on modelling, which is intended to be integrated into many areas of study. Several AHL sub-topics are also present in both topics, most notably vectors are introduced in fourth course study, though graph theory and decision mathematics are not present in any of the CCSSM. Instead of content similar to AI 'Geometry and trigonometry', the CCSSM focus on areas such as similarity, congruence, constructions, circle theorems, and conic sections.

There is a mixture of SL and AHL 'Number and algebra' content present in the CCSSM, which have fourth course standards involving complex numbers and matrices. However, the significant areas of amortization and annuities, eigenvalues and eigenvectors, and laws of logarithms are not included in the CCSSM. In contrast to AI, the CCSSM require the use of proof (for polynomial identities, trigonometric identities, and geometric theorems) and include other AA topics such as the binomial theorem.

For 'Statistics and probability', the CCSSM have strong alignments with the sub-topics of presenting data, measures of central tendency and spread, correlation and linear regression, probability, and the normal distribution – with also some similar ideas involving sampling and populations. However, approximately half of the topic content is not present, including non-linear regression, Binomial and Poisson distributions, transition matrices, and much of hypothesis testing. Overall, the CCSSM have good alignment with both SL and AHL content for 'Functions' and 'Geometry and trigonometry' – though graph theory and decision mathematics are not covered. Also, the CCSSM have a mixture of SL and AHL alignments with 'Number and algebra', less alignment with 'Statistics and probability', and no alignment at all with 'Calculus'. Therefore, AI HL exceeds the content depth of the CCSSM in these respects.

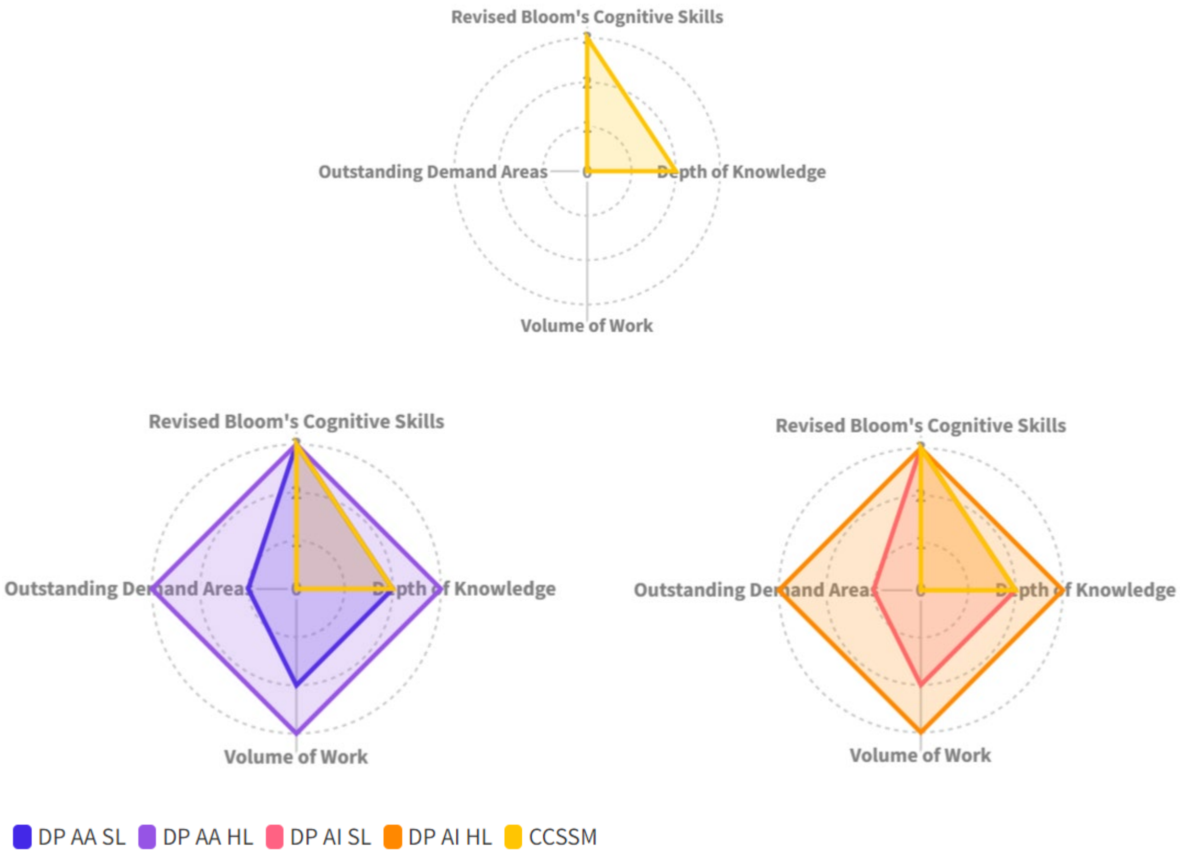
Table 42: CCSSM content which is not covered by DP mathematics subjects

Significant content not in AA (only)	Significant content not in AI (only)
○ Matrices ○ Modelling emphasis ○ Estimating population proportion and mean; developing a margin of error	○ Proof ○ Rational and polynomial functions ○ Binomial theorem ○ Counting principles ○ Absolute value function
Significant CCSSM content not in either DP mathematics subject *	
○ Similarity ○ Congruence ○ Constructions ○ Circle Theorems ○ Conic Sections	

4.7.3 Demand

The DP mathematics curricula and CCSSM were analysed using the same demand tool in order to create a demand profile for DP mathematics: analysis and approaches (SL and HL), DP mathematics: applications and interpretation (SL and HL), and the CCSSM. The CCSSM profile is presented below in the form of a radar diagram, along with two diagrams showing the DP subjects and CCSSM profiles superimposed in one place, enabling immediate visual comparison.

Figure 18: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - DP mathematics subjects received a score of 3.
  - The CCSSM similarly have one set of outcomes and received a score of 3 for their presence of higher order cognitive skills. The standards frequently used evaluation, creation, and analysis through references to critiquing, reflecting, analysing relationships, searching for patterns, making generalisations, reasoning abstractly, improving models, and drawing conclusions.

- Regarding the scores for **Depth of Knowledge**:
  - DP SL subjects received a score of 2 and the HL subjects received a score of 3.

  - The CCSSM were deemed to cover the topics of number, algebra, geometry, trigonometry, statistics, and probability in considerable detail, with evidence of complexity being built and pre-requisite knowledge being used in each. However, the CCSSM are less informative about 'fourth courses', which are available to be taken usually in the last two grades of high school, such as Pre-Calculus, Calculus, and AP Statistics. Hence the topics in the CCSSM alone did not include enough of the material typical of advanced mathematical study to be awarded a score of 3 for depth of knowledge.

- Regarding the scores for **Volume of Work:**
  - DP SL subjects received a score of 2 and the HL subjects received a score of 3.
  - For the CCSSM, discussions acknowledged the difficulty of awarding a score in this category due to the lack of information about teaching hours and the likelihood of variation between states, therefore any score given should be considered as an approximation. Overall, a score of 0 for the volume of work was deemed appropriate for the CCSS, which was based on the standards covered in the first three years of high school, before fourth courses are taken. By the end of the three years, the content covered was deemed comparable to DP SL, though the inclusion of more basic content, due to high school beginning in grade 9, meant that the amount of content is not directly comparable. Regardless of this, the time allocation of three years was judged to be a generous amount of time to cover the standards and lack of contrary evidence meant that the score could not be raised above 0.

- Regarding the scores for **Outstanding Areas of Subject Demand:**
  - DP SL subjects received a score of 1 and HL subjects received a score of 3.
  - In contrast, the CCSSM did not detail much of the content studied in fourth courses, which typically contain more advanced material, thus no areas of outstanding demand were identified and a score of 0 was given for this category.

4.8 France

The school system in France is overseen and regulated by the Ministry for National Education and Youth.[89] Secondary education in France is divided into two cycles – lower and upper secondary education. Upper secondary education lasts three years and is provided to students between the ages of 15 and 18 years old. It is the last three years of upper secondary education that constitutes the *Baccalauréat* (FB).[90]

The general pathway (resulting in the *baccalauréat général*) is the main focus of this study; however, students can also choose:

- the technological pathway, resulting in the *baccalauréat technologique* and;
- the professional pathway, resulting in the *baccalauréat professionnel.*

Study in upper secondary is organised into three years and two pedagogical cycles, namely:

- 1st year: *Seconde* (Grade 10: students aged 15 to 16 years old)
- 2nd year: *Première* (Grade 11: students aged 16 to 17 years old)
- *3rd year: Terminale* (Grade 12: students aged 17 to 18 years old).

Students begin the general pathway in the *Première* year, after completion of the *Seconde* year. The *Seconde* provides an opportunity for students to consolidate and develop their foundational knowledge and skills in order to facilitate a successful transfer from lower to upper secondary education. In the general pathway, students study:

- six common subjects
- three specialty subjects in *Première* and;
- two of these speciality subjects in the *Terminale* year.

Alongside their common and speciality subjects, students can choose one optional subject in the *Première* year and an additional optional subject in the *Terminale* year. Mathematics is offered as a speciality subject in the general pathway and this was the focus of the mathematics subject analysis. See subject description below:

Specialty subjects – FB mathematics (*Première* and *Terminale*)[91,92]

FB mathematics *Première* (FB mathematicsP) and mathematics *Terminale* (FB mathematicsT) are the mathematics specialty subjects offered in the General path of the French Baccalaureate. The subjects follow on from mathematics in the *Seconde* class and students can study FB mathematicsP and FB mathematicsT, or FB mathematicsP only. FB mathematicsP provides a substantial amount of knowledge in the areas of Algebra, Analysis, Geometry, Probability and Statistics and Algorithmics and Programming. FB mathematicsT builds on the knowledge of FB mathematicsT and prepares students for studying mathematics in higher education.

[89] Government of France, Ministry for National Education and Youth. (2023*). Let's build a committed education system together!* Available: https://www.education.gouv.fr/let-s-build-committed-education-system-together-100037

[90] Ibid.

[91] Ministry of National Education and Youth, France. (2019). *Annexe. Programme de mathématiques de première générale* spe632_annexe_1063168.pdf (education.gouv.fr)

[92] Ministry of National Education and Youth, France. (2019). Annexe. Programme de spécialité de mathématiques de terminale Générale. https://eduscol.education.fr/document/24568/download

4.8.1 Learning Outcomes

This section compares and contrasts the learning outcomes of curricula falling within the category of mathematics. For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics applications and interpretation. The learning outcomes for the FB can be drawn from the 'Major Intentions' set out in the curriculum documentation, which include the six 'Math Skills' that are to be developed. The 'Major Intentions' and 'Math Skills' are mostly the same for the *Seconde* class, and the first and final year of the general path, with only slight contextual differences – e.g. *Seconde* class prepares students to enter the general or technology paths, and FB mathematicsT prepares them for higher education. These small differences do not affect the overall alignment to the DP; as such, the analysis has considered the FB learning outcomes as one set. The following summary table demonstrates the learning outcome themes that were extracted from DP mathematics and indicates if and where they were judged to have presence within the learning outcomes of the FB subjects reviewed.

Table 43: Presence of the DP mathematics subject group learning outcome themes in FB curricula

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in FB	
1. Being aware of, and engaging with, mathematics in its wider context		Not present in the Major Intentions
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Not present in the Major Intentions, but some skills are referred to in the 'guidelines for teachers' section
3. Using inquiry-based approaches		Mathematical research is one of the six math skills to be developed
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present in the Major Intentions and math skills
5. Making links and generalisations		Using generalisations and abstraction is present in the Major Intentions. Making links, such as to other subjects, between mathematical concepts, or to real-life is not present
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Reasoning and modelling are math skills to be developed. All math skills are to be utilized during problem-solving
7. Communicating mathematics clearly and in various forms		Communication is one of the six math skills to be developed.
8. Knowing how technology and math influence each other and using technology to develop ideas and solve problems		Software tools referenced in relation to the research math skill

Key:

	This theme is well-evidenced in the learning outcomes of the FB.		*This theme is partially evidenced in the learning outcomes of the FB.*		*This theme is not evident in the learning outcomes of the FB.*

Presence of the DP's Learning Outcome Themes

There is reasonable alignment between the DP's mathematics learning outcomes and those of FB, with most of the DP's themes being well evidenced in its Major Intentions. However, it can be noted that the FB's 'Math Skills' are not described in as much detail as the aims and assessment objectives in the DP.

1. Being aware of, and engaging with, mathematics in its wider context

The DP theme of awareness and engagement with wider contexts is not clearly evidenced in the FB's 'Major Intentions' section. Indeed, the FB skills do not include considering moral, social, and ethical questions, or appreciating the universality of mathematics and multicultural, international and historical perspectives. However, it can be noted that each main topic begins with a 'History of Mathematics' section which teachers may choose to explore with students.

2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work

The DP theme of transferable learning skills is not evidenced in the FB's 'Major Intentions' section. Indeed, there is no reference to reflecting on work, developing a positive and resilient attitude, or working collaboratively. However, it can be noted that some transferable learning skills are referenced in other sections of the FB curriculum, such as in 'Some guidelines for teaching'.

3. Using inquiry-based approaches

'Mathematical research' is one of the 'Mathematics Skills' targeted by the FB curriculum. This aligns well with the DP's learning outcome theme of using inquiry-based approaches throughout the study of mathematics.

4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts

The DP theme of understanding mathematics is evidenced in the FB curriculum through references to applying mathematical techniques and exploring the effectiveness of such mathematical techniques.

5. Making links and generalisations

The DP's theme of making links and generalisations is partially evidenced in the FB curriculum. The use of generalisation and abstraction is explicitly mentioned in the 'Major Intentions' section, which aligns with one of the aims outlined for the DP mathematics subject group. However, making links, such as within mathematics, to other disciplines, or to real-world contexts is not well evidenced in the 'Major Intentions'.

6. Developing critical/creative thinking skills e.g. problem-solving and reasoning

Similarly to the DP, the FB focuses on critical thinking by listing six 'Math Skills' (research, model, represent, calculate, reason, and communicate) that are to be utilised in problem-solving. The presence of critical thinking is further evidenced in the FB in its description that oral communication "allows everyone to change their thinking, even to question it if necessary, to gradually access the truth through proof".[93]

[93] Ibid.

7. Communicating mathematics clearly and in various forms

Communication is well-evidenced in the FB, as it is one of the six 'Math Skills' to be developed. In the FB, students are expected to communicate both orally and in writing. There is also an oral communication section within the 'Major Intentions' section which describes students using oral skills to explain and justify their reasoning and lists various learning situations where students can demonstrate this.

8. Knowing how technology and math influence each other and using technology to develop ideas and solve problems

The use of technology is highlighted through the research skill listed in the FB, as well as linking this to the inquiry-based approach to learning. Furthermore, additional reference to the 'Use of software tools' in the 'Major Intentions' section prescribes the regular use of technology by teachers and students, thus reinforcing that this is a similarly important theme in the FB.

Other Themes in the FB

Most of the themes or skills described in FB are in the DP; however, it can be noted that the skill of representations is highlighted more strongly in the FB. Indeed 'represent' is listed as one of the six 'Math Skills' and describes students being able to choose and change between numerical, algebraic, geometric, and other representations. Furthermore, whilst communication is an important theme in the DP, the FB places an additional explicit emphasis specifically on oral communication.

Summary

Overall, there is a moderate level of alignment between FB and the DP with regards to mathematics learning outcomes. Like the DP, FB promotes critical thinking skills, communication in different contexts, the use of technology in investigation and problem-solving, using generalisation and abstraction, understanding concepts and application, and the use of inquiry approaches. However, themes focused on contemplation of wider contexts and development of transferable learning skills have less emphasis in FB. Conversely, FB makes more explicit reference to the use of mathematical representations and particularly emphasises oral communication of mathematics.

4.8.2 Content

This section compares and contrasts the content of the DP and FB curricula falling within the category of mathematics. The FB speciality mathematics subject content is presented below in a diagram which shows the key topics and subtopics.

Figure 19: FB mathematics content visualiser for the first and final year of mathematics in the general path

FB mathematicsP	Algebra	Numerical sequences, discrete models	Equations, quadratic polynomial functions					
	Analysis	Derivation	Variations and curves representative of functions	Exponential function	Trigonometric function			
	Geometry	Vector calculation and scalar product	Referenced geometry					
	Probability and statistics	Conditional probabilities and independence	Real random variables					
	Algorithms and programming	List						
FB mathematicsT	Algebra and Geometry	Combinatorics and counting	Manipulation of vectors, lines and planes in space	Orthogonality and distances in space	Parametric representations and Cartesian equations			
	Analysis	Suites	Limits of functions	Supplements on derivation	Logarithm function	Sine and cosine functions	Primitive, differential equations	Integral calculus
	Probabilities	Succession of independent proofs, Bernoulli diagram	Concentration, law of large numbers					
	Algorithms and programming	List						

4.8.2.1 Structure

All FB students study mathematics in the *Seconde* class, which is the first year of high school, before choosing whether to enter the general or technological pathway for their final two years. Students in the general pathway of the FB can choose mathematics as one of their three specialty subjects. The study of mathematics in the general pathway is split into mathematics *Première* and mathematics *Terminale*, studied in the first and final year, respectively. Similar to the DP's SL and HL, this offers different levels of study to FB students, as they can choose to study only *Première*, or *Première* and then *Terminale*. Notably, the FB does not offer students a choice of different mathematics subjects with a distinct thematic focus, as the DP does with AA and AI.

Similar to DP mathematics, the content of FB mathematicsT and FB mathematicsP is divided into four or five main topic areas, though, unlike SL and HL, these slightly change between FB mathematicsT and FB mathematicsP. Generally, the main mathematics topics of the general pathway are: Algebra, Analysis, Geometry, Probability and Statistics, and Algorithms and Programming. This is similar to the DP, except that functions and calculus content is not a separate topic area, but rather studied under 'Analysis'. Furthermore, Algorithms and Programming is not a main topic of the DP and is a cross-cutting content area in the FB.

4.8.2.2 Content Alignment

The table below presents a simplified summary of the extent to which FB mathematics aligns with the main topics of the DP's subjects. FB mathematicsT is studied following FB mathematicsP; as such, the alignments for the latter are carried over to the former.

Table 44: Summary of the content alignment FB mathematics has with the main topics in AA

AA topics		FB mathematicsP	FB mathematicsT
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in the FB.		*Partial presence of this topic in the FB.*		*Little or no presence of this topic in the FB.*

* Where applicable, content alignments found in assumed knowledge or pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

Table 45: Summary of the content alignment the FB mathematics has with the main topics in AI

AI topics		FB mathematicsP	FB mathematicsT
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in the FB.		*Partial presence of this topic in the FB.*		*Little or no presence of this topic in the FB.*

* Where applicable, content alignments found in assumed knowledge or pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

FB mathematicsP

<u>*Mathematics: analysis and approaches*</u>

The mapping of content shows that FB mathematicsP has only partial alignment with DP AA SL content across topics, as often several significant subtopics are not covered.

From DP AA SL Number and Algebra content, FB mathematicsP includes content on arithmetic and geometric sequences and series, though presence of financial applications and sum of infinite converging geometric sequence is unclear. Instead, it can be noted that FB mathematicsP introduces limits with sequences and includes finding limits of sequences in simple cases. FB mathematicsP also covers recursively defined sequences. Alignment with the rest of this topic area is limited due to logarithms not being covered until FB mathematicsT. There are no significant alignments with AHL content.

There is reasonable alignment with DP AA SL Functions content, as function concepts, quadratic equations and inequalities, quadratic functions, and exponential functions are covered. However, the level of alignment is reduced by logarithmic, rational, and composite functions not being covered. Moreover, transformations are also not explicitly included in the FB mathematicsP content. Similarly, FB mathematicsP covers some of DP AA SL's Geometry and Trigonometry content, such as circles and radians, definition and exact values of sin and cos, the Pythagorean identity, and functions of sin and cos. However, it does not include the tan function, other identities, or solving trigonometric equations. Instead, FB mathematicsP builds on the vector content introduced in the 2nd class and looks at the scalar product in detail, solving geometric problems, and circles and parabolas. As such, the FB mathematicsP has partial alignment with AHL content in this topic area.

Furthermore, FB mathematicsP covers probability content, including conditional probability and formula, in similar detail to DP AA SL. FB mathematicsP also introduces random variables, including finding the expectation, variance and standard deviation. Thus, there is partial alignment with DP AA SL's Statistics and Probability content. However, FB mathematicsP does not cover correlation and linear regression, or the binomial and normal distributions. Similarly, FB mathematicsP covers some of the DP AA SL's Calculus concepts, such as derivatives of some functions, tangents, and derivative rules, though it does not cover any subtopics related to integration.

A significant area that FB mathematicsP covers, which is not included in DP AA, is Algorithms and Programming. This is a cross-cutting area which is embedded and developed in all content areas of FB mathematicsP – i.e. Algebra, Geometry, and Probability. Students use Python and perform tasks such as writing a Python function that returns the mean of a sample size n or a random variable. Building on the *Seconde* year, FB mathematicsP also consolidates the notions of variable, conditional instruction and loop and the use of functions, as well as introducing the concept of list to students.

Overall, FB mathematicsP has only partial alignment with DP AA SL content in all topics, and also features a few AHL subtopics on vectors. In contrast with the DP AA subject, FB mathematicsP includes a strong focus on algorithms and programming, which are embedded in all content areas.

Mathematics: applications and interpretation

Similarly to DP AA, FB mathematicsP partially aligns with DP AI SL content in most topics and has partial alignment with AHL content in Geometry and Trigonometry due to the inclusion of vectors, radians, sin and cos definitions, and Pythagorean identity. Where there are alignments with the DP AI content, this tends to be content shared between DP AA and DP AI; thus, the subtopics flagged above as being present in the FB mathematicsP remain relevant for DP AI. As such, to avoid repetition, this section will focus on comparison to the specific DP AI content that is not covered by the DP AA.

FB mathematicsP has partial alignment with DP AI SL's Number and Algebra content, as there is some evidence of financial applications in FB mathematicsP, including amortization and annuities. There is no alignment with AHL content as logarithms, complex numbers, and matrices are not included. As noted above, FB mathematicsP instead includes recursive sequences and introduces limits of sequences.

FB mathematicsP has partial alignment with DP AI SL's Functions content due to FB mathematicsP including function concepts, and quadratic, exponential and trigonometric functions. FB mathematicsP includes modelling, though it is not explicit that this is with an as broad range of functions as observed in DP AI SL. FB mathematicsP has no alignment with AHL content as it does not feature composite functions, transformations, and modelling with harder functions.

FB mathematicsP has partial alignment with DP AI's Geometry and Trigonometry SL and AHL content due to FB mathematicsP including trigonometry, circles, radians, sin and cos definitions, the Pythagorean identity, and similar vector content. However, FB mathematicsP does not include Voronoi diagrams, matrix transformations, the cross product, graph theory, or decision mathematics.

There is limited alignment between FB mathematicsP and the DP AI SL's Statistics and Probability content due to FB mathematicsP not including content on correlation and linear regression, the binomial and normal distributions, or hypothesis testing. FB mathematicsP also does not cover DP AI AHL topics in this area. For Calculus, FB mathematicsP has partial alignment with DP AI SL content as it covers derivatives of a few functions, increasing and decreasing functions, tangents, maximum and minimum point, and optimisation problems. However, integration is not introduced in FB mathematicsP.

As noted above, FB mathematicsP has a strong focus on algorithms and programming, which are embedded. Use of technology also has a strong emphasis in DP AI; however, Python is not used.

Overall, FB mathematicsP has partial alignment with DP AI SL content, with a similarly high focus on technology, though differs in its use of algorithms and programming.

*Table 46: FB mathematicsP content which is not covered in the DP**

* Significant content does not include topics which are typically studied *prior* to upper secondary.

Significant content not in AA (only)	Significant content not in AI (only)
	• Limits of sequences (simple cases)
Significant content not in either DP mathematics subject	
• Algorithms and programming. Use of Python.	

FB mathematicsT

Mathematics: analysis and approaches

FB mathematicsT follows FB mathematicsP, meaning that all the subtopic alignments identified above still apply. As such, to avoid repetition, this section focuses on the new content covered by FB mathematicsT.

For DP AA SL's Number and Algebra content, FB mathematicsT's alignment with the DP is increased compared to that of FB mathematicsP through its inclusion of logarithms. FB mathematicsT also has further alignment with the DP AA SL and AHL subtopics of proof and systems of equations; although not a separate topic in themselves, there is evidence students learn some proof techniques over the course and look at solving systems of equations within vectors. That said, the DP AA AHL subtopics relating to complex numbers are not covered in FB mathematicsT. Instead, FB mathematicsT builds further on limits of sequences from FB mathematicsP, including definitions, convergence and divergence, bounded and unbounded sequences, and boundary conditions.

FB mathematicsT also has stronger alignment with the DP AA SL content of Functions as it covers logarithm functions, though most AHL content is not covered, nor are transformations. Trigonometric functions and vectors are revisited, which increases alignment with DP AA SL and AHL content on Geometry and Trigonometry. Indeed, FB mathematicsT covers solving trigonometric equations, vector equations of planes, parametric representations and cartesian equations, and intersections. However, FB mathematicsT does not cover the same range of trigonometric identities, reciprocal functions, or solving harder trigonometric equations as the DP AA.

FB mathematicsT has good alignment with DP AA SL Calculus content as it includes the second derivative, graphical representations of $f, f' and\ f''$, and integration. There is also partial alignment with AHL content as FB mathematicsT covers continuity, differentiability, limits, integration methods, and differential equations. However, it can be noted that implicit differentiation and Maclaurin series are not covered by FB mathematicsT, and the latter covers fewer derivate and antiderivatives of functions, such as $\cot x$, than the DP AA. However, FB mathematicsT covers limits of functions in considerable detail.

For Statistics and Probability, FB mathematicsT covers binomial distributions in detail, but it does not align strongly with DP AA SL as it does not cover correlation and linear regression nor the normal distribution. Instead, FB mathematicsT focuses on sums of random variables – their expectation and variance – and the Bienayme-Chebyshev inequality and the law of large numbers.

As with FB mathematicsP, FB mathematicsT covers Algorithm and Programming content, further consolidating the concepts from FB mathematicsP.

Overall, FB mathematicsT, which represents the cumulative mathematics content of the general pathway, has strong alignment with DP AA SL content in the topics of Number and Algebra, Functions, and Calculus; partial alignment with SL Statistics and Probability and Geometry and Trigonometry; and partial alignment with DP AA AHL content of Number and Algebra, Geometry and Trigonometry, and Calculus. Although not alwaysf fully aligning in content, students studying FB mathematicsT will experience a similar breadth and depth of content to that of DP AA HL. This is due to certain topics covering different concepts, the inclusion of algorithms and programming, and the results and theorems that FB mathematicsT frequently incorporates in several topics.

Mathematics: applications and interpretation

The mapping of content shows that the additional content covered in FB mathematicsT only significantly increases alignment with the DP AI topics of Number and Algebra at SL, Statistics and Probability at SL and Calculus at both SL and AHL. The rest of the alignments remain the same, as FB mathematicsT covers content more similar to that of DP AA, such as limits and continuity, proof, and vectors, in addition to content not in the DP.

As mentioned in regard to DP AA, FB mathematicsT covers the binomial distribution but does not cover correlation and linear regression, the normal distribution, and DP AI-specific subtopics, such as hypothesis testing, nonlinear regression, the Poisson distribution, and transition matrices. Instead, FB mathematicsT covers topics such as the Bienayme-Chebyshev inequality and the law of large numbers. For Calculus, FB mathematicsT covers DP AI AHL subtopics, such as the second derivative, derivative rules, integration, area, differential equations, and Euler's method. FB mathematicsT also covers topics which are not in DP AI, such as integration by parts, and limits and continuity. However, it can be noted that FB mathematicsT does not cover the DP AI subtopics of slope fields, phase portraits, and second order differential equations.

For Number and Algebra, FB mathematicsT covers the DP AI SL content of approximation and errors, as well as the DP AI AHL content on laws of logarithms. FB mathematicsT also covers content not in DP AI, such as combinatorics and limits of sequences. However, FB mathematicsT does not include complex numbers or matrices, which are covered in DP AI.

With regards to other topics, FB mathematicsT does not include graph theory and decision mathematics subtopics that are found in the DP AI AHL Geometry and Trigonometry content, nor does it include modelling with functions covered in AHL.

Table 47: FB mathematicsT content which is not covered in the DP

Significant content not in AA (only)	Significant content not in AI (only)
N/A	• Limits of functions • Continuity and differentiability • Combinatorics • Vector planes • Proof
Significant content not in either DP mathematics subject	
• Algorithms and programming. Use of Python. • Limits of sequences content • The Bienayme-Chebyshev inequality • Law of large numbers	

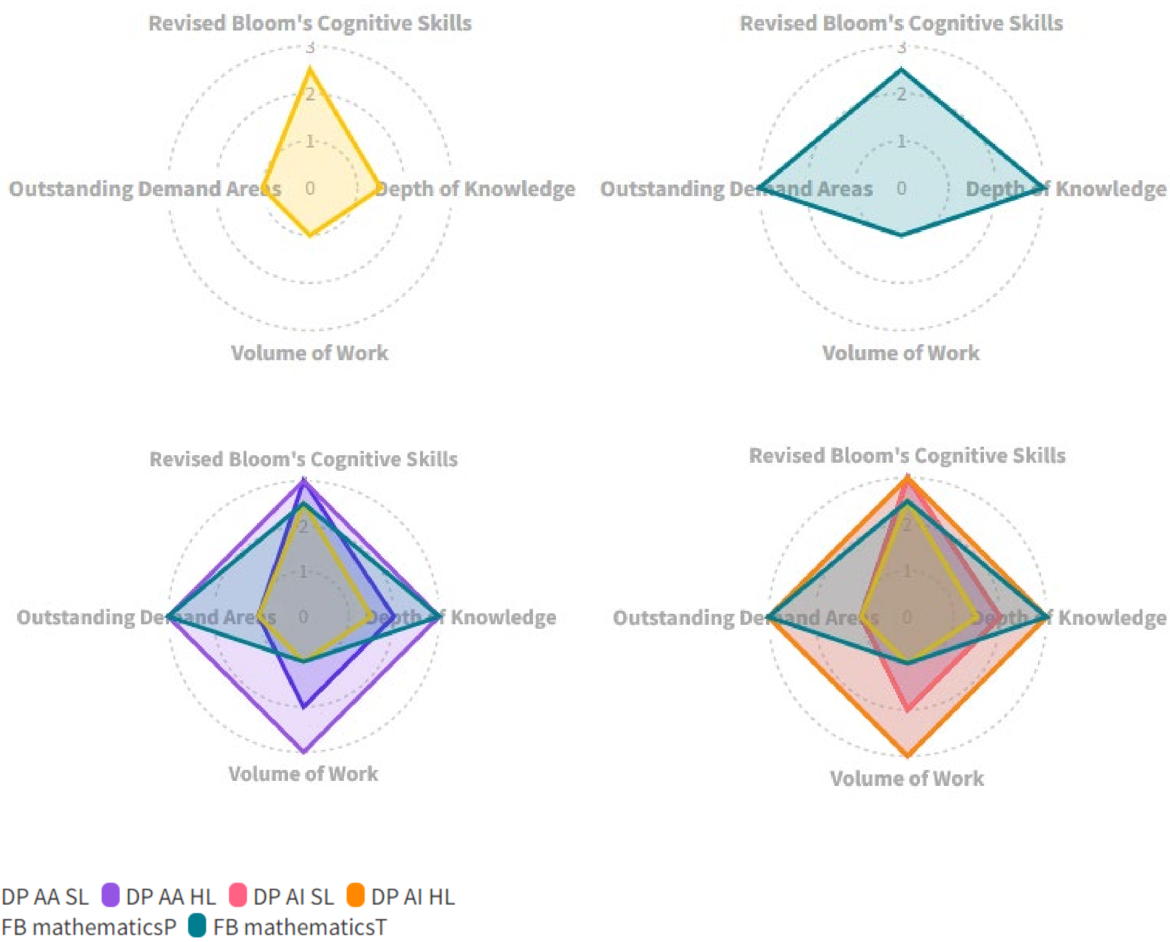
Overall, FB mathematicsT has strong alignment with the DP AI SL's Number and Algebra and Calculus content, and partial alignment with DP AI SL content in other topics, as well as the DP AI AHL content in Geometry and Trigonometry and Calculus. FB mathematicsT covers a significant amount of content that is not in DP AI, some of which is in DP AA (see section above). As such, although content alignment is not strong, the breadth and depth of FB mathematicsT is comparable to that of DP AI HL.

4.8.3 Demand

This section considers the alignment between the DP and FB mathematics curricula in terms of demand.

The DP and FB curricula were analysed using the same demand tool in order to create a demand profile for DP AA (SL and HL), DP AI (SL and HL), FB mathematicsP, and FB mathematicsT. FB mathematicsT represents the demand of mathematics of the general path of the FB. These demand profiles are presented below in the form of radar diagrams, with superimposed diagrams also being featured to enable immediate visual comparison.

Figure 20: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - The DP mathematics subjects scored 3 in this category.
  - The learning outcomes of FB mathematicsP and mathematicsT are the same too, as they share a common list of 'Math Skills'. The panel judged that there was evidence of higher order thinking skills in the documentation, especially with regards to problem solving being the culmination of all these skills. Furthermore, there was reference to investigation and validating or invalidating models. Overall, a score of 2.5 was deemed appropriate for both subjects, as there was not enough evidence to warrant them a 3.
- Regarding the scores for **Depth of Knowledge**:
  - Both DP mathematics subjects (AA and AI) at SL were given a score of 2. At HL, both DP mathematics subjects were awarded a score of 3 for depth of knowledge.
  - For the FB, FB mathematicsP received a score of 1.5 as some topics are studied in considerable depth – e.g. vectors. However, depth of study was not consistent across all topics; other topics, such as probability and statistics, were studied in considerably lesser detail. FB mathematicsT received a score of 3 for depth of

knowledge, as the panellists observed that there was a considerable step up from FB mathematicsP to FB mathematicsT. Each content area in FB mathematicsT was covered in a high level of detail, with the content providing good preparation for university level mathematics-related courses.

- Regarding the scores for **Volume of Work**:
  - Both DP mathematics subjects at SL were deemed to comprise a moderate-heavy volume of work and were given a score of 2. For HL, both DP mathematics subjects were considered to have a heavy volume of work, which merited a score of 3.
  - For the FB, both FB mathematicsP and mathematicsT were deemed to have a moderate volume of work and thus were given a score of 1. FB mathematicsP is taught for four hours every week, for a total of 128 hours in a year, which was judged to be a standard time allocation for the content to be covered. FB mathematicsT features a higher number of complex topics than FB mathematicsP; however, it also features a higher number of teaching hours per week, going from four hours in FB mathematicsP to six in FB mathematicsT. Overall, over the two years, there is a total of 320 teaching hours for mathematics in the FB *général*, which was deemed to be a standard amount of time for the content covered.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - Both DP mathematics subjects at SL contained one area of outstanding demand, which was the ‘mathematical exploration’. The HL subjects had a high number of outstanding areas of demand and thus received a score of 3.
  - For FB subjects, a score of 1 was given to FB mathematicsP for its inclusion of programming with Python and its coverage of vectors. For FB mathematicsT, a score of 3 was given, with proof, sequences, vectors, calculus, and algorithms and programming, all containing subtopics which demonstrated opportunity for challenge and are beyond the usual scope of upper secondary mathematics.

4.9 Spain

The school system in Spain is overseen by the Ministry of Education and Vocational Training, but is decentralised into 17 autonomous regions. As such, while the Ministry oversees the core curriculum and assessment, the regions can determine specific procedures and dictate some subjects within the overall selection.

Overall, the Spanish system is divided into primary school, lower secondary education, and upper secondary education. Upper secondary education is non-compulsory and lasts for two years, being attended by students between the ages of 16 and 18 years old.[94] During upper secondary, students can take the Spanish *Bachillerato* (SB).

The SB is offered through four different streams (i.e. modalities), which students can choose from, in line with their specific interests and career plans:

- the Arts modality (includes two pathways);
- the Sciences and Technology modality;
- the Humanities and Social Sciences modality; and
- the General modality.

Students from all modalities study a range of subjects from different categories, including:

- **Common subjects:** these are subjects taken by all students regardless of their modality.
- **Modality subjects:** these are subjects taken by students in the same modality.
- **Elective subjects:** these are optional subjects that students can choose to study.

In addition to the common and modality subjects above, students in the SB also study elective subjects. The specific subjects offered are determined by the Spanish regional educational authorities but should include at least one second foreign language.[95]

The mathematics studied as part of the Science and Technology modality is the main focus of the mathematics comparison analysis. The below table displays the subjects within this modality.

[94] Government of Spain, Ministry of Education and Vocational Training (n.d). Compulsory Secondary Education, https://educagob.educacionyfp.gob.es/ensenanzas/secundaria.html ; Government of Spain, Ministry of Education and Vocational Training (n.d). Bachiller Certificate, https://educagob.educacionyfp.gob.es/ensenanzas/bachillerato.html

[95] Government of Spain, Ministry of Education and Vocational Training (n.d). General information of the Bachillerato. Elective subjects. https://educagob.educacionyfp.gob.es/ensenanzas/bachillerato/informacion-general/organizacion.html

Table 48: Subjects offered in the Science and Technology modality

Modality	Year of study	Subjects
Science and Technology	Year 1	Mandatory subject: Mathematics I
		Two subjects to be chosen from: • Biology, geology and environmental sciences • Technical drawing I • Physics and chemistry Technology and engineering I
	Year 2	Mandatory subject: Mathematics II or Mathematics Applied to Social Sciences II
		Two subjects to be chosen from: • Biology • Technical drawing II • Physics • Geology and environmental sciences • Chemistry Technology and engineering II

The following subjects from the Science and Technology Modality were used in the mathematics subject comparisons.

SB mathematics I[96]

Mathematics I (SB mathematics I) is studied only by students in the Science and Technology modality of the SB. The subject is compulsory in the first year and prepares students for the second year, where either mathematics II or mathematics applied to social sciences II is studied. The mathematics content studied in this subject covers a broad range of topics that will contribute to preparing students for mathematics and science courses in higher education.

SB mathematics II[97]

Mathematics II (SB mathematics II) is only offered in the Science and Technology modality and is studied in the second year. Building on SB mathematics I, this subject revisits topics to cover more depth and advanced concepts, as well as introducing new topics. On completing SB mathematics II, students will be well prepared for higher education courses in mathematics and the sciences. The cumulative study of SB mathematics I and II aims to provide conceptual understanding through problem-solving, reasoning, and mathematical research. These subjects also include applications and analysis of issues relating to science and technology.

4.9.1 Learning Outcomes

This section compares and contrasts the learning outcomes of curricula falling within the category of mathematics.

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics: applications and interpretation. The learning outcomes for the SB mathematics subjects are represented by nine 'specific

[96] Ministry of Education, Spain. (2023). *Math*. Available from: https://educagob-educacionyfp-gob-es.translate.goog/curriculo/curriculo-lomloe/menu-curriculos-basicos/bachillerato/materias/matematicas/desarrollo.html?_x_tr_sl=es&_x_tr_tl=en&_x_tr_hl=en-US&_x_tr_pto=wapp

[97] Ibid.

competences', each of which have some 'evaluation criteria'. The specific competences and their evaluation criteria are mostly the same for all mathematics subjects.[98] The following summary table demonstrates the learning outcome themes that were extracted from DP mathematics and indicates if and where they were judged to have presence within the learning outcomes of the SB mathematics subjects reviewed.

Table 49: Presence of the DP mathematics subject group learning outcome themes in SB curricula

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the SB	
1. Being aware of, and engaging with, mathematics in its wider context		Particularly present in evaluation criteria 6.2, which looks at the contribution of mathematics to progress and to solutions for challenges in society
2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		Strongly present in specific competence 9 which centers on personal and social skills
3. Using inquiry-based approaches		Particularly present in specific competence 3 which speaks to formulating and investigating conjectures
4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts		Present in several competences which refer to mathematical ideas, concepts, and contexts
5. Making links and generalisations		Strongly present in the specific competences 5 and 6 which refer to demonstrating an integrated mathematical vision and making connections to real world and other areas of knowledge
6. Developing critical/creative thinking skills e.g. problem-solving and reasoning		Strongly present in several competences, with frequent references to solving problems, using reasoning, modelling, modifying, and creative thinking
7. Communicating mathematics clearly and in various forms		Particularly present in specific competence 8 which centers on communication
8. Knowing how technology and SB mathematics Influence each other and using technology to develop ideas and solve problems		Present in evaluation criteria 1.1 and specific competence 3, both referring to use of technological tools

Key:

	This theme is well-evidenced in the learning outcomes of the SB.		*This theme is partially evidenced in the learning outcomes of the SB.*		*This theme is not evident in the learning outcomes of the SB.*

Presence of the DP's Learning Outcome Themes

There is strong alignment between the DP mathematics learning outcomes and those of the SB, with each of the DP's themes being well evidenced in the specific competences and evaluation criteria.

[98] SB mathematics II, uses slightly different adjectives from those used in the first year subject SB mathematics I, in order to show a progression in skills; for example, it uses 'select' instead of 'use'. These small nuances do not affect the overall alignment to the DP.

1. Being aware of, and engaging with, mathematics in its wider context

Similar to the DP, the SB learning outcomes encourage consideration of the wider contexts of mathematics. Indeed, the SB refers to contexts such as sustainability, responsible consumption, and equity. Furthermore, all subjects analyse the contribution of mathematics to progress and in proposing solutions to solving issues in society – with SB mathematics I and II focusing on scientific and technological challenges. It can be noted that this is described as 'reflecting' on the contribution of mathematics in SB mathematics I, whilst in SB mathematics II it is described as 'valuing' the contribution of mathematics. However, unlike the DP, it can be noted that the outcomes do not explicitly state that ethical questions or multiple perspectives will be considered.

2. Developing learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work

The DP's theme of transferable learning skills is well evidenced in the SB, primarily in specific competence 9, which centres on personal and social skills. Similarly to the DP, this competence includes skills such as learning from mistakes (reflecting), perseverance, having a positive attitude, learning from criticism, and working effectively with others. It can be noted here that, in the first year, students will identify the most conducive social skills, while, in the second year, they will apply them.

3. Using inquiry-based approaches

In addition, the use of inquiry approaches is also present in the SB, with specific competence 3 describing that students will acquire new knowledge through formulating conjectures and investigation. It can be noted here that this will be done in a guided way in the first year subjects and autonomously in second year subjects.

4. Understanding the concepts, principles and nature of mathematics and applying concepts and procedures to a range of contexts

Many competences, including 5 and 6, also evidence the theme of understanding mathematics and applying it, as students interrelate different concepts and procedures. Application to specific contexts is especially present, with reference to solving problems in mathematical contexts, in daily life, in various situations, in the real-world, and in the field of science and technology.

5. Making links and generalisations

The DP theme of making links is an especially strong theme in the learning outcomes of the SB. Specific competence 5 centres on establishing connections between mathematical concepts and procedures, such that students should demonstrate an integrated mathematical vision and apply connections whilst solving problems. Moreover, specific competence 6 centres on making links to other areas of knowledge and the real world. It can also be noted that SB mathematics II expects students to be able to generalise algorithms.

6. Developing critical/creative thinking skills, e.g. problem-solving and reasoning

Similar to the DP, critical thinking skills are strongly present in the SB learning outcomes. Most of the nine specific competences incorporate critical thinking skills such as problem solving, reasoning, analysing, evaluating, creativity, and being innovative. Some specific ways that students will demonstrate critical thinking skills include: evaluating the efficiency of strategies and tools, verifying the validity of solutions, selecting the most appropriate solution/tools/strategies according to context, structuring reasoning, modifying and creating algorithms, assessing usefulness of representations, and modelling and solving problems of daily life and of science and technology.

7. Communicating mathematics clearly and in various forms

Similar to the DP, the SB also expects students to communicate mathematics clearly in various contexts, using appropriate notation and terminology. Indeed, specific competence 8 requires students to demonstrate organisation, precision, and rigor in communication, as well as different contexts, both individually and collectively.

8. Knowing how technology and maths influence each other and using technology to develop ideas and solve problems.

Specific competence 3 describes that students will acquire new knowledge through formulating conjectures and investigation which links to the DP theme of technology, as it states that students will use technology in the formulation or investigation of conjectures or problems. Technology is also present in specific competences 1 and 7, which describe students selecting and using appropriate tools in modelling and solving problems. Alignment with the DP here is unsurprising, as SB mathematics I and II reside within the Science and Technology modality.

Other Themes in the SB

Most of the specific competences in SB subjects relate to a DP theme; however, there are a few skills in the SB which are more explicit or emphasised than in the DP. These are primarily located in specific competences 4, 5, 7, and 9, which focus on computational thinking, connections, representations, and personal and social skills.

Firstly, specific competence 4 has a more explicit focus on students using computational thinking to model and solve problems, though it can be noted that the DP similarly describes students using logical thinking in problem solving. Moreover, whilst the DP learning outcomes promote links to the real world and other subjects, the SB additionally emphasises making links between mathematical concepts and processes, which encourages deeper conceptual understanding and flexible thinking. The SB also has more explicit focus on representations, as specific competence 7's evaluation criteria expects students to 'select and use various forms of representation, assessing their usefulness for sharing information'.[99] Finally, the SB extends the personal and social skills also mentioned in the DP to include managing emotions, promoting group wellbeing, and making decisions – though it can be noted here that these skills are reflected in the IB learner profile.

Summary

Overall, there is a high level of alignment between the SB and DP subjects with regards to mathematics learning outcomes. Like the DP, the SB promotes critical thinking skills, consideration of global issues and contexts, transferable learning skills, clear communication, use of technology, making connections, and use of inquiry approaches. Thus, the SB takes a similarly holistic approach to the DP with regards to mathematics learning. The SB specific competences are more detailed than the DP's, which occasionally contribute to more skills

[99] Ministry of Education, Spain. (2023). *First year – Mathematics I. Specific skills*. Available from: https://educagob-educacionyfp-gob-es.translate.goog/curriculo/curriculo-lomloe/menu-curriculos-basicos/bachillerato/materias/matematicas/criterios-eval-primer-curso.html?_x_tr_sl=es&_x_tr_tl=en&_x_tr_hl=en-US&_x_tr_pto=wapp

being described, though most of these relate to a DP theme. However, it can be noted that the SB places more emphasis than the DP on computational thinking, representations, making connections within mathematics, and personal and social skills. The small differences in wording of SB mathematics II evaluation criteria shows a progression in skills from SB mathematics II; however, this does not affect overall alignment with the DP themes.

4.9.2 Content

This section compares and contrasts the content of the DP and SB curricula falling within the category of mathematics. The SB's mathematics I and II content is presented below in the diagrams showing the key topics and subtopics included in each.

Figure 21: SB mathematics: content visualiser for mathematics I

Mathematics I	A. Number sense	1. Sense of operations	2. Relationships			
	B. Meaning of measure	1. Measurement	2. Change			
	C. Spatial sense	1. Two-dimensional geometric shapes	2. Location and systems of representation	3. Visualization, reasoning and geometric modeling		
	D. Algebraic sense	1. Patterns	2. Mathematical model	3. Equality and inequality	4. Relations and functions	5. Computational thinking
	E. Stochastic sense	1. Organization and analysis of data	2. Uncertainty	3. Inference		
	F. Socio-affective sense	1. Beliefs, attitudes and emotions	2. Teamwork and decision making	3. Inclusion, respect and diversity		

Figure 22: SB mathematics: content visualiser for mathematics II

Mathematics II	A. Number sense	1. Sense of operations	2. Relationships			
	B. Meaning of measure	1. Measurement	2. Change			
	C. Spatial sense	1. Geometric forms of two and three dimensions	2. Location and systems of representation	3. Visualization, reasoning and geometric modeling		
	D. Algebraic sense	1. Patterns	2. Mathematical model	3. Equality and inequality	4. Relations and functions	5. Computational thinking
	E. Stochastic sense	1. Uncertainty	2. Probability distributions			
	F. Socio-affective sense	1. Beliefs, attitudes and emotions	2. Decision making	3. Inclusion, respect and diversity		

4.9.2.1 Structure

The SB modality that students choose dictates the options they have for the study of mathematics. Students in the General modality study General Mathematics in the first year; those in the Humanities and Social Sciences modality may choose to study mathematics applied to social sciences I and II in the first and second year, respectively; and students in the Science and Technology modality are required to study SB mathematics I in the first year, and either SB mathematics II or mathematics applied to social sciences II to be studied in the second year. No mathematics is offered in the Arts modality. Therefore, unlike the DP, the SB has pathways in which the study of mathematics is optional and not required.

Focusing on the Science and Technology modality, students may choose a purer mathematics route by studying SB mathematics II, or take a more applied route by studying mathematics applied to social sciences II. This has similarities to the offering of AA and AI in the DP, as both routes have a different focus, whilst sharing content. However, the content size of mathematics applied to the social sciences II is smaller than SB mathematics II, hence this differs to the DP, where AA and AI are designed to be of similar size to one another.

Like the DP, the Science and Technology modality mathematics content is structured into the same five main topic areas for both SB mathematics I and SB mathematics II. Like DP HL and SL, SB mathematics II builds on SB mathematics I content in each topic area. For the Science and Technology modality, the five main content areas are: A. Number sense, B. Meaning of measure, C. Spatial sense, D. Algebraic sense, and E. Stochastic sense. It should be noted here that the official and publicly available curriculum for the SB describes the mathematics content in broader terms than the DP syllabi and does not detail all the specific concepts studied.

4.9.2.2 Content Alignment

The table below presents a simplified summary of the extent to which SB mathematics aligns with the main topics of the DP's subjects. As noted above, the mathematics content of the SB is only broadly defined in the official Ministry of Education and Vocational Training's publicly available curriculum; as such, to facilitate comparison, the content mapping was supported by the use of textbooks sourced online. The sources included official textbooks from the Spanish ministry of education and independent textbooks which follow the Spanish curriculum.[100] All sources were cross-checked with the curriculum and official textbooks.

As indicated in the content structure section above, SB mathematics II is studied following SB mathematics I; as such, the alignments shown for these subjects demonstrate the cumulative content learnt by students on this pathway in the Science and Technology modality.

[100] Pascual. L.G., Menéndez. A. V., et al. (2022). MATEMÁTICAS II 2º de Bachillerato. Available from: Matematicas II (apuntesmareaverde.org.es); Muñoz. J., Moya. P., et al. (2022). MATEMÁTICAS I 1º de Bachillerato. Available from Matematicas I (apuntesmareaverde.org.es); Rodríguez del Río, Roberto. (i.d). Matemáticas I. 1º bachillerato. Bachillerato a distancia. Available from https://sede.educacion.gob.es/publiventa/matematicas-i-1-bachillerato-bachillerato-a-distancia/bachillerato-matematicas/14452

Table 50: Summary of the content alignment between the DP mathematics: analysis and approaches (AA) topics and SB mathematics subjects

AA topics		SB mathematics I	SB mathematics II
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Table 51: Summary of the content alignment between the DP mathematics: applications and interpretation (AI) topics and SB mathematics subjects

AI topics		SB mathematics I	SB mathematics II
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in the SB.		*Partial presence of this topic in the SB.*		*Little or no presence of this topic in the SB.*

* Where applicable, content alignments found in assumed knowledge or pre-requisite subjects are carried forwards and combined with new alignments to represent the cumulative content covered.

SB mathematics I

<u>DP Mathematics: analysis and approaches (AA)</u>

The mapping of content shows that SB mathematics I has alignment with DP AA SL in all topics, though some DP AA SL topics are covered in less depth, while others go beyond the DP AA SL coverage, including AHL content.

SB mathematics I includes a large amount of content from Number and Algebra, with a strong presence of DP AA SL content and a partial presence of AHL content. Indeed, SB mathematics I covers standard form, sequences and series, rational exponents, laws of logarithms, complex numbers, De Moivre's theorem, and systems of linear equations in similar depth and detail to the DP. SB mathematics I does not include the DP subtopics regarding proof, but instead covers limits of sequences. Another topic where SB mathematics I has alignment with both SL and AHL content is Geometry and Trigonometry. Indeed, SB mathematics I includes similar detail on trigonometry of right angles, cosine and sine rules, radians, exact values, trigonometric identities and relationships, trigonometric functions, reciprocal trigonometric functions, compound and double angle formula, symmetry properties, properties of vectors, scalar products, vector equations of lines, and coincident and parallel lines. However, SB mathematics I uses two dimensional vectors only and does not cover all AHL, such as the cross product and equations of planes. Instead, SB mathematics I covers conics, including ellipses, hyperbolas, parabolas and equilateral hyperbolas, which are not covered in the DP.

SB mathematics I has a strong presence of DP AA SL Functions content, covering domain and range, composite and inverse functions, quadratic equations and inequalities, and graphs and key features of quadratic, exponential, logarithmic, reciprocal, and rational functions. SB mathematics I also included a few sections of AHL subtopics, such as odd and even functions, absolute value function, and the remainder theorem for polynomials. However, SB mathematics I does not cover transformations or further AHL content, though it does cover root and piecewise functions.

SB mathematics I only has partial alignment with DP AA SL Statistics and Probability and Calculus content, due to significant areas not being covered. Indeed, SB mathematics I covers presenting data, measures of central tendencies and dispersion, correlation and regression, probability and Bayes theorem, but does not cover discrete random variables, or the binomial and normal distributions. Similarly, SB mathematics I covers in similar detail DP AA SL content regarding derivatives, such as a wide range of derivatives, increasing and decreasing functions, tangents, derivative rules, and maxima and minima, but does not cover subtopics related to integration. However, SB mathematics I covers in detail limits and continuity, including finding limits of functions, differentiating from first principles, and studying the continuity of functions, which aligns with some DP AA AHL content.

Overall, SB mathematics I covers a considerable amount of DP AA content from the DP curriculum. In general, it covers a similar amount of content as DP AA SL, though goes into more depth in some topics and less in others. For DP AA SL content, SB mathematics I is strongly aligned with Number and Algebra, Functions, and Geometry and Trigonometry, and partially aligned with Calculus and Statistics and Probability. For DP AA AHL content, SB mathematics I has partial alignment with Number and Algebra and Geometry and Trigonometry.

DP Mathematics: applications and interpretation (AI)

Similar to DP AA, SB mathematics I has the most alignment with the DP AI's topics of Number and Algebra and Geometry and Trigonometry. Indeed, SB mathematics I covers most of the DP AI's Number and Algebra content, except amortization and annuities, matrices, and eigenvalues and eigenvectors. It can be noted that SB mathematics I covers complex numbers in similar depth to DP AA – i.e. in more detail than DP AI's coverage. As noted above, SB mathematics I also includes limits of sequences. Regarding Geometry and Trigonometry, with the exception of Voronoi diagrams, SB mathematics I covers all DP AI SL content, as well as AHL subtopics of radians, definitions of sin, cos and tan, the Pythagorean identify, vectors, equations of vector lines, and the scalar product. However, SB mathematics I does not cover graph theory, matrix transformation, adjacency matrices, or decision mathematics subtopics.

As noted above, SB mathematics I covers further trigonometric identities, reciprocal trigonometric functions, and conic sections which are not in DP AI.

SB mathematics I has partial alignment with DP AI SL Functions content. Although covering a wide range of functions, SB mathematics I does not focus on the modelling of these functions. SB mathematics I covers some AHL subtopics such as composite and inverse functions. SB mathematics I includes piece-wise functions also.

SB mathematics I also has partial alignment with DP AI SL Calculus, as it does not cover integration, optimisation, or the trapezoidal rule. Instead, SB mathematics I covers AHL subtopics of further derivatives, derivative rules, second derivatives, as well as limits and continuity content which is not in DP AI. With regards to Statistics and Probability, SB mathematics I has limited alignment to DP AI, as it does not cover, discrete random variables, the binomial and normal distributions, or hypothesis testing, nor any AHL subtopics.

Overall, SB mathematics I has the most alignment with Number and Algebra and Geometry and Trigonometry content in the DP AI, covering nearly all SL subtopics and some AHL subtopics. Following this, SB mathematics I has partial alignment with DP AI SL Functions and Calculus content, occasionally including some AHL subtopics. Alignment with Statistics and Probability is limited.

Table 52: SB mathematics I content which is not covered in the DP

Significant content not in AA (only)	Significant content not in AI (only)
• Piecewise functions	• Limits and continuity • De Moivre's theorem • Reciprocal trigonometric functions • Compound angle identities and double angle formula
Significant content not in either DP mathematics subject	
• Limits of sequences • Half angle formula • Conics (Ellipse, hyperbola, parabola, equilateral hyperbola) • Root functions, supply and demand functions • Derivatives of hyperbolic functions • Solving systems of trigonometric equations	

SB mathematics II

DP Mathematics: analysis and approaches (AA)

SB mathematics II must be studied following SB mathematics I, hence the content found in the analysis for SB mathematics I applies for SB mathematics II. SB mathematics II introduces new topics, recaps topics, and extends SB mathematics I topics. With regards to DP AA, SB mathematics II has increased alignment with AHL content in all five main topics, as well as some SL content, especially for Statistics and Probability and Calculus.

SB mathematics II revisits vectors and includes vectors in three-dimensions, also introducing the cross product. SB mathematics II also covers vector equations of planes and intersections and angles, which are AHL content in DP AA. In addition to these, SB mathematics II includes concepts not in DP AA, such as studying linear independence of vectors, different forms of plane equations, mixed product, bundles of planes, orthogonal projections, symmetrical points, and different distances in space.

SB mathematics II also includes both more DP AA SL and AHL content from Calculus. Indeed, SB mathematics II introduces integration, covering a wide range of anti-derivatives, definite and indefinite integrals, area under a curve, volumes of revolution and integration methods of substitution, by parts, and use of partial fractions. SB mathematics II also includes L'Hôpital's rule for limits. However, it can be noted that differential equations and the Maclaurin series are not present. Instead, SB mathematics II goes deeper into limits and continuity, increasing the rigour and level of problems than SB mathematics I. SB mathematics II also covers Rolle's and mean value theorems, the fundamental theorem of integral calculus, and more cases of partial fractions.

SB mathematics II combines knowledge from SB mathematics I, as well as the new content learnt regarding limits, continuity and derivatives, to cover a topic on representing functions. These include graphing difficult functions using knowledge to find domain, range, axis intersections, symmetries, asymptotes, parabolic branches, monotony, critical points, concavity and convexity, inflection points, and regions of the plane in which it is defined. The types of functions required to be graphed align with AHL Functions content.
SB mathematics II further aligns with both DP AA SL and AHL content in Statistics and Probability by covering probability in similar detail, including Bayes Theorem and other formulae, as well as discrete random variables, continuous random variables and the binomial and normal distributions. SB mathematics II also includes permutations and combinations and the binomial expansion, which increases alignment with AHL Number and Algebra – though methods of proof are not covered.

SB mathematics II has a substantial amount of content involving matrices, which is not an area covered in DP AA. This includes operations of matrices, properties of determinants, adjoint matrices, and various methods of calculating determinants and inverse matrices. Furthermore, SB mathematics II also covers the Rouché–Frobenius theorem, Gauss method, and Cramer's rule.

Overall, SB mathematics II has a strong presence of DP AA SL and AHL content in all topics, as well as including additional content not in DP AA. SB mathematics II goes into more detail regarding limits and continuity and vectors than DP AA and covers a substantial amount of content around matrices. Therefore, the breadth and depth of SB mathematics II somewhat exceeds that of DP AA HL and considerably surpasses that of DP AA SL.

DP Mathematics: applications and interpretation (AI)
As with DP AA, SB mathematics II has more alignment with the DP AI's areas of Statistics and Probability and Calculus than SB mathematics I, due to covering discrete random variables, and distributions. However, SB mathematics II does not cover hypothesis testing or any AHL Statistics and Probability content, such as nonlinear regression, linear combinations, the central limit theorem, Poisson distribution, or transition matrices and Markov chains. Instead, within this topic area, SB mathematics II covers continuous random variables and permutations and combinations. For Calculus, SB mathematics II covers more DP AI SL and AHL subtopics such as optimisation, integrals, integration methods, area and volumes of revolution, thought does not include differential equations, slope fields, Euler's methods and

numerical solutions, or phase portraits. SB mathematics II instead revisits limits and continuity in more depth and covers more antiderivatives and methods of integration.

As mentioned with DP AA, SB mathematics II includes a substantial amount of content on matrices. Like DP AI HL, SB mathematics II covers the definition of a matrix, operations, determinants and inverse matrices, and solving systems of equations. However, SB mathematics II does not cover eigenvalues and eigenvectors, adjacency matrices, matrix transformations, or transition matrices. Instead, SB mathematics II covers more details regarding adjoint matrices, properties of determinants, calculating determinants, calculating inverses, matrix rank, and matrix expression of systems of equations (Rouché–Frobenius theorem, Gauss method, and Cramer's rule).

Compared to SB mathematics I, SB mathematics II does not significantly increase its alignment with the DP AI in the content area of Geometry and Trigonometry, as it focuses more on vectors (see DP AA for details), whereas DP AI covers graph theory and decision mathematics. No further alignments are found with Functions, as SB mathematics II does not emphasise modelling.

*Table 53: SB mathematics II content which is not covered in the DP mathematics subjects**

<table>
<tr><th>Significant content not in AA (only)</th><th>Significant content not in AI (only)</th></tr>
<tr><td>Matrices
Confidence intervals</td><td>Further forms of equations of a line
General systems of linear equations (homogeneous and equivalent systems)
Vector equations of planes
Intersecting planes (a line and plane, two and three planes)
Combinations and permutations and binomial expansion
Limits and continuity
Integration methods and partial fractions</td></tr>
<tr><th colspan="2">Significant content not in either DP mathematics subject</th></tr>
<tr><td colspan="2">Matrices content (properties and further methods of calculating determinants and inverse matrices; Adjoint matrix; Matrix rank; Rouche-Frobenius theorem, Gauss method, Cramer's rule)
Vectors content (linear independence of vectors, parametric equations of a plane, segmental equations of a plane, equation of a plane from three points, conditions for four points to be coplanar; mixed product of vectors; bundles of planes; orthogonal projections; symmetrical points)
Distance (point to plane, between planes, line to plane, between lines)
Rolle and Mean Value Theorems
Formal definitions and rigor used in the topics of limits, continuity, and calculus
Further cases of partial fractions
Mean Value theorem and Fundamental Theorem of integral calculus</td></tr>
</table>

Overall, SB mathematics II is strongly aligned with DP AI Number and Algebra content, covering the majority of DP AI SL and AHL subtopics. Following this, SB mathematics II is strongly aligned with DP AI SL Geometry and Trigonometry and Calculus content, and partially aligned with the AHL content in these topics. SB mathematics II is also partially aligned with the DP AI SL content of Functions and Statistics and Probability. In addition, SB mathematics II includes a substantial amount of content that is not present in DP AI (see table below), some of which is covered in DP AA. Overall, the breadth and depth of SB mathematics II somewhat exceeds that of DP AI HL, and considerably surpasses that of DP AI SL.

4.9.3 Demand

This section considers the alignment between the DP and SB mathematics curricula in terms of demand.

The DP and SB curricula were analysed using the same demand tool in order to create a demand profile for DP AA (SL and HL), DP AI (SL and HL), SB mathematics I, and SB mathematics II. These demand profiles are presented below in the form of radar diagrams, with superimposed diagrams also being featured to enable immediate visual comparison.

Figure 23: Visual representations of subject demand

The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - The DP mathematics subjects scored a 3 in this category.
  - The mathematics learning outcomes of the SB were drawn from the specific competences and evaluation criteria for SB mathematics I and SB mathematics II.

There is little variation in the learning outcomes for these subjects, hence both were awarded the same score. The panel judged that higher-order thinking skills, such as evaluation, creative thinking and analysis, were present in several learning outcomes, hence a score of 3 was awarded. Specifically, there were references to modifying and creating algorithms, developing critical, creative and innovative skills, assessing the usefulness of representations, formulating and investigating conjectures, and using reasoning, and creativity to generating new mathematical knowledge. Whilst there is enough evidence of higher order thinking skills for both SB mathematics I and SB mathematics II to receive a score of 3, it can be noted that SB mathematics II contained slightly more evidence, demonstrating a progression from SB mathematics I. In particular, SB mathematics II included further skills of making decisions and acting more autonomously.

- Regarding the scores for **Depth of Knowledge**:
  - Both DP mathematics subjects at SL were given a score of 2 and HL subjects received a score of 3.
  - For SB, mathematics I received a score of 1.5, as some topics are studied in considerable depth, in cases comparable to that of DP SL; however, there was not enough evidence of depth to warrant a score of 2. For SB mathematics II, a score of 3 was given as all topics were studied in very high detail – comparable to the DP's HL subjects. The level of detail studied on matrices, for example, was deemed particularly high.

- Regarding the scores for **Volume of Work**:
  - DP SL subjects received a score of 2 and DP HL subjects received a 3.
  - SB mathematics I and II each have minimum teaching hours of 87.5 hours, which is less than the recommended hours for both DP SL and HL. Based on the size and depth of the content in each of these subjects, SB mathematics I was given a score of 2 for its moderate-heavy volume of work and SB mathematics II was given a score of 3 for its heavy volume of work. Indeed, the amount studied in SB mathematics I and II combined is of similar size to DP HL, yet the minimum number of teaching hours altogether is 175 hours, which was deemed a very short time allocation to cover the amount and complexity of the content.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - Both DP SL subjects received a score of 1 and DP HL subjects had a high number of outstanding areas of demand, and thus were given a score of 3.
  - For SB subjects, a score of 1 was given to SB mathematics I, as its coverage of complex numbers was deemed to be particularly advanced, going into De Moivre's theorem for example. For SB mathematics II, a score of 3 was given, with complex numbers, matrices, vectors, limits and continuity, and continuous random variables all containing subtopics which demonstrated opportunity for stretch and are beyond the usual scope of upper secondary mathematics.

4.10 Brazil

The education system in Brazil is overseen by the Ministry of Education (*Ministério da Educação*, MoE). Broadly, the Brazillian education system is divided into basic education (*educação básica*) and higher education (*ensino superior*). Basic education is compulsory and spans three stages: early childhood education (*ensino infantil*); elementary school (*ensino fundamental*), which encompasses both primary and lower secondary; and high school (*ensino médio)*.

The upper secondary stage of education, the *ensino médio*, spans three years (grades 1-3) and is delivered in general or technical institutions.[101] This stage acts as a continuation from primary education, and as a preparatory stage prior to higher education or vocational training.

The *National Guidelines for High School Education*, *National Common Curricular Base (BNCC)*, and *Curricular References for the Preparation of Formative Itineraries* are key to the organisation and structure of high school education in Brazil and are referred to collectively here as the Brazilian High School Curriculum (BHSC).

As part of the BHSC, the BNCC describes the essential learning and minimum standards to be achieved for all pupils. At school level, the curriculum is set by state education secretariats and municipal education authorities. Providing they meet the standards outlined in the BNCC, educational institutions and networks may construct their curriculum as they feel is pertinent to their context. The BNCC thus underpins all curriculum offerings, but rather than being a curriculum in itself, it provides guidance for content planning, allowing for flexibility at the state and school-level.[102]

Learning in the BHSC is structured into basic general education and formative itineraries. Basic general education is guided by the BNCC, which arranges the essential learning into the four areas of knowledge below:

- Language and Technology (including Portuguese)
- Mathematics and Technology
- Natural Sciences and Technology
- Applied Human and Social Sciences. [103]

The other component of the BHSC is the formative itineraries, which schools may offer based on regional and local needs, interests and resources. Formative itineraries focus on deepening and expanding learning in one of the areas of knowledge, or on technical and professional training.[104] Integrated itineraries may also be offered which combine the study of one area of knowledge with another area, or with technical and professional training.

[101] OECD. (2021). *Education Policy Outlook in Brazil: With a focus on national and subnational policies. OECD Education Policy Perspectives, No. 38.* OECD Publishing, Paris. Available from: Education Policy Outlook in Brazil: With a focus on national and subnational policies | OECD
[102] OECD. (2021). Education in Brazil: An International Perspective. The Brazilian education system. OECD Publishing, Paris. Available from: https://doi.org/10.1787/c61f9bfb-en
[103] Ministry of Education. National Education Council. Basic Education Chamber. (2018). *Resolution No.3, of November 21, 2018*. p. 5-6 (art.11). Available from: http://portal.mec.gov.br/docman/novembro-2018-pdf/102481-rceb003-18/file
[104] Ministry of Education. National Education Council. Basic Education Chamber. (2018). *Resolution No.3, of November 21, 2018*. p. 7 (art.12)

As such, the analysis uses the following subjects for comparison to DP mathematics.

BHSC Mathematics and Technology (Basic General Education) – BHSC MAT (BGE)
In the BHSC, Mathematics and Technology is a compulsory area of knowledge for the basic general education component of high school. This area of knowledge is designed to consolidate and deepen the mathematics learnt in elementary school and aims for high school students to become more aware of the interrelatedness of mathematics, through the perspective of application to real-world contexts. BHSC MAT (BGE) is based upon the specific competencies and skills prescribed by the BNCC.[105] To support the content analysis, the Rio de Janeiro Referential Curriculum (RJRC)[106] is also consulted to provide additional insight into the typical mathematics topics and subtopics covered in BHSC MAT (BGE).

BHSC Mathematics and Technology (Formative Itinerary) – BHSC MAT (FI)
In the BHSC, Mathematics and Technology is an area of knowledge that students may choose to specialise in for their formative itinerary component of high school. Formative itineraries are curricular units which are designed for students to deepen their knowledge and prepare for further studies or careers. BHSC MAT (FI) is based upon the 'Curricular references for the preparation formative itineraries', which guide the development of this subject by each state.[107] Again, the RJRC is consulted to provide further insights into the type of content covered in BHSC MAT (FI).[108]

4.10.1 Learning Outcomes

This section compares and contrasts the learning outcomes of curricula falling within the category of mathematics.

For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics: applications and interpretation.

BHSC Mathematics and Technology (MAT) learning outcomes are presented as specific competencies and specific skills. For formative itineraries specifically, additional skills are given, which are based on the formative itinerary structuring axes. Moreover, the BHSC also articulates general competencies for high school education which have also been considered when relevant here.

The following summary table demonstrates the learning outcome themes that were extracted from DP mathematics and indicates if and where they were judged to have presence within the learning outcomes of BHSC Mathematics and Technology (MAT).

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105 Brazil, Ministry of Education. (2018). *5.2.1. The area of Mathematics and Technology: Specific competencies and skills*. BNCC. Available from: National Common Curricular Base - Education is the Base (mec.gov.br)
106 Rio de Janeiro State Government, Department of Education. (2022). *Mathematics and Technology*. High School Referential Curriculum for the State of Rio de Janeiro. p. 68-70. Available from: 1- Revisão CURRÍCULO ENSINO MÉDIO.cdr (educacao.rj.gov.br)
107 Brazil, Ministry of Education. (n.d.). *Curricular References for the Preparation of Formative Itineraries*. Available from: Referenciais-Curriculares-para-Elaboracao-de-Itinerarios-Formativos-1-1.pdf (sedu.es.gov.br)
108 Rio de Janeiro State Government, Department of Education. (2022). *Curricular Organisation of Formative Itineraries*. Available from: Trails (educacao.rj.gov.br)

Table 54: Presence of the DP mathematics subject group learning outcome themes in BHSC Mathematics and Technology (MAT)

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in BHSC Mathematics and Technology (MAT)	
1. Be aware of, and engage with, mathematics in its wider context		Present in BHSC MAT – contexts such as socio-economic issues, sustainability, world challenges, and community issues are considered.
2. Develop learning skills; have a positive and resilient attitude, work both independently and collaboratively, be reflective and evaluate work		Present in BHSC MAT – usually articulated in competencies that are cross-cutting for all areas of knowledge.
3. Use inquiry-based approaches		Investigation and its related skills are present in BHSC MAT.
4. Understand the concepts, principles and nature of mathematics and apply concepts and procedures to a range of contexts		BHSC MAT expects students to understand and apply mathematical concepts and procedures to a diverse range of contexts.
5. Make links and generalisations		Making connections to other areas of knowledge is particularly present in the BHSC MAT.
6. Develop critical/creative thinking skills e.g. problem-solving and reasoning		Present in BHSC MAT – problem-solving and reasoning is featured. Creativity and innovation skills are particularly highlighted for formative itineraries.
7. Communicate mathematics clearly and in various forms		Present in BHSC MAT – for example, accurately using a range of representations is expected.
8. Know how technology and mathematics influence each other and use technology to develop ideas and solve problems		Use of technology is expected during investigations and problem-solving in BHSC MAT.

Key:

	This theme is well-evidenced in the learning outcomes of BHSC MAT		*This theme is partially evidenced in the learning outcomes of BHSC MAT*		*This theme is not evident in the learning outcomes of BHSC MAT*

Presence of the DP's Learning Outcome Themes

There is a strong alignment between the DP's mathematics learning outcomes and the learning outcomes articulated for BHSC Mathematics and Technology (MAT), with all the DP's themes being well-evidenced in the specific competencies and specific skills, and further evidenced in the additional skills for the formative itinerary (FI). The presence of each DP theme is discussed in more detail below.

1. Be aware of, and engage with, mathematics in its wider context

The DP's theme of awareness and engagement with wider contexts is well evidenced in BHSC MAT. Indeed, the BHSC promotes the importance of students engaging with contexts that are relevant and meaningful to their lives, such as socioeconomic situations, sustainability, contemporary world challenges, and community issues. This particularly echoes the DP's aim for its students to apply mathematics skills to local and global developments. Furthermore, students are expected to engage critically with media and make ethical and socially responsible decisions when investigating world challenges, which has similarities with the DP's aim of developing an awareness of moral, social and ethical questions that have arisen regarding mathematics. Lastly, in line with the DP, the BHSC indicates a consideration of multicultural perspectives as it states that "reflection should also be considered about the different roles that mathematics education can play in socio-political and cultural contexts."[109]

The FI skills further demonstrate this theme, as students are expected to research information regarding how mathematics has contributed to explaining scientific, social, professional, and cultural phenomena. During this, students should consider different points of view, which aligns with the DP's international outlook. Furthermore, several FI skills are dedicated to diverse sociocultural and environmental issues, including identifying and explaining these and proposing and testing mediation/intervention actions and strategies. Overall, Brazilian high school students are similarly expected to engage with mathematics in wider contexts.

2. Develop learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work

The DP theme of transferable learning skills is well evidenced in BHSC MAT; however, it can be noted that the learning outcomes relevant to this theme are primarily located in the general competencies for high school, rather than in the specific competencies for MAT. That said, an introduction written in the BHSC for MAT confirms that transferable learning skills are expected to be developed within the area of knowledge. Indeed, self-esteem, perseverance in problem-solving, respectfulness of others, and a pre-deposition to work in groups are identified as attitudes to be developed in mathematics.

The FI skills similarly do not articulate transferable learning skills for MAT specifically, instead these are included in skills which are cross-cutting for all areas of knowledge. These skills include being persistent, working well with others, and reflecting on personal development.

3. Use inquiry-based approaches

The DP theme of using inquiry-based approaches is well-evidenced in the learning outcomes of BHSC MAT. Both the DP and BHSC outline expectations that students will investigate real-life scenarios and analyse information to draw conclusions. Furthermore, both expect students to make and investigate conjectures and to assess the validity of these.

In the FI skills there is further evidence of the use of inquiry-based approaches. Indeed, one of the four structural axes of formative itineraries is scientific research, which in MAT corresponds to students being expected to make hypotheses, propose solutions, and test solutions and strategies - including ethical, creative, and innovative solutions, and mediation and intervention strategies to solve multicultural and environmental problems.

4. Understand the concepts, principles and nature of mathematics and apply concepts and procedures to a range of contexts

The DP theme of understanding and applying mathematics is well-evidenced in MAT. As outlined in the BHSC's specific competencies, students are expected to use mathematical strategies, concepts, and procedures to interpret, build models, and solve problems in a range of contexts.

109 Brazil, Ministry of Education. (2018). *5.2.1. The area of Mathematics and Technology: Specific competencies and skills.* BNCC.

This theme is also evidenced in the FI skills, as students are expected to apply mathematical knowledge and skills to sociocultural and environmental issues and to create new knowledge and approaches. This implies that students choosing a formative itinerary in MAT will need to have a deep understanding of mathematical concepts and a strong competency in applying these.

5. Make links and generalisations

The DP's theme of making links and generalisations is evidenced in BHSC MAT. Like in the DP, the BHSC includes an expectation that links will be made to other areas of knowledge, such as Natural Sciences and Humanities, and that students are able to apply mathematics in these areas. Moreover, the BHSC indicates an expectation of making connections between mathematical concepts, such as when making judgements. Making generalisations is also present, as one specific competency includes students observing patterns, and several skills linked to the same competency refer to making conjectures for generalisation.

By design, the formative itineraries can provide opportunities for making links between subjects, as they allow students to pursue studies which integrate more than one area of knowledge. Furthermore, the FI skills include considering possibilities for generalisation when using models and developing new knowledge and approaches.

6. Develop critical/creative thinking skills e.g. problem-solving and reasoning

Similarly to the DP, developing critical and creative thinking skills is a strong focus in BHSC MAT. Indeed, critical and creative thinking skills are clearly shown through expectations for students to critically interpret real-world information, perform analysis, solve problems, evaluate the plausibility of results, make decisions, and build an argument. Notably, in the specific skills, the BHSC often requires students to 'solve and elaborate problems',[110] with the intention that students reflect and question what would happen if a part of the problem was changed.

This theme is also represented in the FI skills. Indeed, students are expected to continue to analyse, evaluate, and make decisions. Moreover, creative thinking, as one of the structuring axes of all formative itineraries, is particularly emphasised, with students expected to use this skill to come up with innovative solutions to real problems.

7. Communicate mathematics clearly and in various forms

Communicating mathematics is a skill to be developed in both DP mathematics and BHSC MAT. Indeed, the BHSC requires students to use different mathematical representations flexibly and accurately in the communication of results to problems. Furthermore, although not highly emphasised in the specific competencies, the introduction to MAT highlights communication as a key competence, stating that students should be able to provide mathematical arguments with correct language and notation, as well as to communicate in reports and oral presentation. Therefore, like the DP, students are expected to communicate in various forms.

Similarly, the FI skills require students to communicate accurately when presenting their actions and reflections regarding findings, interpretations, and arguments.

[110] Ibid.

8. Know how technology and mathematics influence each other and use technology to develop ideas and solve problems

As the name suggests, technology is an important part of BHSC MAT. The BHSC outlines that students are expected to be aware of the implications of technology, including its issues, as well as using it as an alternative method to solve problems, make models, and explore concepts. Indeed, several skills require the use of digital technology.

This theme is also present in the FI skills, as students are required to conduct research into how mathematics has contributed to technology, which aligns strongly with the DP expectation that students should be aware of how mathematics and technology influence one another.

Other Themes in BHSC MAT Learning Outcomes

Most of the themes and skills described for BHSC MAT are in the DP; however, it can be noted that there are a few FI skills which have a greater emphasis. Indeed, the FI skills have a greater focus on proposing mediation and intervention strategies to sociocultural and economic problems than the DP. There is also a greater emphasis on proposing innovative solutions and generating new knowledge and approaches, which reflects the structural axis of entrepreneurship that the formative itineraries are based on.

Summary

Overall, there is a high level of alignment between DP mathematics and BHSC Mathematics and Technology (MAT) with regards to learning outcomes. In particular, the mathematics learning outcomes for BHSC MAT have strong similarities to DP mathematics with regards to considering and using mathematics in wider contexts, such as global challenges, which reflects the international outlook of the DP's mathematics curriculum. Furthermore, both encourage inquiry-based approaches by expecting students to investigate conjectures, analyse information, and draw conclusions. Critical and creative thinking skills are also a key emphasis in both curricula, as well as the use of technology, effective and accurate communication, making connections and generalisations, and understanding and application of mathematical concepts and procedures. Though broadly they share very similar learning outcome themes, it can be noted that the FI skills have a greater focus on involving entrepreneurship and proposing mediation and intervention strategies than DP mathematics.

4.10.2 Content

This section compares the content of DP mathematics subjects with BHSC MAT. For the content analysis of BHSC MAT, several sources have been used. For BHSC MAT (BGE), the BNCC's specific competencies and skills have been used, as well as the Rio de Janeiro Referential Curriculum (RJRC) for mathematics. For BHSC MAT (FI), the RJRC's specialisation pathways for MAT formative itineraries have been used. The mathematics content from the BNCC and RJRC are presented in the following diagrams.

Figure 24: Visualiser of BHSC Mathematics and Technology. (Source – BNCC).

Mathematics and Technology		
Specific Competencies		
1. Use mathematical strategies, concepts, and procedures to interpret situations in different contexts, whether they are daily activities or facts of the Natural Sciences and Humanities, of socio-economic issues or disseminated by different means, in order to contribute to the general training.	2. Propose or participate in actions to investigate world challenges and make ethical and socially responsible decisions, based on the analysis of social problems, such as those related to sustainability, the implications of technology in the world of mobilizing and articulating concepts and procedures and languages specific to Mathematics.	3. Use mathematical strategies, concepts, definitions, and procedures to interpret, build models, and solve problems in various contexts, analysing the plausibility of the results and the adequacy of the proposed solutions, in order to build consistent argumentation.
Specific Skills*		
EM13MAT101 - EM13MAT106	EM13MAT201 - EM13MAT203	EM13MAT301 - EM13MAT316
Specific Competencies		
4. Understand and use, flexibly and accurately, different mathematical representations (algebraic, geometric, statistical, computational studies, etc.), in the search for solutions and communication of results.	5. Investigate and establish conjectures regarding different concepts and mathematical properties by employing strategies and resources such as; observing patterns, experiments and different technologies, and identifying the need, or not, for a validation of these conjectures.	
Specific Skills*		
EM13MAT401 - EM13MAT407	EM13MAT501 - EM13MAT511	

*See Appendix D for the Mathematics and Technology Specific Skills in full detail.

Figure 25: Visualiser of BHSC Mathematics and Technology (Source - RJRC)

Basic General Education	Formative Itinerary		
	Integrated Core	Specialisation Pathways	
Mathematics (based on the BNCC and organised using the Specific Competencies and Skills)	Life Project	Connected Mathematics	On Your Own
	Elective 1	Gamification	Fiscal Mathematics
	Elective 2	Robotics	Fiscal Mathematics
	Elective 3 (from Catalogue of Electives)	Logic	Financial Citizenship

4.10.2.1 Structure

For BHSC MAT (BGE), the BNCC outlines specific competencies and specific skills that all states are required to cover in their curricula. The specific skills are organised under the five specific competencies or are categorised into the topic areas of Number and Algebra, Geometry and Measurements, and Probability and Statistics. The DP also organises content into topic areas (Number and algebra, Functions, Geometry and trigonometry, Statistics and probability, and Calculus). It can be noted that the DP has Functions as a separate topic, while the BNCC includes functions-related content in Number and Algebra. Each state in Brazil can decide how to organise their mathematics curriculum, thus both the BHSC and DP allow for flexibility in how the curriculum is organised, without demanding a particular sequence of teaching.

Like with the option to do a HL course in DP mathematics subjects, there is opportunity for Brazil's high school students to study further mathematics (beyond what is compulsory in basic general education) through choosing to specialise in Mathematics and Technology in their formative itinerary – BHSC MAT (FI). Using Rio de Janeiro as an example, MAT formative itineraries are organised into specialisation pathways that contain curricular components. These curricular components are not as broad as DP topics. For example, one of the curricular components in Rio de Janeiro is Financial Citizenship, which is not covered by DP mathematics and has a narrower scope than DP topics, such as Number and algebra. Furthermore, MAT formative itineraries may not offer routes allowing for a choice of pure or applied mathematics focuses, unlike the DP's offering of AA and AI. Instead, MAT formative itineraries appear to strongly lean towards an applied thematic focus. However, it should be noted that formative itineraries, and overall curriculum structure, can vary from state to state in Brazil.

4.10.2.2 Content Alignment

This section will analyse the alignment of mathematics content in DP mathematics and BHSC MAT. The tables below present a simplified summary of the extent of content alignment that BHSC MAT (BGE) and BHSC MAT (FI) have at the topic level with DP mathematics.

Table 55: Summary of the content alignment BHSC MAT has with the main topics in DP AA.

AA topics		BHSC MAT (BGE)	BHSC MAT (FI)*
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in BHSC MAT		*Partial presence of this topic in BHSC MAT*		*Little or no presence of this topic in BHSC MAT*

* Content alignments found for basic general education (BGE) are carried forwards and combined with, where applicable, new alignments identified in the formative itinerary (FI), to represent the cumulative content covered.

Table 56: Summary of the content alignment BHSC MAT has with the main topics in DP AI.

AI topics		BHSC MAT (BGE)	BHSC MAT (FI)*
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in BHSC MAT		*Partial presence of this topic in BHSC MAT*		*Little or no presence of this topic in BHSC MAT*

* Content alignments found for basic general education (BGE) are carried forwards and combined with, where applicable, new alignments identified in the formative itinerary (FI), to represent the cumulative content covered.

BHSC Mathematics and Technology (Basic General Education) – BHSC MAT (BGE)

The content of BHSC MAT (BGE) shows partial alignment with DP SL content in all main topics except Calculus (which is not an area of mathematics covered in the BHSC). Indeed, the skills cover, or indicate coverage of, several DP SL subtopics in Number and algebra, Functions, Geometry and trigonometry, and Statistics and probability. For the most part, these tend to be subtopics which are common to both DP mathematics subjects (AA and AI), though the strong application focus of the BHSC MAT (BGE) reflects similarities with the applied thematic focus of DP AI. The below discusses how BHSC MAT (BGE) aligns with SL content in each DP topic, distinguishing between DP AA and DP AI content where necessary.

Number and algebra

With regards to Number and algebra SL content, BHSC MAT (BGE) covers mostly subtopics which are common to DP AA and DP AI, such as standard form, arithmetic and geometric sequences, and financial applications. Coverage of exponential and logarithmic functions in BHSC MAT (BGE) indicates that some of the other subtopics in DP AA and DP AI regarding rational exponents and laws of logarithms may be covered, but it is not explicit. Other SL subtopics are not included, as neither are any of the AHL subtopics.

Functions

With regards to Functions SL content, BHSC MAT (BGE) skills cover the concepts of domain and range, graphing of functions, and key features of graphs. The types of functions included are linear, quadratic, exponential and logarithmic. Like DP AI, BHSC MAT (BGE) expects modelling with linear and quadratic functions, and indicates that similar modelling skills are covered, though modelling with other functions listed in DP AI SL (or AHL) is not explicitly included. From DP AA SL, BHSC MAT (BGE) does not cover composite and inverse functions, quadratic inequalities, reciprocal and rational functions, transformations, and only indicates some coverage regarding graphical and analytical solutions. However, it can be noted that BHSC MAT (BGE) often references similar real-life contexts, links to other subjects, and uses of technology, that appear in the suggested 'Connections' parts of the DP AA and DP AI syllabuses for Functions.

Geometry and trigonometry
With regards to Geometry and trigonometry content, BHSC MAT (BGE) skills cover mostly subtopics shared between DP AA and DP AI, such as solving problems regarding volume, area, and trigonometry. Again, several real-life contexts are referenced in BHSC MAT (BGE) which are similar to those described in the DP's 'Connections' sections, such as mapmaking and design. BHSC MAT (BGE) skills include comparing phenomena with cosine and sine functions, which are covered in DP AA SL and appear in the 'Connections' suggested for modelling in DP AI Functions. However, BHSC MAT (BGE) does not indicate that sine and cosine functions are covered in as much depth as DP mathematics subjects. BHSC MAT (BGE) also does not indicate that it covers other SL content (such as radians, trigonometric identities, and Voronoi diagrams) or any AHL content.

Statistics and probability
With regards to Statistics and probability content, again BHSC MAT (BGE) skills cover mostly subtopics which are shared between DP AA and DP AI, such as sampling, presenting data, measures of central tendency and dispersion, and basic probability concepts. There is an indication that correlation is covered, as well as further probability concepts (such as tree diagrams). However, BHSC MAT (BGE) does not include other SL content such as linear regression, or binomial and normal distributions, nor any AHL content from DP AA or DP AI.

Other BHSC MAT (BGE) Content
As shown in the following table, BHSC MAT (BGE) does not include a significant amount of content which is not covered in DP mathematics. The only noteworthy difference is that BHSC MAT (BGE) includes some content on algorithms and programming, though this appears to be at an introductory, rather than in-depth, level.

*Table 57: BHSC MAT (BGE) content which is not covered in the DP**

<table>
<tr><th>Significant content not in AA (only)</th><th>Significant content not in AI (only)</th></tr>
<tr><td>• Modelling with 1st and 2nd degree functions</td><td>• Counting principles</td></tr>
<tr><th colspan="2">Significant content not in either DP mathematics subject</th></tr>
<tr><td colspan="2">• Algorithms and programming – two of the skills in BHSC MAT (BGE) relate to this area and include using flowcharts, algorithms, and concepts of programming languages
• Conics</td></tr>
</table>

* Significant content does not include topics which are typically studied *prior* to upper secondary.

Summary
Overall, BHSC MAT (BGE) partially aligns with DP SL content and covers a lesser depth and breadth of content. There is no alignment with AHL content, as would be expected from a common curriculum, thus BHSC MAT (BGE) has considerably less breadth and depth compared to DP HL mathematics. It can be noted that BHSC MAT (BGE) often includes similar contexts, interdisciplinary links, and uses of technology to those suggested in the 'Connections' sections of the DP subject guides, which reinforces the learning outcome findings that BHSC MAT places similar importance on these aspects.

BHSC Mathematics and Technologies (Formative Itinerary) – BHSC MAT (FI)
BHSC MAT (FI) represents the pathway of studying MAT in basic general education and then specialising in MAT in the formative itinerary. The Rio de Janeiro curriculum (RJRC) is consulted to provide examples of the types of content covered in BHSC MAT (FI).

Rio de Janeiro offers two specialised pathways for MAT formative itineraries, namely ‘Connected Mathematics’ and ‘On your own’.[111] The ‘Connected Mathematics’ specialisation pathway offers the three curricular components of Gamification, Robotics, and Logic. The ‘On Your Own’ specialisation pathway offers the three curricular components of Fiscal Mathematics, Financial Mathematics, and Financial Citizenship.

These components further develop skills which are present in DP mathematics, such as problem-solving, reasoning, logical thinking, evaluation, inquiry-approaches, and use of technology skills. There is also a particular emphasis on decision-making and creating strategies, which is less specifically targeted in DP mathematics. However, the content of MAT formative itineraries in Rio de Janeiro does not align with DP mathematics.

*Table 58: BHSC MAT (FI) content which is not covered in DP mathematics**

Significant content not in AA (only)	Significant content not in AI (only)
N/A	N/A
Significant content not in either DP mathematics subject	
• Logic • Gamification • Robotics • Fiscal mathematics • Financial mathematics (some different coverage to the DP) • Financial citizenship	

*It should be noted that these topics have been drawn from the RJRC and therefore should be considered as examples only. In practice, the content of formative itineraries can vary state to state.

Summary

The examples from Rio de Janeiro indicate that BHSC MAT (FI) does not present a stronger alignment with DP mathematics than BHSC MAT (BGE), as no further SL subtopics or AHL subtopics could be identified. Instead, BHSC MAT (FI) includes a strong focus on application and covering areas that are not a focus of DP mathematics, such as robotics, gamification, logic, fiscal mathematics, and financial citizenship. By thematic focus, BHSC MAT (FI) is more akin to the application focus of DP AI, rather than DP AA. However, the areas of application are different, and a significant amount of DP AI content is not present. Generally, the documentation indicates that BHSC MAT (FI) incorporates more concepts from the area of application, rather than focusing on covering additional and more complex mathematical content.

Overall, it appears that BHSC MATH (FI) has significantly less depth in mathematics content compared to DP HL mathematics subjects. It is particularly notable that DP AHL content was not covered in BHSC MAT (FI). However, it can be noted that the extent of the complexity of mathematical concepts involved in the formative itinerary curricular components is somewhat difficult to confidently ascertain from the level of detail in the documentation.

It is challenging to directly compare the breadth and depth of BHSC MAT (FI) with DP SL mathematics, due to the significant differences in their content and approach. Students who study BHSC MAT (FI) will experience a breadth of mathematics content, focusing on

[111] Rio de Janeiro State Government, Department of Education. (2022). *Curricular Organisation of Formative Itineraries*.

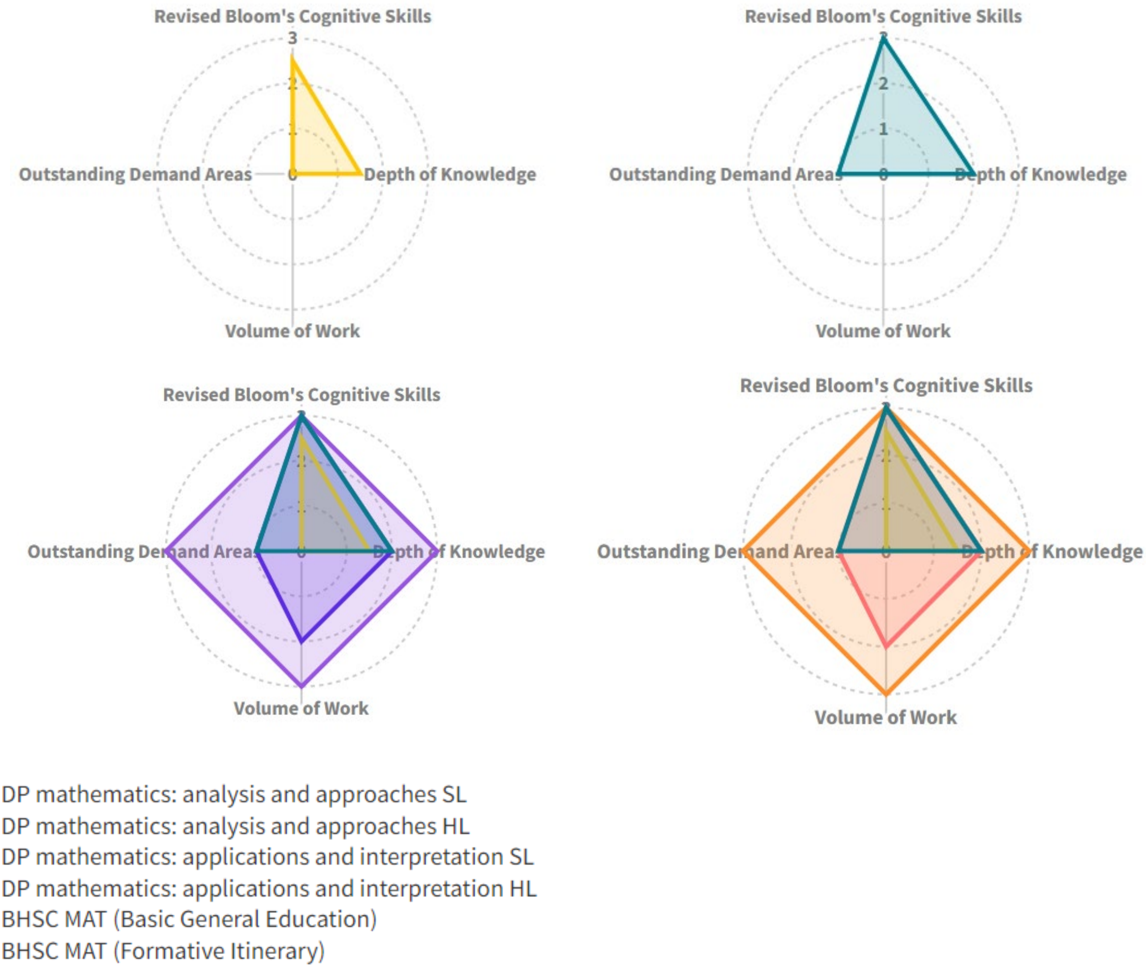
applications, as well as some SL content from most DP topics. However, it can be noted that calculus – a main topic in DP mathematics – is not studied at all in BHSC MAT.

4.10.3 Demand

This section considers the alignment between DP mathematics and BHSC Mathematics and Technology in terms of demand.

Using the same demand tool for the analysis of all subjects, a demand profile was created for DP AA (SL and HL), DP AI (SL and HL), BHSC MAT (BGE), and BHSC MAT (FI). BHSC MAT (FI) represents the cumulative demand of studying mathematics in the basic general education and then specialising in MAT in the formative itinerary. These BHSC demand profiles are presented below in the form of radar diagrams, with superimposed diagrams featured last to enable the immediate visual comparison of all DP and BHSC profiles.

Figure 26: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
    - All DP mathematics subjects received a score of 3 in this category.

  - The learning outcomes of BHSC MAT (BGE) demonstrate that higher order thinking skills are encouraged throughout, evidenced by skills requiring critical analysis, elaboration of problems, and critical interpretation of economic and social situations. However, there was some doubt with regards to the extent these skills would be realised considering the somewhat low demand of mathematics content covered, therefore a score of 2.5, rather than 3, was awarded. Building on the skills of BHSC MAT (BGE), there was further evidence of higher order skills for BHSC MAT (FI). Indeed, skills such as developing and evaluating models, proposing creative and innovative solutions, and testing strategies contributed to a score of 3 being given overall.

- Regarding the scores for **Depth of Knowledge**:
  - Both DP SL subjects were given a score of 2 and both DP HL subjects were awarded a score of 3.
  - BHSC MAT (BGE) covers some upper-secondary level content in topics such as functions. However, in contrast to DP SL and HL, not many topics are covered in considerable detail and topics requiring strong pre-requisite knowledge, such as calculus, are not included. Alone, this would merit a score of 1. However, due to the skills that are to be combined with this content, a consensus among panellists was reached that students would be consistently engaging with the content beyond habitual responses and carrying out tasks which require higher cognitive demands, thus a score of 1.5 was awarded to acknowledge this. For BHSC MAT (FI), a score of 2 was deemed appropriate. Whilst it does not appear that this subject covers more complex and abstract mathematical concepts than BHSC MAT (BGE), the skills for formative itineraries suggest that further time would be spent on cognitively demanding tasks such as planning, creating, and extended thinking.

- Regarding the scores for **Volume of Work**:
  - Both DP mathematics subjects at SL were given a score of 2. For HL, both DP mathematics subjects merited a score of 3.
  - For BHSC MAT (BGE), it was deemed that the number of hours allocated to cover its content (400 hours) was generous, given the light number of topics and concepts involved. Similarly, the time allocated for BHSC MAT (FI) was also deemed to be generous. Therefore, both profiles received a score of 0 in this category.[112]

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - A score of 1 was awarded to both DP SL subjects and a score of 3 was awarded to both HL subjects.
  - For BHSC MAT (BGE), no areas which went beyond typical upper-secondary mathematics education could be identified, thus a score of 0 was awarded. For BHSC MAT (FI), a score of 1 was tentatively awarded, as the documentation suggested that time will be spent on developing projects, which could have scope for independent research and exploration (similar to the DP mathematical exploration). However, the level of detail on formative itineraries presents challenges for assessing the demand of the tasks.

[112] The teaching hours were drawn from the Rio de Janeiro curriculum and may be different to other states.

4.11 Mexico

Education in Mexico is overseen by the Secretariat of Public Education and is compulsory from the age of three to 18. The school system is divided into primary school (ages six to 11) and secondary school (ages 11 to 18). Secondary education in Mexico is divided into lower and upper secondary, each lasting three years.

Upper secondary (*Educación Media Superior)* is offered through three main pathways:

- *Bachillerato General*
- *Bachillerato Tecnológico*
- *Educación Profesional Técnica.*

The Mexican *Bachillerato General* (MBG) is a traditional, academic pathway designed to prepare students for higher education, as well as developing the competencies necessary for the world of work.[113] The *Bachillerato Tecnológico*, combines general and vocational education to enable students to pursue higher education or a more professional/technical route.[114] Finally, the *Educación Profesional Técnica* is purely vocational and provides access to the labour market and employment.[115] Within the *Educación Media Superior,* the MBG is the focus in this report, as this programme is the most aligned with the DP.

The MBG lasts for three years and is divided into six semesters, with each year consisting of two semesters. During the MBG, students complete five curriculum components, namely the:

- Core component (‘*Componente de Formación Fundamental’*)
- Mandatory Extended Core component *(‘Componente de Formación Fundamental Extendido Obligatorio’*)
- Extended Core component *(‘Componente de Formación Fundamental Extendido’*)
- Work Experience component *(‘Componente de Formación Laboral’*)
- Expanded Curriculum *(‘Currículum Ampliado’*).[116]

Students study units from all five of these components. Units in the Core, Mandatory Extended Core, and Extended Core components are organised into Socio-cognitive Resources and Areas of Knowledge, which are outlined below:

Socio-cognitive Resources:

- Language and Communication
- Mathematical Thinking
- Historical Consciousness
- Digital Culture

Areas of Knowledge:

- Natural Sciences, Experimental Sciences and Technology
- Social Sciences
- Humanities

The MBG units used for comparison to DP mathematics are outlined below.

[113] Government of Mexico. (n.d.) *Bachillerato General.* Available from: Bachillerato General (sep.gob.mx)
[114] Mexican Secretariat of Education, Science, Technology and Innovation. (2024). *Bachillerato Tecnológico*. Available from: Bachillerato Tecnológico | Secretaría de Educación, Ciencia, Tecnología e Innovación (edomex.gob.mx)
[115] Government of Mexico. (n.d.). *Servicios educativos*. Available from: Subsecretaría de Educación Media Superior: Servicios educativos (sep.gob.mx)
[116] Mexican Secretariat of Public Education. (2024). *Base Document for the Bachillerato General*.

Mathematical Thinking[117]

Mathematical Thinking (MT) is one of the four Socio-cognitive Resources and includes common learning for all high school students. Socio-cognitive Resources play transversal roles, designed to support and enhance learning in the knowledge areas and other Socio-cognitive Resources. The curriculum structure is such that all the Curricular Learning Units offered for mathematics come under the Socio-cognitive Resource of Mathematical Thinking. As detailed below, some of the Curricular Learning Units are common for all Mexican high school students, while others are compulsory and optional for the Bachillerato General specifically.

Compulsory *Mathematical Thinking units in the Bachillerato General:*

- **Mathematical Thinking I, II, and III**[118]
  Mathematical Thinking I, II, and III are part of the Core component, described by the Mexican Common Curriculum Framework for Higher Secondary Education and are common learning for all high school students. These first three curricular units in Mathematical Thinking place emphasis on developing the skills associated with mathematical thinking, such as problem-solving and reasoning, which will benefit students regardless of their future career path. Each curricular unit corresponds to the first three semesters of high school.
- **Selected Topics in Mathematics I and II**[119]
  Selected Topics in Mathematics I and II are part of the Mandatory Extended Core component and are compulsory units for all institutions offering the Bachillerato General. These units follow Mathematical Thinking I, II, and III and are designed to broaden and deepen knowledge mathematical knowledge through covering additional topics and concepts.

Optional *Mathematical Thinking units in the Bachillerato General:*[120]

- **Probability and Statistics I** and **Probability and Statistics II**
- **Differential Calculus** and **Integral Calculus**
- **Financial Mathematics I** and **Financial Mathematics II**
- **Drawing I** and **Drawing II**

The optional mathematics units listed above are part of the Extended Core curriculum component and, if chosen, are studied in the last two semesters of high school. Students of the Bachillerato General choose eight optional units, from a range of Areas of Knowledge and Socio-cognitive Resources, and therefore the number of above optional units studied will vary

[117] Mexican Secretariat of Public Education. (2023). *Learning progressions for the socio-cognitive resource Mathematical Thinking.* Available from: Progresiones de Aprendizaje - Pensamiento Matematico.pdf (sep.gob.mx)
[118] Ibid.
[119] Undersecretary of Higher Secondary Education. (2021). *Selected Topics in Mathematics I*. Available from: https://www.cobachbcs.edu.mx/Content/Files/Docentes/programas-de-asignatura/semestre-5/componente-propedeutico/03-ciencias-exactas-e-ingenierias/temas-selectos-de-matematicas.pdf; *Selected Topics in Mathematics II*. Available from: https://www.cobachbcs.edu.mx/Content/Files/Docentes/programas-de-asignatura/semestre-6/componente-propedeutico/03-ciencias-exactas-e-ingenierias/temas-selectos-de-matematicas-II.pdf
[120] Undersecretary of Higher Secondary Education. (2018). *Programs of Study for the Class of 2022 – 2025. Propaedeutic Training Component.* Available from: Programs of Study for the Generation 2023 - 2026 and Subsequent. (sep.gob.mx)

between students. Each unit deepens students' knowledge of a particular area of mathematics.

The analysis will consider, and distinguish between, MT units which are compulsory in the Bachillerato General, and those which are optional.

4.11.1 Learning Outcomes

This section compares and contrasts the learning outcomes of curricula falling within the category of mathematics. For its mathematics learning outcomes, the DP sets out aims and assessment objectives for all subjects within the mathematics subject group – hence the extracted themes are the same for mathematics: analysis and approaches and mathematics: applications and interpretation.

The mathematics learning outcomes for the Bachillerato General are drawn from the Learning Progressions for Mathematical Thinking and the Programmes of Study for the optional units. From Learning Progressions for Mathematical Thinking, the trajectory learnings, learning goals, and categories and sub-categories of Mathematical Thinking are the main sources for learning outcomes, though other areas in the learning progression document are also considered. From the Programmes of Study of optional subjects, the General Competencies, and the Extended Disciplinary Competencies (for Mathematics), are used as learning outcomes, and are the same across all optional mathematics units.
The following summary table presents the learning outcome themes extracted from DP mathematics and indicates if, and where, they are judged to have presence within the learning outcomes of the Bachillerato General.

Table 59: Presence of the DP mathematics subject group learning outcome themes in the Bachillerato General.

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the Bachillerato General	
1. Be aware of, and engage with, mathematics in its wider context		Not strongly present in the Bachillerato General's learning outcomes for mathematics
2. Develop learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work		These skills are intended to be developed in the compulsory units of Mathematical Thinking and in the optional units' General Competencies
3. Use inquiry-based approaches		Although there is not a strong emphasis on using inquiry-based approaches, the ability to observe and make conjectures is expected in Mathematical Thinking I, II, and III
4. Understand the concepts, principles and nature of mathematics and apply concepts and procedures to a range of contexts		Present in the trajectory learning, learning goals, and 'Procedural' category of Mathematical Thinking I, II, and III, as well as the Extended Disciplinary Competencies of the optional units
5. Make links and generalisations		This theme is evidenced by the focus on 'transversality' in the Bachillerato General. For example, Mathematical Thinking is expected to be integrated with Areas of Knowledge and other Socio-cognitive Resources

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the Bachillerato General	
6. Develop critical/creative thinking skills e.g. problem-solving and reasoning		Present in the trajectory Learning, learning goals, and categories of Mathematical Thinking I, II, and III. Also present in the Extended Disciplinary Competencies of optional units
7. Communicate mathematics clearly and in various forms		Present in the trajectory learning for Mathematical Thinking I, II, and III
8. Know how technology and mathematics influence each other and use technology to develop ideas and solve problems		Use of technology is present, but not as strongly as in the DP

Key:

	This theme is well-evidenced in the learning outcomes of the Bachillerato General		*This theme is partially evidenced in the learning outcomes of the Bachillerato General*		*This theme is not evidenced in the learning outcomes of the Bachillerato General*

Presence of the DP's Learning Outcome Themes

There is reasonably strong alignment between the DP's and Bachillerato General's learning outcomes for mathematics, with most of the DP's themes being evidenced. The following discusses in more detail the presence of each DP theme in the Bachillerato General's learning outcomes.

1. Be aware of, and engage with, mathematics in its wider context

The DP's mathematics theme of awareness and engagement with wider contexts is not a strong focus of the Bachillerato General's mathematics learning outcomes. Indeed, there are no similar expectations to the DP, which include applying mathematics to developments in local and global communities, appreciating the moral, social, and ethical questions arising from mathematics, and appreciating multicultural and international perspectives. That said, teachers of the Bachillerato General could this involve this theme, as the Learning Progressions document for Mathematical Thinking I, II, and III contains suggestions for integrating mathematical thinking into Historical Awareness, including learning the history of how concepts were developed.

Furthermore, in the optional units' Programmes of Study there are references to using contexts from the real-world, such as sustainable development. Moreover, while not specifically for mathematics, it can be noted that the General Competencies somewhat reflect this theme through expectations that students engage with multiple perspectives and global issues regarding the environment.

2. Develop learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work

The DP mathematics theme of developing transferable learning skills is well evidenced in the Bachillerato General. Indeed, within Learning Progressions for Mathematical Thinking it is expressed that, similarly to the DP, students should develop positive and curious attitudes to mathematics, perseverance in problem solving, and the ability to work independently and collaboratively. Furthermore, while not specific to mathematics, the General Competencies for optional units include similar transferable learning skills, such as collaborative working and learning autonomously.

3. Use inquiry-based approaches

The DP mathematics theme of using inquiry-based approaches is somewhat evidenced in the learning outcomes of the Bachillerato General. Indeed, the expectation of performing investigations is not highlighted as a requirement in the mathematics units. However, students may use some skills related to inquiry-based approaches, as the Bachillerato General identifies the ability to observe and make conjectures as a sub-category of Mathematical Thinking.

4. Understand the concepts, principles and nature of mathematics and apply concepts and procedures to a range of contexts

The DP mathematics theme of understanding and applying mathematics is well-evidenced in the Bachillerato General. One of the four trajectory learnings of Mathematical Thinking focuses on the application of procedures to various contexts, such as knowledge areas and everyday life. Furthermore, several of the learning goals display this theme, as they include using algorithms and procedures, and establishing strategies and visualisations, to aid understanding. Furthermore, the Extended Disciplinary Competencies in the optional units also include the application of concepts to various contexts.

5. Make links and generalisations

The DP mathematics theme of making links and generalisations is well-evidenced in the Bachillerato General. Indeed, ‘transversality’ is a key focus in the curriculum and, as outlined in Learning Progressions for Mathematical Thinking, the area is designed as a Socio-cognitive Resource, which has the purpose of supporting learning in the Areas of Knowledge, which in turn deepens mathematical learning. Furthermore, the categories and sub-categories in Mathematical Thinking have been designed with the intention that connections are made within areas of mathematics, as well as for facilitating metacognition, reflection, and abstraction. Moreover, the Extended Disciplinary Competencies in optional units include transversal themes, namely social, environmental, health, and reading, which promote the application of mathematics in these contexts.

6. Develop critical/creative thinking skills e.g. problem-solving and reasoning

Similarly to the DP, developing critical and creative thinking skills is a strong focus in the Bachillerato General. Problem-solving and reasoning are particularly key skills in the Bachillerato General, and are the focus of several trajectory learnings, categories of MT, and subsequent learning goals. Specifically, the Bachillerato General expects students to develop processes of intuition and reasoning, which include making conjectures and approximations, and producing rigorous arguments. Furthermore, like the DP, the Bachillerato General learning outcomes expect students to propose solutions and models to problems set in a range of theoretical and practical contexts. This theme is also present in the Extended Disciplinary Competencies of the optional units, which include the skills of building models and using different approaches in solving problems.

7. Communicate mathematics clearly and in various forms

The DP theme of communicating mathematics is clearly evidenced in the mathematics learning outcomes of the Bachillerato General. Indeed, mathematical communication is the focus of one of the categories of MT, within which students are expected to use correct mathematical language, interpret and generate expressions and representations, and communicate ideas, concepts, and conjectures with others. Moreover, the learning goals also reflect this theme, as they expect students to use rigorous mathematical language, as well as to socialise and debate with peers. The theme is also present in the Extended Disciplinary Competencies of optional units, which expect students to interpret various representations and to communicate solutions using mathematical language, both verbally and in written form.

8. Know how technology and mathematics influence each other and use technology to develop ideas and solve problems

This DP theme involving the use of technology is not strongly represented in the Bachillerato General mathematics learning outcomes. Indeed, the use of technology is only referenced once within the learning goals, in the context of checking a procedure taken for solving a problem. Indeed, there is not the same emphasis as the DP, which strongly promotes the use of technology to solve problems and develop ideas.

Other Themes in the MBG

The skills described for mathematics in the Bachillerato General all relate to the DP's learning outcome themes. However, it can be noted one or two specific skills have a more explicit emphasis in the Bachillerato General. These include intuitive thinking and using heuristic strategies.

Summary

Overall, there is reasonably strong alignment between the DP and Bachillerato General with regards to mathematics learning outcomes. In particular, they have strong similarities with regards to developing critical thinking skills (such as problem solving and reasoning), making links with other areas of knowledge, communicating mathematics in various formats, and developing transferable learning skills. However, the use of technology receives a significantly lesser focus in the Bachillerato General compared to the DP, as does the use of inquiry-based approaches. Furthermore, engaging with wider contexts is not a present theme in mathematics units learning outcomes specifically, however it is an intended outcome for the Bachillerato General in general. Although no significantly different themes emerge from the Bachillerato General, it can be noted that there is more explicit emphasis on developing intuitive thinking and heuristics strategies as part of students' reasoning skills.

4.11.2 Content

This section compares the content of DP mathematics subjects with the mathematics content in the Bachillerato General. The mathematics content from the Mexican Bachillerato General is presented in the following visual.

Figure 27: Visualiser of Mathematical Thinking units in the Bachillerato General.

	Component of the Bachillerato General	Curricular Learning Units			
Mathematical Thinking	Core curriculum component (Compulsory Units)	**Mathematical Thinking I** (Statistical and probabilistic thinking) Learning Progressions 1-15	**Mathematical Thinking II** (Arithmetic, algebraic, and geometric thinking) Learning Progressions 1-14	**Mathematical Thinking III** (Variational thinking) Learning Progressions 1-15	
	Mandatory Extended Core curriculum component (Compulsory Units)	**Selected Topics in Mathematics I** Block I: Strategies to solve algebraic problems Block II: Linear equations I Block III: Linear equations II Block IV: Quadratic equations Block V: System of inequalities	**Selected Topics in Mathematics II** Block I: Properties of polygons Block II: Theorems of Thales and Pythagoras Block III: Trigonometric functions Block IV: Laws of sines and cosines Block V: Rectilinear segments and equations of lines		
	Extended Core curriculum component (Optional Units)	**Drawing Mathematics I** Block I: Historical background Technical Drawing Block II: Two-dimensional representation techniques Block III: Three-dimensional applications of Technical Drawing	**Financial Mathematics I** Block I: Basic foundation of Financial Mathematics and its application Block II: Sequences and series Block III: Simple interest Block IV: Personal finances	**Probability and Statistics I** Block I: Statistical elements Block II: Graphic description of a data set Block III: Statistical measurements Block IV: Behaviour of two variables	**Differential Calculus** Block I: Limits Block II: The derivative Block III: Applications of the derivative
		Drawing Mathematics II Block I: Application of Technical Drawing in mechanics Block II: Application of technical drawing in electrical systems Applications of technical drawing in construction	**Financial Mathematics II** Block I: Compound interest and inflation Block II: Annuities Block III: Amortization of credits	**Probability and Statistics II** Block I: Probability Block II: Probability distributions Block III: Probabilistic models	**Integral Calculus** Block I: Differentials Block II: Indefinite integrals Block III: Integration methods Block IV: Definite integrals and applications

4.11.2.1 Structure

The MBG organises mathematics into units, with some in each of the curriculum components (Core, Mandatory Extended Core, and Extended Core). In contrast, DP mathematics is structured into subjects rather than units, namely mathematics: analysis and its approaches (AA) and mathematics: applications and interpretation (AI). Each unit in the MBG lasts one semester, whereas the subjects in the DP are studied for the duration of the programme (two years).

Students of both the DP and MBG are required to study mathematics content as part of their respective programmes. Indeed, all DP students must take at least a SL course in mathematics (either AA or AI) and MBG students must study the compulsory units (Mathematical Thinking I, II, and III and Selected Topics in Mathematics I and II). In addition, both the DP and MBG allow students to specialise in mathematics, with the offer of doing a HL course in either AA or AI in the DP and the choice of optional units from the Extended Core curriculum component of the MBG.

Regardless of the chosen subject and level, all DP students study the same mathematics topics, namely Number and algebra, Functions, Geometry and trigonometry, Statistics and probability, and Calculus. For HL courses, the additional high level (AHL) content extends and adds to the SL content in each topic. In contrast, while all students of the MBG will study all the same topics as part of Mathematical Thinking and Selected Topics in Mathematics units, students wanting to specialise further in mathematics can choose the areas (and thus topics) of mathematics they study. Hence, at the specialisation level, students in the MBG have more flexibility than the DP with regards to the amount and type of mathematics topics that they continue to study.

Lastly, while the DP is flexible with regards to the sequencing of content in teaching, the learning progressions of MT units are given in the order that they should be taught in. Instead, autonomy is given to MBG teachers with regards to the amount of time allocated to each progression and the depth to which it is covered.

4.11.2.2 Content Alignment

This section will compare the alignment of mathematics content in the DP and MBG. The following tables present a simplified summary of the content alignment that MBG units have with each DP topic. The MBG Mathematical Thinking units are organised into those that are compulsory and those that are optional.

Table 60: Summary of the content alignment the Bachillerato General has with the main topics in AA.

AA topics		Presence in compulsory units	Presence in compulsory and optional units
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Table 61: Summary of the content alignment the Bachillerato General has with the main topics in AI.

AI topics		Presence in compulsory units	Presence in compulsory and optional units
SL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		
AHL	1. Number and algebra		
	2. Functions		
	3. Geometry and trigonometry		
	4. Statistics and probability		
	5. Calculus		

Key:

	Strong presence of this topic in the MBG		*Partial presence of this topic in the MBG*		*Little or no presence of this topic in the MBG*

Compulsory units:
Mathematical Thinking I, II, and III and Selected Topics in Mathematics I and II
Optional units:
Probability and Statistics I and II, Differential Calculus, Integral Calculus, Financial Mathematics I and II, and Drawing Mathematics I and II.

In the last columns, the alignments found in the compulsory units are carried over and combined with those in the optional units, to represent the cumulative mathematics content offered in the Bachillerato General
It should be noted that the programmes of study are currently being updated and new versions were only available for Mathematical Thinking I, II and III.

Compulsory Mathematical Thinking Units in the Bachillerato General

(Mathematical Thinking I, II, and III and Selected Topics in Mathematics I and II)

As shown in the tables, the compulsory units combined generally have partial alignment with DP SL content and no alignment with DP AHL content. Indeed, except for Number and algebra, the compulsory units have partial alignment with SL content in all topics. That said, it can be noted that the partial alignment with Statistics and probability and Calculus content is stronger compared to Functions and Geometry and Trigonometry content. Most of the DP SL content that is covered by the compulsory units are sub-topics which are common to both AA and AI. The following discusses the alignment the units have with each DP topic in more detail, identifying differences that arise for alignment with AA and AI.

Number and algebra

The MBG's compulsory units do not align with the DP SL or AHL content in Number and algebra. Aside from rational exponents and some counting principles, the concepts in this DP topic are not covered by the units. This includes sub-topics that are common to AA and AI, such as arithmetic and geometric sequences and series, financial applications, laws of logarithms, and complex numbers, as well as content specific to each DP subject, such as binomial theorem and proof (AA) and amortization and annuities, matrices, and eigenvectors (AI).

Functions

With regards to Functions content, the compulsory units cover linear, quadratic, exponential, and logarithmic functions, including the key features of their graphs. However, it is not explicit that concepts such as domain, range and inverse functions are covered. With regards to AA-specific content, the compulsory units do not include other SL concepts such as composite functions, inverse functions, quadratic inequalities, rational functions, and transformations, nor any AHL content involving more complex functions and analytical solutions. With regards to AI-specific content, the compulsory units cover modelling, but with fewer functions than both SL and AHL. Indeed, the compulsory units' model linear and quadratics functions, but not others, such as sinusoidal, or exponential growth and decay, models. Furthermore, the units do not explicitly outline the range of modelling skills that DP AI does. As noted for AA, transformations, composite, and inverse functions, which are AHL content in AI, are not covered by the compulsory units.

Geometry and trigonometry

With regards to Geometry and trigonometry content, the compulsory units only cover some DP SL content, which is common to both AA and AI. This includes some of the more basic geometry and trigonometry content such as solving problems regarding area and volume, right-angled triangles, and Pythagoras Theorem. Furthermore, sine and cosine functions are also considered, but in less depth, with transformations also not covered. Rather, a large proportion of content is not covered from AA and AI. Indeed, AA SL content of radians, circles, and trigonometric equations and identities is not covered, nor is any AHL content such as reciprocal functions and vectors. Furthermore, other AI SL content is not covered either, such as Voronoi diagrams, or any AHL content such as radians, matrix transformations, vectors, graph theory, adjacency matrices, or decision mathematics.

Statistics and probability

With regards to Statistics and probability content, the compulsory units cover a good amount of the DP SL content which is shared by AA and AI. This includes sampling, presenting data, measures of central tendency and dispersion, correlation, probability, and the normal distribution. However, linear regression, discrete random variables, and the binomial distribution are not covered. The compulsory units also cover a few concepts that are not common to AA and AI, such as Bayes Theorem (AA) and hypothesis testing (AI SL). However, Bayes Theorem is a suggested extension in the compulsory units, and the coverage of hypothesis testing is significantly less in-depth than DP AI SL and AHL content (for example, it does not include chi-squared tests, t-tests, critical regions, or testing for proportion and population mean). Other AHL content from AA and AI is not covered, such as continuous random variables in AA HL and non-linear regression, Poisson distribution, Central Limit Theorem, transition matrices, and Markov chains in AI HL.

Calculus

With regards to Calculus content, the compulsory units cover a reasonable amount of DP SL content. As AI SL has less calculus content than AA SL, the compulsory units align more closely with AI SL. The units cover the concept of limits, increasing and decreasing functions, derivatives or polynomials, differentiation rules, maximum, minimum and inflexion points, and optimisation problems. However, the units do not cover any integration content, which particularly impacts alignment with AA SL. Other SL content which is not covered is the second derivative and kinematic problems (AA) and trapezial approximation (AI SL).

In terms of AHL content, the compulsory units cover concepts of continuity and differentiability (AA), but no other AA AHL content, including further derivatives, evaluation of limits, implicit differentiation, further integration methods, and differential equations. The units also do not cover AI AHL content such as second derivatives, integration methods, differential equations, slope fields, Euler's method, phase portraits, and second order differentiation educations. However, it can be noted that students may set up for differential equations to represent contexts, but not solve them.

Other content in the MBG compulsory units

Most content in the compulsory units is covered by DP mathematics or is outlined as prior learning for the DP. However, linear programming is covered in Mathematical Thinking II, which is not an area focused on by either DP mathematics subject.

*Table 62: Content in the compulsory mathematics units of the MBG that is not covered in the DP**

Significant content not in AA (only)	Significant content not in AI (only)
• Some hypothesis testing content	• Counting principles • Continuity and differentiability
Significant content not in either DP mathematics subject	
• Linear programming	

* Significant content does not include topics which are typically studied *prior* to upper secondary.

Summary

Overall, the compulsory units have partial alignment with DP SL mathematics content, though overall represent less breadth and depth. It can be noted that the compulsory units generally have the same degree of alignment with DP AA and with DP AI, as they often cover content which is common to both. The compulsory units contain very little DP mathematics AHL content, and do not cover many other significant areas, they therefore have significantly less breadth and depth than DP HL mathematics.

Optional Mathematical Thinking Units in the Bachillerato General

(Statistics and Probability I and II, Differential Calculus, Integral Calculus, Financial Mathematics I and II, and Drawing Mathematics I and II)

This section will focus on how the inclusion of optional units impacts the alignment with DP mathematics content. As shown in the tables, the optional units increase alignment with DP

SL content in Number and algebra, Statistics and probability, and Calculus, as well as DP AHL content in Calculus. With certain topics, there is occasionally stronger alignment with one DP mathematics subject compared to another. Indeed, the optional units align more strongly with AI SL content for Number and algebra, but more strongly with AA (SL and AHL) content for Calculus. The optional units do not contain any DP Functions or Geometry and trigonometry content, therefore the alignment is the same as the compulsory units in these topics. The below discusses in the detail how the optional units align with each DP topic.

Number and algebra

With regards to Number and algebra content, the optional units Financial Mathematics I and II, increase the alignment with DP SL in this topic. Indeed, Financial Mathematics I includes concepts which are covered in both DP AA and AI, such as arithmetic and geometric sequences and series and financial applications. Furthermore, Financial Mathematics II presents further alignment with DP AI SL, as it includes an in-depth coverage of amortization and annuities. However, it can be noted that the optional units did not include DP AHL content such as proof, complex numbers, and matrices.

Functions and Geometry and trigonometry

As mentioned above, the optional units did not contain any further DP sub-topics from Functions and Geometry and trigonometry. Therefore, the Bachillerato General overall includes fewer types of functions than those covered in AA and modelled in AI. Furthermore, a significant amount of trigonometry content which is in AA is not covered, such as trigonometric equations and identities, and reciprocal trigonometry. Moreover, no vectors content is covered from AA or AI AHL, as is none of the graph theory and decision maths from AI AHL content.

Statistics and probability

With regards to Statistics and probability content, alignment with DP SL increases with the optional units. Indeed, Statistics and Probability I includes linear regression, which appears in both DP AA and AI. The same unit also references chi-squared tests, suggesting some further hypothesis testing content is covered from DP AI SL. Statistics and Probability II covers the binomial and normal probability distributions, as well as a small amount of AHL content involving Bayes Theorem (AA only) and the Poisson distribution (AI only). Other AHL content is not covered from either AA or AI.

Calculus

With regards to Calculus, the optional units Differential Calculus and Integral Calculus increase the alignment with DP SL and AHL content in this topic for both DP mathematics subjects. Differential Calculus covers concepts that are in both AA and AI, such as derivatives of transcendental functions, differentiation rules, higher order derivatives, and kinematic applications. The unit also presents further alignment with AA as it covers evaluating limits and the use of L'Hopital's rule. The Integral Calculus unit introduces integration, covering concepts common to AA and AI such as indefinite and definite integrals, area under a curve, integration methods, and volumes of revolution. Again, the unit displays some stronger alignment with AA than AI by including integration by parts and the use of partial fractions. However, it can be noted that the optional units do not cover as many derivatives and integrals as AA, nor do they cover implicit differentiation, solving differential equations, and Maclaurin series. Moreover, they also do not cover AI AHL content such as slope fields, Euler's method, phase portraits, and second order differential equations.

Other content in the MBG optional units

The Bachillerato General also offers two further optional units - Drawing I and II. These units focus on technical drawing, which is not an area covered by either DP mathematics subject. Drawing I involves the learning of theoretical, practical and regulatory elements of technical drawing, as well as the techniques and different representations. Drawing II focuses on the applications of technical drawing, such as for mechanics, electrical systems and constructure. Students use traditional and digital instruments, including CAD software.

Table 63: Mathematics content in the optional units of the Bachillerato General, that is not covered in DP mathematics subjects

Significant content not in AA (only)	Significant content not in AI (only)
• Poisson distribution • Hypothesis testing and chi-squared tests • Amortization • Annuities	• Counting principles • Bayes Theorem • Further integration methods • Evaluation of limits and L'Hopitals rule
Significant content not in either DP mathematics subject	

• Bernoulli distribution
• The content of Drawing I and II, which includes:
  ○ Theoretical, practical and regulatory elements of technical drawing
  ○ Two-dimensional representation techniques
  ○ Three-dimensional applications of technical drawing
  ○ Applications of technical drawing in mechanics
  ○ Applications of technical drawing in electrical systems
  ○ Applications of technical drawing in construction	

Summary

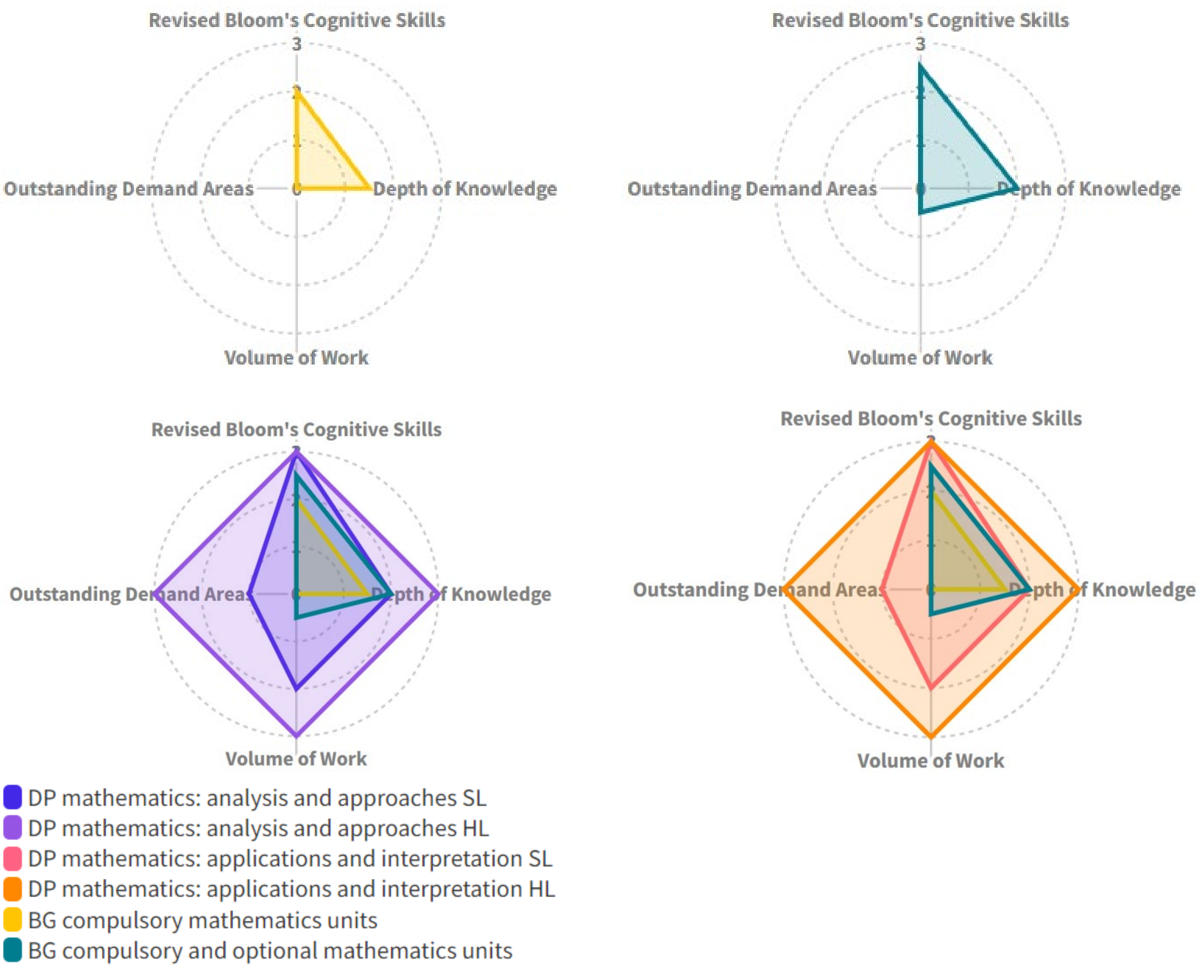
Combined, the mathematics units offered in the Bachillerato General have somewhat greater breadth and depth than DP SL mathematics, but less than DP HL mathematics. However, it should be noted that students choosing to specialise in mathematics need not take all the optional units, therefore, the breadth and depth of actual mathematics study is variable. For example, a student who studies the Statistics and Probability and Financial Mathematics units, without calculus, will experience a breadth and depth that is more akin to DP SL mathematics.

Overall, alignment is most strong with Calculus content, as the optional units of Differential Calculus and Integral Calculus include a significant proportion of DP SL and AHL content. Following this, alignment is most strong with SL Statistics and probability content, and there is also good alignment with SL Number and algebra content (particularly from AI) and SL Functions content. Apart from for Calculus, very little DP AHL content is covered in the Bachillerato General. Furthermore, Geometry and trigonometry is by far the Bachillerato General's weakest area of alignment with the DP, with very few SL and AHL topics covered. Instead, it can be noted that the Bachillerato General offers Drawing Mathematics I and II which focus on technical drawing, using traditional and digital instruments and applying to a range of different areas. Finally, the units mostly include content that is common to AA and AI, though occasionally align more strongly with one of these subjects for certain topics.

4.11.3 Demand

This section considers the alignment between the DP and MBG mathematics in terms of demand. The DP and MBG curricula were analysed using the same demand tool in order to create a demand profile for DP AA (SL and HL), DP AI (SL and HL), MBG compulsory mathematics units, and MBG compulsory and optional units. These demand profiles are presented below in the form of radar diagrams, with superimposed diagrams at the end also being featured to enable immediate visual comparison.

Figure 28: Visual representations of subject demand



The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - The DP mathematics subjects scored a 3 in this category.
  - The MBG compulsory units and optional units currently differ in the learning outcomes that they describe, and therefore there is a slight difference in scores. The compulsory units' outcomes focus on problem-solving and reasoning, with some evidence of higher-order thinking skills through defending reasoning and proposing conjectures. However, overall, the skills were deemed to be at the analysis level and a score of 2 was awarded. The optional units, building on similar

skills in the compulsory units, contain further competencies which offer some more evidence of higher-order thinking, such as using critical thinking in Drawing Mathematics and decision-making in Financial Mathematics, therefore, a slightly higher score of 2.5 was awarded to compulsory and optional units combined to acknowledge this.

- Regarding the scores for **Depth of Knowledge**:
  - Both DP SL subjects received a score of 2 and HL subjects received a score of 3.
  - The MBG compulsory units do not have a significant breadth of content, but some topics, such as calculus, statistics and probability, and functions are covered in considerable detail, requiring pre-requisite knowledge and a progression of learning. However, taking into account the number of topics studied in depth, and also noting that the topics were not studied in quite the same level of detail as DP SL, resulted in a 1.5, rather than a 2, being awarded. If optional units are also studied, then there is a greater depth of knowledge, as these build and extend the knowledge learnt in the compulsory units. However, units such as Probability and Statistics only add concepts such that the coverage is similar to DP SL, whilst others, such as Drawing Mathematics, represent further breadth, rather than depth. The topic of calculus is the only area which the units cover significantly more complex content. Therefore, taking into account the number of topics, as well as noting that the number and type of optional units students take is their choice, a score of 2, rather than a 3 was deemed appropriate for the compulsory and optional units combined.

- Regarding the scores for **Volume of Work**:
  - DP SL subjects received a score of 2 and DP HL subjects received a 3.
  - For MBG Mathematical Thinking I, II, and III units, a total of 192 hours is allocated, and for Selected Topics units, the current allocation is 96 hours. As mentioned, the compulsory units do not represent a considerable breadth of content or level of complexity, and therefore it was deemed that this was a generous amount of time to cover the content, resulting in a score of 0. The optional units are each allocated 48 hours, which was again deemed a generous amount of time, though a score of 0.5 was awarded to acknowledge the increase in complexity of content being covered.

- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - DP SL subjects received a score of 1 and DP HL subjects received a score of 3.
  - For all MBG units, the skills and content covered was deemed to not require any substantial demand beyond what is typical for upper-secondary mathematics.

4.12 Japan

The Japanese education system includes six years of elementary school followed by six years of secondary school, which is divided into three years of lower secondary (grades 7-9) and three years of upper secondary (grades 10-12).[121] MEXT – the Ministry of Education, Culture, Sports, Science and Technology – regulates the education system from kindergarten to upper secondary school and develops curriculum standards known as the *National Course of Study*.[122] The *Course of Study* is revised approximately once every 10 years; the most recent revision took place in 2018 and was implemented from 2020 to 2022.[123]

The courses offered in high schools providing upper-secondary education are classified as general, specialised, or integrated. General courses, and their associated subjects, are suitable for those wishing to progress to higher education or employment, but who do not intend to pursue a specific vocational area. In contrast, specialised courses provide vocational or specialist education for those who intend to pursue a particular vocational area or career. There are two types of subjects offered in specialised courses: industry and non-industry. Integrated courses, introduced in 1994, offer a variety of subjects that draw from both general and specialised courses.[124]

A minimum of 74 credits is required to graduate high school[125] and students are required to study compulsory subjects from the following areas:

- Japanese Language
- Geography and History
- Civics
- Mathematics
- Science
- Health and Physical Education
- Arts
- Foreign Language
- Home Economics
- Information
- Comprehensive Inquiry or Inquiry-based Study of Science and Mathematics.

In addition to compulsory subjects, students study electives in order to reach the minimum credit requirement. Optional subjects are offered in the same areas as listed above. There is also a wide range of industry and non-industry specialised subjects to choose from. Industry specialised subjects have the following subject areas: Agriculture, Industry, Business, Fisheries, Home Economics, Nursing, Informatics, and Welfare.[126] Non-industry specialised subjects are organised into the areas of: Science and Mathematics, Physical Education,

[121] National Information Centre for Academic Recognition Japan (NIC-Japan). (n.d.). *Overview of the Japanese Education System*. Available from: https://www.nicjp.niad.ac.jp/en/japanese-system/about.html
[122] MEXT. (2018). *High School Course of Study. Chapter VI Institutions of Higher Learning.* p. 5-6. Available from: 高等学校学習指導要領（平成30年告示）
[123] MEXT. (2018). *High School Course of Study. Supplementary Provisions (Ministry of Education, Culture, Sports, Science and Technology Decree No. 13 on March 30, 2006).* p. 11.
[124] MEXT. (n.d.). *Principles Guide Japan's Educational System.* Available from: https://www.mext.go.jp/en/policy/education/overview/index.htm
[125] MEXT. (2018). *High School Course of Study. Chapter VI Institutions of Higher Learning.* p.5-6.
[126] MEXT. (n.d.). *Principles Guide Japan's Educational System.*

Music, Art and Design, and English. Students studying vocational courses must have a minimum of 25 credits from specialised subjects and schools are to determine the credits for each subject.

The mathematics subjects from the Japanese High School Curriculum (JHSC) which were used in the analysis are described below.

General Subjects

- **Mathematics I, II, and III:** The first of these, Mathematics I (3 credits), is compulsory and designed to accommodate both students who complete high school mathematics with this subject alone and those who study further subjects. Mathematics II (4 credits) is studied after Mathematics I and comprises the core content of high school mathematics. Mathematics III (3 credits) follows Mathematics II and is designed for students who wish to study mathematics in depth and potentially enter specialised fields which require mathematics.[127]
- **Mathematics A, B, and C:** These subjects offer students the opportunity to broaden their mathematics study. Mathematics A complements the content of Mathematics I and can be taught in parallel. Mathematics B and C cover more advanced content than Mathematics I, thus should be studied after. These subjects can be studied individually, without the need to study the other. In each subject there is enough content for 3 credits, but content is usually selected such that each subject is worth 2 credits.[128]
- **Inquiry-Based Study of Science and Mathematics:** There are two subjects, Science and Mathematics Basic Inquiry and Science and Mathematics Inquiry**.** It is compulsory for students in specialised science and mathematics departments to study the inquiry-based subjects.[129]

Specialised Subjects

Each of the below subjects comprise a compilation of content from the general subjects, with certain areas of content extended. Mathematics I and II (SMC) are compulsory for all students in science and mathematics departments, as are the above-mentioned inquiry-based subjects.[130]

- **Mathematics I for the Science and Mathematics Course (SMC)**
- **Mathematics II for the Science and Mathematics Course (SMC)**
- **Advanced Mathematics for the Science and Mathematics Course (SMC).**[131]

High school students in Japan can choose a variety of mathematics subjects and combinations. Given the flexibility in the JHSC, four pathways (below) have been selected for the comparative analysis with DP mathematics and will be particularly relevant for the content and demand alignment analysis. These pathways aim to represent two main groups of students: those who pursue a broad and balanced study of mathematics in upper-secondary without a focus on further specialisation, akin to SL students, and those with a strong interest in mathematics who may pursue further studies, similar to HL students.

[127] MEXT. (2018). *High School Course of Study. Mathematics*. p. 91-97. Available from: 高等学校学習指導要領（平成30年告示）

[128] Ibid. p. 97-102.

[129] MEXT. (2018). *High School Course of Study. Science and Mathematics*. p. 196-198.

[130] MEXT. (2018). *High School Course of Study. Science and Mathematics*. p. 435-437.

[131] Advanced Mathematics is also referred to as Special Topics in Mathematics.

- **General Subjects Pathway I (GSPI)**: Mathematics I, II, A and B and Basic Inquiry
- **General Subjects Pathway II (GSPII)**: Mathematics I, II, III, A, B and C, Basic Inquiry and Inquiry
- **Specialised Subjects Pathway I (SSPI)**: Mathematics I and II (SMC), Basic Inquiry and Inquiry
- **Specialised Subjects Pathway II (SSPII)**: Mathematics I and II (SMC), Advanced Mathematics (SMC), Basic Inquiry and Inquiry.

4.12.1 Learning Outcomes

This section compares and contrasts the learning outcomes of curricula falling within the category of mathematics. Ecctis extracted learning outcome themes from the DP mathematics subject group's aims and assessment objectives, which are the same for both AA and AI.

The learning outcomes for JHSC mathematics subjects are primarily drawn from the three main objectives articulated for the general mathematics subjects and the three (similar) objectives outlined for the specialised mathematics subjects. The objectives for the inquiry-based science and mathematics subjects are considered in addition to these.

The following summary table presents the learning outcome themes extracted from DP mathematics and indicates if, and where, they are judged to have presence within the learning outcomes of the JHSC.

Table 64: Presence of the DP mathematics subject group learning outcome themes in the JHSC.

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the JHSC	
1. Be aware of, and engage with, mathematics in its wider context.		This theme is present in some subjects' objectives, namely Mathematics A and B and the inquiry-based studies.
2. Develop learning skills; having a positive and resilient attitude, working both independently and collaboratively, being reflective and evaluating work.		This theme is present in the third objective which describes recognising the merits of mathematics, persistent thinking, and reflecting on and evaluating work.
3. Use of inquiry-based approaches.		This theme is present in some subjects' objectives, namely the inquiry-based studies.
4. Understand the concepts, principles and nature of mathematics and apply concepts and procedures to a range of contexts.		This theme is strongly present in the objectives, particularly the first objective which focuses on knowledge, understanding and application.
5. Make links and generalisations.		This theme is present in the objectives, particularly in the second objective which describes understanding relationships between phenomena and developing an integrated approach.
6. Develop critical/creative thinking skills e.g. problem-solving and reasoning.		This theme is strongly present in the objectives, with the third objective emphasising problem-solving, reasoning, creativity, and making decisions.
7. Communicate mathematics clearly and in various forms.		This theme is present in the objectives, with the second objective highlighting the importance of clear and accurate expression.

Themes extracted from the learning outcomes in the DP mathematics subject group	Presence in the JHSC	
8. Know how technology and mathematics influence each other and use technology to develop ideas and solve problems.		The use of technology is not explicitly mentioned in any of the objectives. While its use is certain, these skills are emphasised less in the objectives.

Key:

	This theme is well-evidenced in the learning outcomes of JHSC.		*This theme is partially evidenced in the learning outcomes of the JHSC.*		*This theme is not evidenced in the learning outcomes of the JHSC.*

Presence of the DP's Learning Outcome Themes

There is a reasonably strong alignment between the DP and JHSC learning outcomes for mathematics, with most of the DP's themes being evidenced. In the JHSC, the first mathematics objective (for both general and specialised mathematics subjects) focuses on understanding concepts, principles, and laws and applying these to a variety of contexts, which strongly aligns with the same theme in the DP (theme 4). Moreover, the second objective focuses on understanding relationships between phenomena and the use of mathematical communication, thus aligning with the DP themes of making links and generalisations, and accurate communication. The third objective emphasises valuing mathematics, actively engaging with it and demonstrating perseverance. It also focuses on utilising problem-solving, reasoning, reflection, evaluation, creativity, and decision-making skills. This objective therefore aligns closely with the DP themes of developing learning skills and employing critical and creative thinking skills.

The DP theme of engaging with mathematics in a wider context is partially present in the JHSC objectives. The objectives of general subjects describe students engaging with phenomena (their consideration, interpretation, mathematisation, and expression), though does not specify that these phenomena would link to, for example, global issues. The presence of this theme is more explicit in Mathematics A and B and in the inquiry-based subjects. Indeed, Mathematics A and B include units named *Mathematics and human activities* and *Mathematics and society*, respectively, which include considerations related to this DP theme. Moreover, the inquiry-based subjects can include the exploration of topics such as the environment.

The DP theme of the use of inquiry-based approaches is present in some subjects' objectives. Indeed, the inquiry-based subjects naturally have a strong focus on the use of these approaches. Furthermore, the specialised subjects' objectives indicate this theme may be present, stating "the aim is to develop the following qualities and abilities necessary for inquiry through mathematical activities, observations and experiments".[132] The DP theme of technology is not well-evidenced in the JHSC objectives. Indeed, there is no explicit mention to technology skills and thus technology receives a lesser emphasis compared to the DP learning outcomes.

[132] MEXT. (2018). *High School Course of Study. Section 9 Science and Mathematics.* p. 435.

Summary

Most DP themes are evidenced in the JHSC objectives, including understanding and application, mathematical communication, learning skills and critical thinking. However, the DP themes of engaging with mathematics in a wider context and the use of technology are only partially or minimally present. Inquiry-based approaches are present in some JHSC subjects, but not others. No significantly different themes emerge from the JHSC objectives, though the inquiry-based and specialised subjects have more emphasis on making links between science and mathematics specifically.

4.12.2 Content

This section compares the mathematics content of the DP and JHSC subjects. The content of the JHSC mathematics subjects is presented in the following diagrams, displaying the key topics and sub-topics in each.

Figure 29: JHSC mathematics content visualiser.

General subjects	Mathematics	Mathematics I	Numbers and formula	Shapes and measurement	Quadratic functions	Data analysis				
		Mathematics II	Various expressions	Geometry and equations	Exponential functions and logarithmic functions	Trigonometric functions	Differentiation and integration			
		Mathematics III	Limits of sequences and functions	Differentiation	Integration methods					
		Mathematics A	Properties of figures	Number of cases and probability	Mathematics and human activities					
		Mathematics B	Numerical sequences	Statistical inference	Mathematics and social life					
		Mathematics C	Vectors	Curves on the plane and the complex plane	Ingenuity in mathematical expression					
	Inquiry-Based Study of Science and Mathematics	Basic-inquiry	*To acquire the following knowledge and skills:* (a) Understanding the significance of inquiry. (b) Understanding the process of inquiry. (c) An understanding of research ethics. (d) (Basic) skills in observation, experiments, surveys, etc. (e) (Basic) skills for analysing phenomena. (f) (Basic) skills for summarising and presenting the results of research.							
		Inquiry	*Students will acquire the basic knowledge and skills shown in the content through the process of exploring the following:* (A) Matters related to natural and social phenomena. (B) Matters relating to cutting-edge science and interdisciplinary fields. (C) Matters concerning the natural environment. (D) Matters relating to science and technology. (E) Matters related to mathematical phenomena.							
Specialised subjects	Science and Mathematics	Mathematics I (SMC)	Numbers and formula*	Shapes and measurement	Quadratic functions	Exponential functions and logarithmic functions	Data analysis	Number of cases and probability		
		Mathematics II (SMC)	Various expressions	Numerical sequences	Trigonometric functions and the complex plane	Geometric figures and equations*	Limits	Differential calculus	Integration*	Statistical inference*
		Advanced Mathematics (SMC)	Vectors*	Matrices and their applications*	Discrete mathematics*	Mathematics and daily life and society				

An asterisk (*) indicates the topics where the general subjects' content has been extended to include new material.

Structure

The JHSC organises mathematics into subjects, both general and specialised. Students of both the DP and JHSC are required to study mathematics content as part of their respective programmes. Indeed, the DP requires students to study at least a SL course in mathematics (either AA or AI) and the JHSC requires students to study at least Mathematics I, or to have achieved equivalent outcomes in another subject (such as a specialised subject). Students studying in the science and mathematics department must take the specialised subjects Mathematics I (SMC) and Mathematics II (SMC).

The JHSC general subjects are more elective in style than the DP subjects. Indeed, each individual subject is smaller in comparison to DP subjects and focuses on a narrower range of content. Students can tailor their mathematics study by choosing from the general subjects Mathematics I, II, III, A, B, and C, whereas, in the DP, students choose only one subject to study. Furthermore, in the subjects Mathematics A, B and C, students can select the content that they wish to study to attain two credits. This contrasts to the DP subjects in which all content is compulsory. Hence, the general subjects offer more flexibility and optionality for mathematics study than the DP. However, both are similar in the regard that they provide opportunity to specialise in mathematics, with the offer of doing a HL course in either AA or AI in the DP and the offer of Mathematics III and Mathematics C in the JHSC.

The structure of the JHSC specialised subjects is more similar to that of the DP. The three specialised subjects cover all the content of the six general subjects and expand on it in certain areas (see Figure 29). As a result, they tend to encompass more material, making them more akin to DP subjects. In addition to being fewer and larger, all content within the specialised subjects is compulsory, offering less flexibility for customisation. Like DP HL subjects, the specialised subjects allow for a deeper study of mathematics. However, since Mathematics I and II (SMC) are compulsory for students in science and mathematics departments, they do not offer a less specialised pathway, such as DP SL subjects.

Finally, the DP differs to JHSC by offering stand-alone subjects with distinct thematic focuses, such as AA and AI.

Content Alignment

This section will compare the alignment of mathematics content in the DP and JHSC. The following tables present a simplified summary of the content alignment that the four selected JHSC pathways have with the SL and additional higher level content (AHL) of each DP topic.

Table 65: Summary of the content alignment between JHSC pathways and the main topics in AA.

	AA topics	GSPI	GSPII	SSPI	SSPII
SL	1. Number and algebra				
	2. Functions				
	3. Geometry and trigonometry				
	4. Statistics and probability				
	5. Calculus				
AHL	1. Number and algebra				
	2. Functions				
	3. Geometry and trigonometry				
	4. Statistics and probability				
	5. Calculus				

Table 66: Summary of the content alignment between JHSC pathways and the main topics in AI.

	AI topics	GSPI	GSPII	SSPI	SSPII
SL	1. Number and algebra				
	2. Functions				
	3. Geometry and trigonometry				
	4. Statistics and probability				
	5. Calculus				
AHL	1. Number and algebra				
	2. Functions				
	3. Geometry and trigonometry				
	4. Statistics and probability				
	5. Calculus				

Key:

	Strong presence of this topic in the JHSC pathway.		*Partial presence of this topic in the JHSC pathway.*		*Little or no presence of this topic in the JHSC pathway.*

- General Subjects Pathway I (GSP1): Mathematics I, II, A, B and Basic Inquiry
- General Subjects Pathway II (GSP2): Mathematics I, II, III, A, B and C and Inquiry subjects
- Specialised Subject Pathway I (SSP1): Mathematics I and II (SMC) and Inquiry subjects
- Specialised Subject Pathway II (SSP2): Mathematics I and II (SMC), Advanced Mathematics (SMC) and Inquiry subjects.

General Subjects Pathway I

(The study of Mathematics I, II, A, and B and Basic Inquiry)

This pathway has strong alignment with SL mathematics content in the DP, slightly more so with AA than AI. The JHSC subjects in this pathway cover key topics such as sequences and series, quadratic functions, exponential and logarithmic functions, trigonometric relationships and identities, presenting data, summary statistics, probability, discrete random various, the binomial and normal distribution, derivatives and integrals of polynomials, and maximum and minimum points. This leads to at least partial alignment with the SL content in all DP topics, with strong alignment for most.

Alignment is stronger with AA's *Number and algebra* content than AI's, as the pathway includes binomial theorem and proof but not financial applications such as amortization and annuities. Similarly, alignment is stronger with AA's *Functions* content, as the JHSC subjects do not explicitly focus on the modelling of functions. In contrast, alignment is stronger with AI's

Calculus content, as the pathway does not include AA SL content such as differentiation rules, the second derivative, and integration by inspection and substitution.

While alignment is overall judged to be strong with the SL content of most topics, not all SL subtopics are necessarily covered. For example, reciprocal and rational functions, transformations, and Voronoi diagrams are not included. Furthermore, whilst hypothesis testing is introduced, it is not covered to the same extent as AI's SL or AHL content.

The JHSC pathway has very little alignment with AHL content. However, it can be noted that there are couple of subtopics covered, including counting principles (AA), proof by induction (AA), factor theorem (AA) and a few other subtopics that are SL content in AA and AHL content in AI.

Lastly, there is some content in the JHSC subjects which is not covered in one, or both, DP mathematics subjects (see Table 67). Most significantly, Mathematics II covers some different geometry content such as equations of circles, loci and more coverage of the properties of geometrical figures. Moreover, Mathematics A and B include the units *Mathematics and Human Activities* and *Mathematics and Social Life*. While these are not distinct subtopics in DP mathematics, their content shares similarities with the links and considerations to be embedded in DP mathematics, such as the social, cultural and historical contexts of mathematics.

Table 67: Content in covered in the JHSC pathway (GSPI) that is not covered in the DP subjects.

Significant content not in AA (only)	Significant content not in AI (only)
• Concepts of hypothesis testing (Math B)	• Binomial theorem (Math II) • Proof (Math B) • Counting principles (Math A) • Double angle identities (Math II) • Factor theorem (Math II)
Significant content not in either DP mathematics subject	
• Linear programming (Math II) • Loci (Math II) • Equation of a circle (Math II) • Euclidean algorithms (Math A) • More properties of geometric figures (Math A) • Mathematics and human activities (as a distinct subtopic) (Math A) • Mathematics and social life (as a distinct subtopic) (Math B)	

Overall, General Subjects Pathway I has strong alignment with SL mathematics content and has similar breadth and depth to SL subjects. There is limited alignment with AHL content, and the pathway has significantly less breadth and depth than HL subjects. Lastly, the Basic Inquiry subject in the JHSC pathway covers inquiry skills but does not require students to be as independent as the DP mathematical exploration, resulting in only partial alignment.

General Subjects Pathway II
(The study of Mathematics I, II, III, A, B, and C and Inquiry subjects)

This pathway includes the study of all the subjects in GSPI, as well as Mathematics III, Mathematics C, and the Inquiry subject. Therefore, this section will focus on how these additional subjects impact alignment with DP mathematics.

Mathematics III covers further differentiation and integration content, thus alignment with the DP's *Calculus* SL and AHL content increases. Indeed, the subject covers further derivatives, differentiation rules, the second derivative, area under a curve, and volumes of revolution. Moreover, Mathematics III has a unit that covers limits of functions, which aligns with AA's AHL content regarding continuity, differentiability and limits. The same unit also covers limits of sequences which is not covered by DP mathematics. Mathematics III also covers some more DP *Functions* content such as rational functions, and composite and inverse functions.

Mathematics C includes coverage of vectors; thus, it increases the alignment with the DP's *Geometry and trigonometry* AHL content. However, the extent of coverage is less, particularly in comparison to AA. Mathematics C also covers similar content regarding complex numbers, which increases alignment with AHL content in *Number and algebra*. Moreover, Mathematics C covers matrices and discrete graphs, which are subtopics in AI.

Generally, Mathematics III and C align with DP AHL content, though there are several subtopics which are not covered, including most AA and AI AHL content from *Statistics and probability*. Conversely, there are a few areas that are covered in Mathematics III and C which are not in one, or either, of the DP subjects (see Table 68). Most notably these are limits of sequences and conics.

Table 68: Mathematics content in covered in the JHSC pathway (GSPII) that is not covered in the DP subjects.

Significant content not in AA (only)	**Significant content not in AI (only)**
• Concepts of hypothesis testing and confidence intervals (Math B) • Matrices (Math C) • Discrete graphs (Math C)	• Binomial theorem (Math II) • Proof (Math B) • Counting principles (Math A) • Double angle identities (Math II) • Factor theorem (Math II) • Rational functions (Math III) • Limits of functions (Math III)
Significant content not in either DP mathematics subject	
• Linear programming (Math II) • Loci (Math II) • Equation of a circle (Math II) • Euclidean algorithms (Math A) • More properties of geometric figures (Math A)	• Recurrence sequences (Math B) • Mathematics and human activities (as a distinct subtopic) (Math A) • Mathematics and social life (as a distinct subtopic) (Math B) • Limits of sequences (Math III) • Conics (Math C)

Overall, General Subjects Pathway II (al general subjects combined) has reasonable content alignment with DP HL subjects. Since this pathway covers a substantial amount of DP content, along with other content, the breadth and depth of mathematics content can be considered comparable to that of DP HL mathematics. Lastly, the Inquiry subject in the pathway develops inquiry skills and requires students to conduct independent research, thus it aligns strongly with the DP's mathematical exploration.

Specialised Subjects Pathway I

(The study of Mathematics I (SMC), Mathematics II (SMC), and the Inquiry subjects)

Most of the specialised subjects' content is drawn from the general subjects. Specialised subject Mathematics I (SMC) covers all content from Mathematics I, some content from

Mathematics II and Mathematics B, and a very small amount from Mathematics III. Specialised subject Mathematics II (SMC) covers the rest of the content from Mathematics II and III, most of Mathematics B, and some of Mathematics C. Therefore, this pathway encompasses a considerable amount of content that is in General Subjects Pathway II and thus has somewhat similar alignment to DP mathematics, though with a few notable differences.

Whilst some content from Mathematics C is covered, the specialised subjects Mathematics I and II (SMC) do not cover the vectors, matrices or discrete graphs subtopics. Therefore, there is, in comparison to General Subjects Pathway II, less alignment with AHL content in *Geometry and trigonometry* and AI's AHL content in *Number and algebra*. This pathway also does not cover the topics of *Mathematics and Human Activities* (from Mathematics A) and *Mathematics and Social Life* (from Mathematics B).

Instead, a few topics from the general subjects are expanded on. Indeed, Mathematics II (SMC) extends the *Statistical Inference* topic from Mathematics B to include continuous random variables, uniform distributions, and probability density functions, which increases alignment with AA's *Statistics and probability* AHL content. The extension also includes more hypothesis testing content, such as chi-squared tests and t-tests, which aligns with AI's *Statistics and probability* SL content. Furthermore, solving simple first order differential equations is added, which is also present in DP *Calculus* AHL content.

Table 69 presents the content in this pathway that is not covered in one, or both, DP subjects.

Table 69: Mathematics content in covered in the JHSC pathway (SSPI) that is not covered in the DP subjects.

Significant content not in AA (only)	**Significant content not in AI (only)**
• Hypothesis testing, confidence intervals, chi-squared tests and t-tests (Math II SMC)	• Binomial theorem (Math II) • Proof (Math B) • Counting principles (Math A) • Double angle identities (Math II) • Factor theorem (Math II) • Rational functions (Math III) • Limits of functions (Math III) • Continuous random variables (Math II SMC)
Significant content not in either DP mathematics subject	
• Linear programming (Math II) • Loci (Math II) • Equation of a circle (Math II) • Euclidean algorithms (Math A)	• More properties of geometric figures (Math A) • Recurrence sequences (Math B) • Limits of sequences (Math III) • Conics (Math C)

Overall, Specialised Subjects Pathway I has reasonable alignment with DP HL subjects. Indeed, most SL content is covered, as well as AHL content from several topics. As the pathway covers a considerable amount of DP content, in addition to other subtopics, the breadth and depth of mathematics content can be considered similar to that of DP HL mathematics. The alignment is slightly stronger with AA content than AI content.

Specialised Subjects Pathway II

(Mathematics I (SMC), Mathematics II (SMC), Advanced Mathematics (SMC) and Inquiry subjects)

The specialised subjects combined encompass all the content from the general subjects. Therefore, the alignment to DP mathematics is similar to the General Subjects Pathway II. That said, the specialised subjects extend the content in certain areas, which leads to a slightly stronger alignment. Advanced Mathematics (SMC) covers the vectors content of Mathematics C and extends it to include vector equations of lines and places, thus increasing alignment with DP *Geometry and trigonometry* content. However, it can be noted that AA's coverage of vectors remains more extensive. Advanced Mathematics (SMC) also extends the content of Mathematics C regarding matrices and discrete graphs. It covers more of what is addressed in DP AI, though still to a lesser extent.

It can be noted that some AHL content is not present in either the general or specialised subjects. Notably, this includes eigenvalues and eigenvectors (AI), modelling functions (AI), reciprocal trigonometric ratios (AA), cross product and further vector content (AA), non-linear regression (AI), Poisson distribution (AI), transition matrices and Markov chains (AI), Maclaurin series (AA), slope fields and phase portraits (AI), and second order differential equations (AI).

Table 70 presents the content in this pathway that is not covered in one, or both, DP subjects.

Table 70: Mathematics content in covered in the JHSC pathway (SSPII) that is not covered in the DP subjects.

Significant content not in AA (only)	Significant content not in AI (only)
• Hypothesis testing, confidence intervals, Chi-squared tests and t-tests (Math II SMC) • Discrete graphs and applications (Advanced Math SMC) • Matrices (Advanced Math SMC)	• Binomial theorem (Math II) • Proof (Math B) • Counting principles (Math A) • Double angle identities (Math II) • Factor theorem (Math II) • Rational functions (Math III) • Limits of functions (Math III) • Continuous random variables and probability density functions (Math II SMC) • Vector equations of planes (Advanced Math SMC)
Significant content not in either DP mathematics subject	
• Linear programming (Math II) • Loci (Math II) • Equation of a circle (Math II) • Euclidean algorithms (Math A) • More properties of geometric figures (Math A)	• Recurrence sequences (Math B) • Limits of sequences (Math III) • Conics (Math C) • Mathematics and human activities (as a distinct subtopic) (Math A) • Mathematics and social life (as a distinct subtopic) (Math B)

Overall, Specialised Subjects Pathway II has considerable alignment with DP mathematics content. The specialised subjects encompass all the general subjects' content and extends it. Therefore, the breadth and depth of this pathway's content can be considered similar, if not somewhat greater than, that of DP HL subjects.

4.12.3 Demand

This section considers the alignment between the DP mathematics subjects and the selected JHSC pathways with regards to demand. The DP and JHSC subjects were analysed using the same demand tool in order to create a demand profile for DP AA (SL and HL), DP AI (SL and HL), JHSC General Subjects Pathway I, JHSC General Subjects Pathway II, JHSC Specialised Subjects Pathway I, and JHSC Specialised Subjects Pathway II. These demand profiles are presented below in the form of radar diagrams, with superimposed diagrams at the end also being featured to enable immediate visual comparison.

Figure 30: Visual representations of subject and pathway demand.

DP mathematics: analysis and approaches SL
DP mathematics: analysis and approaches HL
DP mathematics: applications and interpretation SL
DP mathematics: applications and interpretation HL
JHSC General Subject Pathway I JHSC General Subject Pathway II
JHSC Specialised Subject Pathway I JHSC Specialised Subject Pathway II

The panel of experts carried out a detailed analysis of each course and reached a consensus on the scores shown in the profiles above. The following points were particularly important within the panel discussion:

- Regarding the scores for **Bloom's Cognitive Skills**:
  - Both DP mathematics subjects, at SL and HL, received a score of 3.
  - All JHSC pathways received a score of 3 due to the objectives for both general and specialised subjects consistently featuring higher-order thinking skills, such as critical thinking, making decisions based on mathematical reasoning, reflecting on and evaluating problem-solving and inquiry processes, and developing creativity.

- Regarding the scores for **Depth of Knowledge**:
  - Both DP mathematics subjects at SL were given a score of 2. At HL, both DP mathematics subjects were given a score of 3.
  - General Subject Pathway I received a score of 2 due to a wide range of topics being covered in a considerable, although not high, level of detail. The rest of the pathways received a score of 3, due to most topics being studied in very high detail. Furthermore, the objectives also contributed to the pathway scores, as they emphasised an integrated and expansive approach to learning, as well as indicating opportunities for extended thinking in the inquiry-based subjects.

- Regarding the scores for **Volume of Work**:
  - Both DP mathematics subjects at SL were given a score of 2. At HL, both DP mathematics subjects were given a score of 3.
  - General Subjects Pathway I scored a 1 due to the content being spread over several subjects which overall led to a standard time allocation for the content, thus representing a moderate workload. General Subjects Pathway II and Specialised Subjects Pathway II both received a score of 1.5. These pathways similarly spread the content across multiple subjects, however, the breadth and depth of the subjects meant that the workload was deemed to be slightly more than moderate. Finally, Specialised Subjects Pathway II was given a score of 2. Indeed, the additional content and complexity led to it being judged as having a moderate to heavy workload.

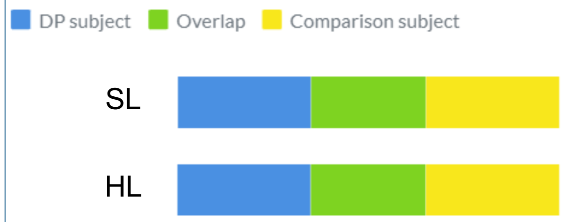
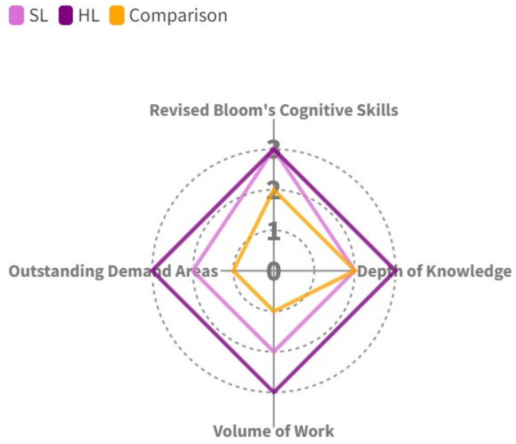
- Regarding the scores for **Outstanding Areas of Subject Demand**:
  - Both DP mathematics subjects at SL received a score of 1. DP mathematics subjects at HL received a score of 3.
  - General Subjects Pathway I received a score of 1 as the Basic Inquiry subject and inclusion of proof methods were deemed enough to be considered as one area of outstanding demand. General Subjects Pathway II was given a score of 2 for its inclusion of the Inquiry subject, proof, limits of sequences and functions, coverage of complex numbers, and polar coordinates. Specialised Subjects Pathway II

received a score of 3 for these areas in addition to continuous random various and probability density functions and differential equations. Specialised Subjects Pathway II also received a score of 3 for these areas as well as its coverage of vectors.

5. Key Findings

This section summarises the alignment and main similarities and differences found between the DP and the comparison subjects of each of the eleven comparison programmes in this study.

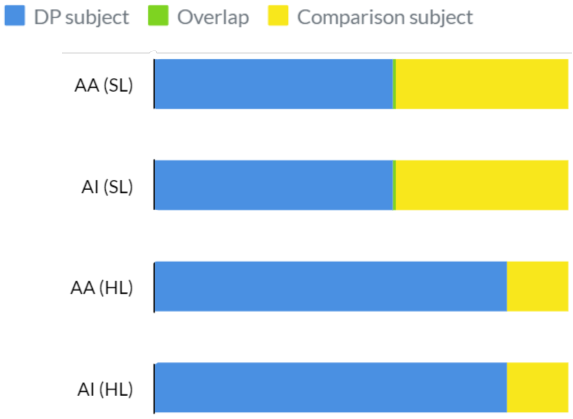
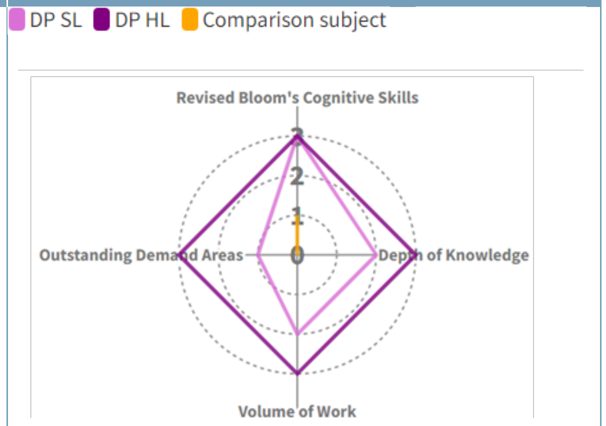
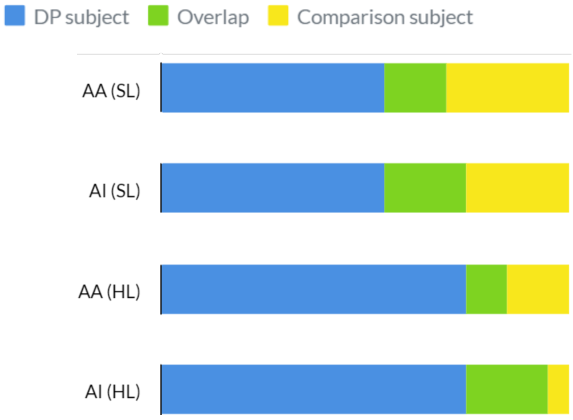
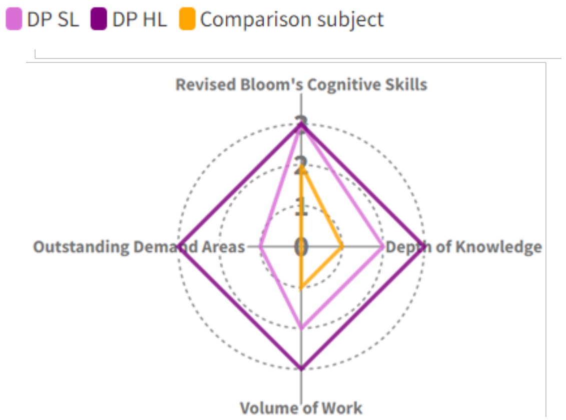
Key:

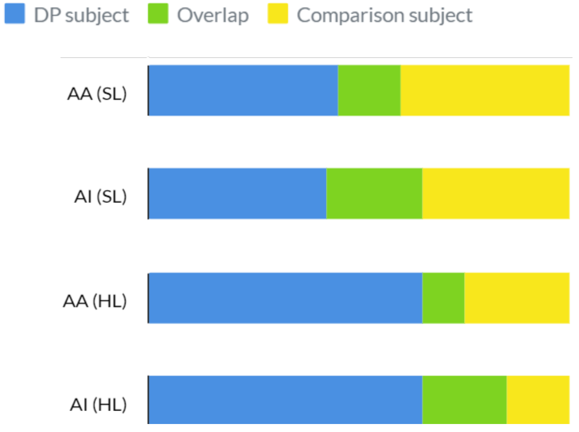
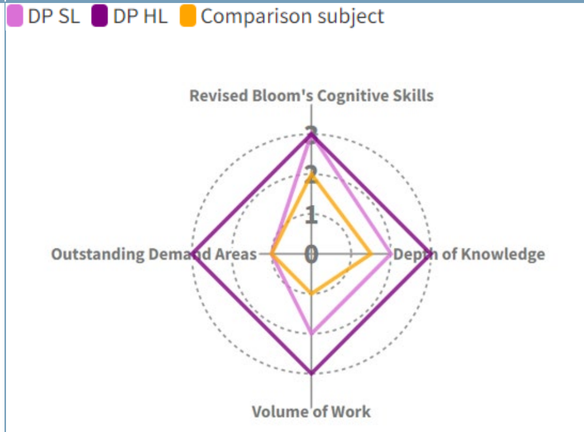
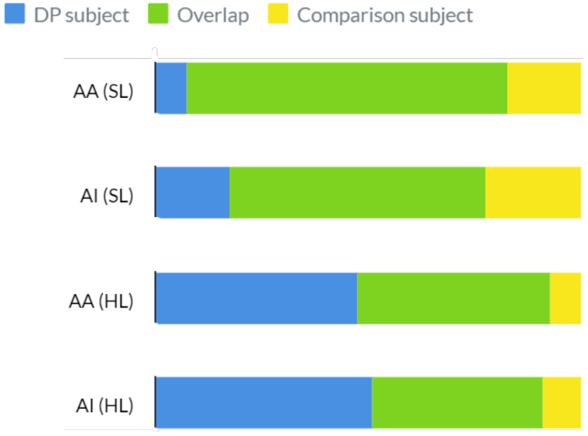
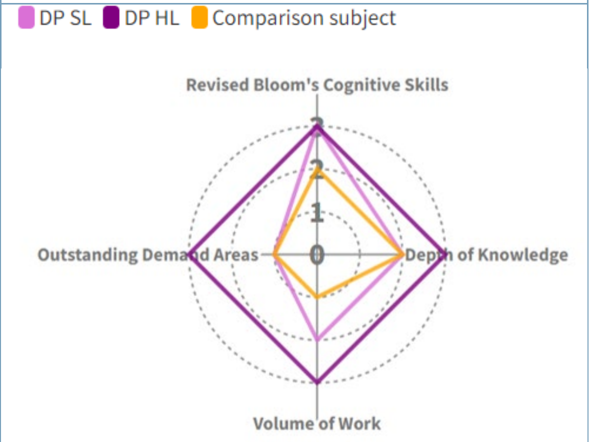
Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Displays the name of the comparison subject	Low **Moderate** High This represents the learning outcome alignment between the DP subject and the comparison subject. A black border is placed around the selected judgement – 'Moderate' in this example.	 These bars represent the content alignment between the DP subject and the comparison. There is one bar showing alignment with SL content and another for HL content (inclusive of SL content). The **green** section of the bar represents the overlap of content between the subjects. The **blue** section represents content which was in the DP subject only. The **yellow** section represents content which was in the comparison subject only. Therefore, if, say, the blue section was longer than the yellow, this can be interpreted as DP subject having more content unique to itself than the comparison did. A large green bar would indicate that a substantial proportion of content overlaps between the DP and the comparison subject.	 This radar diagram displays the demand judgement scores for the comparison subject(s) and the DP subject – both SL and HL. It can be noted that mathematics: analysis and approaches (AA) and mathematics: applications and interpretation (AI) score the same as each other for SL and the same as each other for HL.

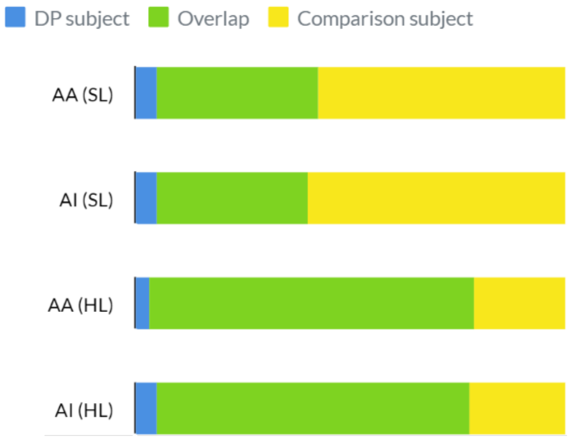
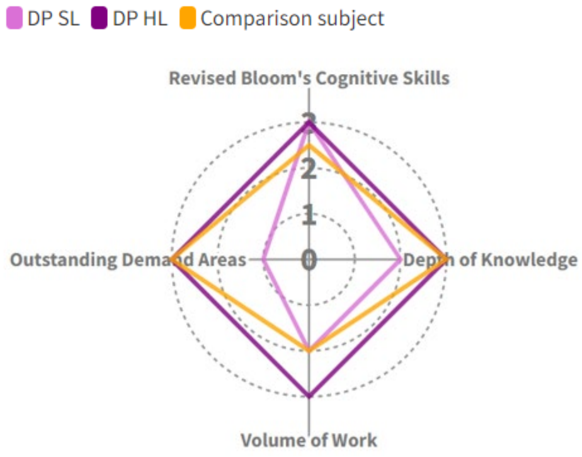
5.1 Australia (Victoria)

The subject level alignment between the DP and VCE mathematics subjects is represented below:

Figure 31: Visual representations of subject-level alignment (mathematics subjects).

<table>
<tr><th>Subject name</th><th>Learning outcomes alignment</th><th>Content alignment</th><th>Demand alignment</th></tr>
<tr><td>Foundation Mathematics (FM)</td><td>Low
Moderate
High</td><td>
</td><td>
</td></tr>
<tr><td>General Mathematics (GM)</td><td>Low
Moderate
High</td><td>
</td><td>
</td></tr>
</table>

<table>
<tr><th>Subject name</th><th>Learning outcomes alignment</th><th>Content alignment</th><th>Demand alignment</th></tr>
<tr><td>Further Mathematics (FRM)</td><td>Low
Moderate
High</td><td></td><td></td></tr>
<tr><td>Mathematics Methods (MM)</td><td>Low
Moderate
High</td><td></td><td></td></tr>
</table>

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Specialist Mathematics (SM)	Low **Moderate** High		

- **Learning outcomes alignment**: the level of alignment between both DP mathematics subjects, at both SL and HL, and the VCE units' learning outcomes is varied. For Foundation Mathematics, General Mathematics, and Further Mathematics the alignment was low, increasing to moderate for Mathematical Methods and Specialist Mathematics. No courses are considered to have high alignment with the DP's learning outcomes.

- **Content alignment**: the level of content alignment between DP mathematics subjects and VCE mathematics subjects varied. Greater overlap is found with courses offering Units 3 & 4 than with those offering Units 1 & 2 only, namely Foundation Mathematics and General Mathematics. Further Mathematics, though offering Units 3 & 4, also has limited alignment with DP subjects, though slightly more with AI than AA, due to its applied nature. Mathematical Methods has strong alignment with DP SL content, especially AA, though limited overlap with DP HL, which exceeds Mathematical Methods content in breadth and depth. Finally, Specialist Mathematics has significant overlap with both HL subjects, also including more additional content than the DP subjects.

- **Demand alignment**: with the exception of Specialist Mathematics, all VCE subjects are surpassed by the demand of DP SL mathematics. Foundation Mathematics and General Mathematics score considerably lower in all categories, whilst Further Mathematics and Mathematical Methods both score lower for volume of work and cognitive skills but are generally more comparable to SL. DP HL considerably surpasses in demand all subjects except Specialist Mathematics, which is more comparable due to scoring the same, or similar, in all categories except volume of work.

The **key similarities** identified were the following:

- **Similarities in learning outcomes:** there are some overlaps in the learning outcomes of DP and VCE mathematics subjects. Out of the eight learning outcome themes extracted from the DP mathematics subject group, five are present in Foundation Mathematics and six are present for all other VCE subjects. The strongest similarities identified are themes of understanding mathematical concepts, applying critical thinking skills in analysis and problem-solving, using investigative skills, and the emphasis on the use of technology. Specialist Mathematics has the strongest similarity with DP outcomes, as it includes more focus on reasoning and making generalisations, closely followed by Mathematical Methods.

- **Similarities in content:** the level of similarity with DP content varies significantly. The courses most closely aligned to DP subjects are Mathematical Methods and Specialist Mathematics. Mathematical Methods has strong similarities with DP SL mathematics content, particularly AA. Mathematical Methods covers a similar range of topics and most of its content is present in the DP. Therefore, the breadth and depth of Mathematical Methods is comparable to DP SL. Specialist Mathematics has strong similarities with DP HL mathematics content, as it covers a considerable amount of

AHL sub-topics found in both AA and AI. Therefore, Specialist Mathematics is comparable to DP HL for content depth.

- **Similarities in demand**: Mathematical Methods is strongly aligned with DP SL mathematics subjects with regard to demand – scoring similarly in most categories, especially for depth of knowledge. Further Mathematics is less aligned with DP SL mathematics subjects but has some similarities – scoring similarly in most categories, especially for outstanding areas of demand. Specialist Mathematics is the only subject with strong alignment with the demand of DP HL mathematics subjects and scores very similarly, especially in the categories of depth of knowledge and outstanding areas of demand.

The **key differences** identified were the following:

- **Differences in learning outcomes**: in contrast to the DP mathematics curricula, VCE sets out specific outcomes for all its units, as well as having a small number of 'aims' which are applicable to all. Not all extracted DP themes are present, as VCE mathematics outcomes do not focus on wider contexts of mathematics or transferable learning skills. Furthermore, DP themes which are present are often not emphasised or described in similar ways to the DP. Generally lacking in comparable emphasis are: reasoning, using generalisation, making links, and communicating in various forms. Though references to these skills are made in the VCE learning outcomes, they are often very specific in nature, rather than being presented as general expectations to be continually developed.

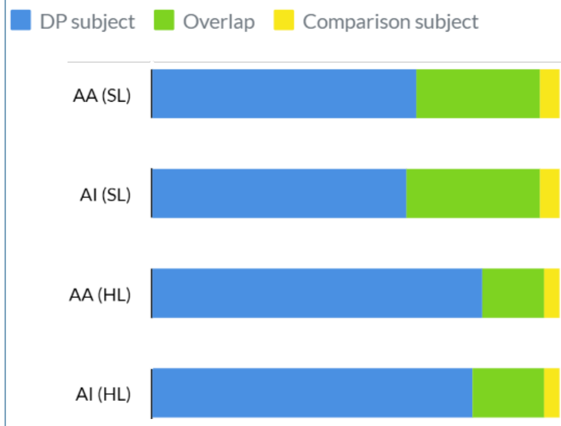
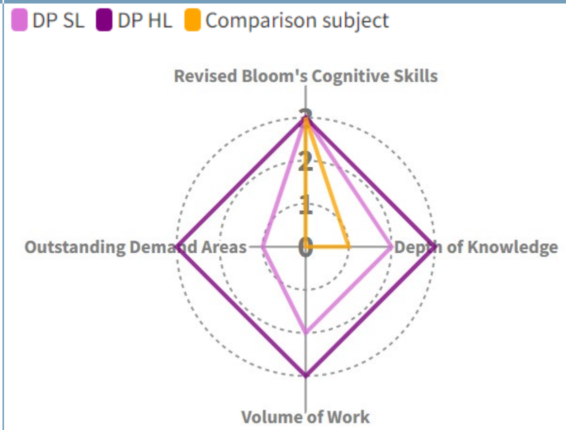
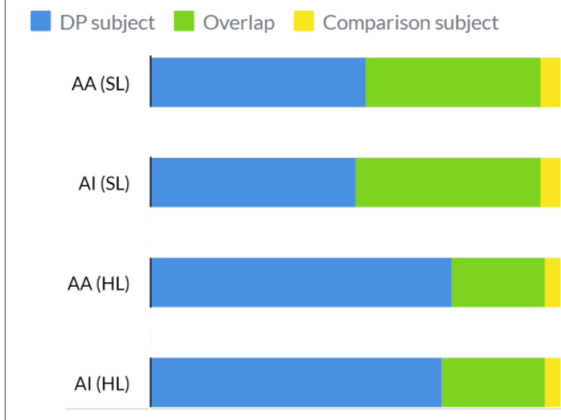
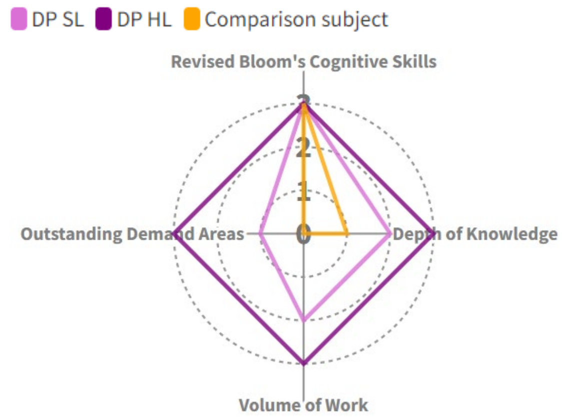
- **Differences in content:** There are some structural differences between DP and VCE mathematics subjects. Most significantly, General Mathematics and Specialist Mathematics (Units 1 & 2) have optional content and thus have an element of flexibility not present in DP mathematics. Furthermore, whilst both DP and VCE curricula cater for more advanced mathematical study, VCE also offers a more basic mathematics subject in the form of Foundation Mathematics. Moreover, VCE mathematics also offers a more strongly applied-focused subject in the form of Further Mathematics, which focuses largely on statistics and finance. In terms of content alignment, Foundation Mathematics has no alignment with DP mathematics content as it generally covers less-demanding concepts. General Mathematics and Further Mathematics contain a small number of sub-topics from the DP curricula, though they generally have limited or no alignment with the main topics in the DP. Finally, Specialist Mathematics exceeds the content size of HL subjects, as it includes topics from both AA and AI, as well as additional topics such as mechanics and logic.

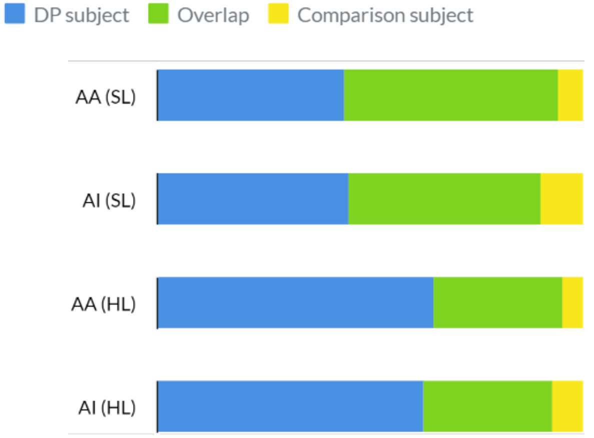
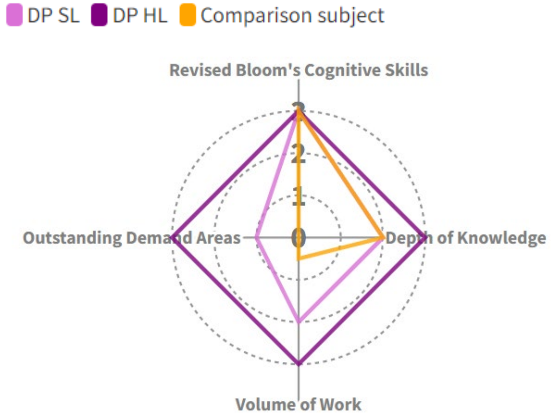
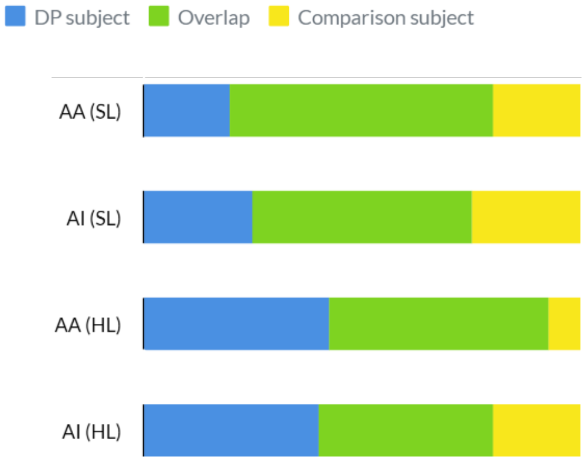
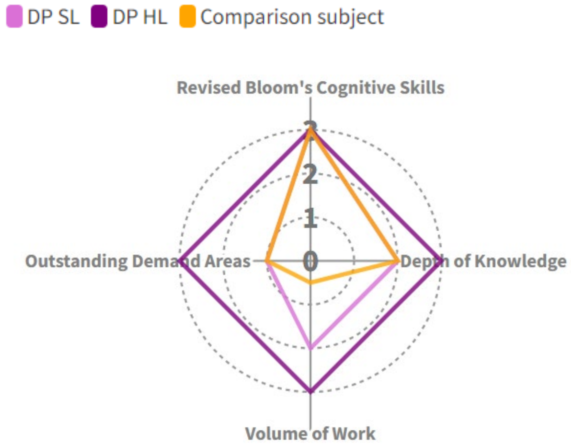
- **Differences in demand:** all VCE subjects do not score as highly as the DP for presence of higher order cognitive skills. Indeed, Foundation Mathematics scores significantly less than DP SL and HL subjects in all demand categories. Furthermore, with the exception of Specialist Mathematics, no other subject in VCE mathematics has similarity in demand to DP HL, particularly in the categories of volume of work and outstanding areas of demand.

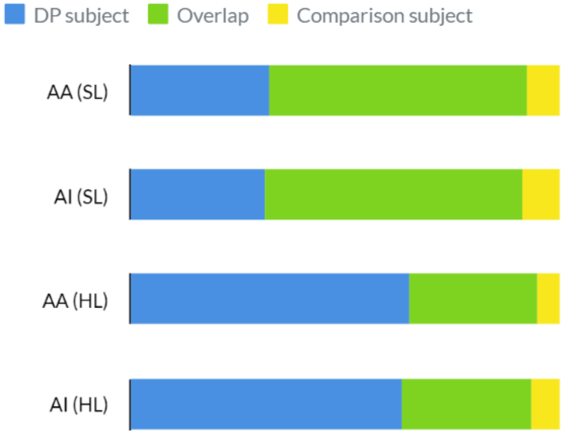
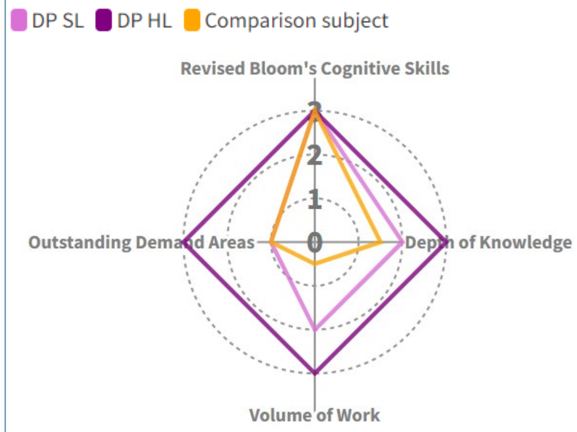
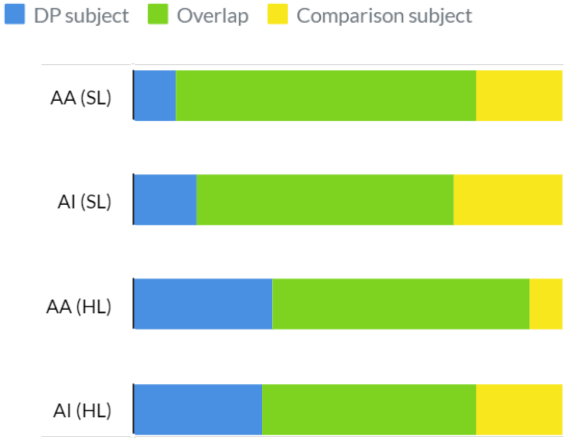
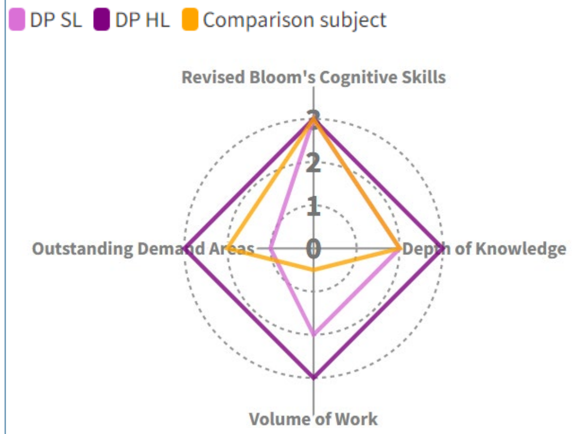
5.2 Canada (Ontario)

The subject level alignment between the DP and OSSD mathematics subjects is represented below:

Figure 32: Visual representations of subject-level alignment (mathematics subjects)

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Grade 11 Functions and Applications (F & A)	Low Moderate **High**		
Grade 11 Functions	Low Moderate **High**		

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Grade 12 Advanced Functions (AF)	Low Moderate **High**		
Grade 12 Calculus and Vectors (CV)	Low Moderate **High**		

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Grade 12 Mathematics of Data Management (DM)	Low Moderate **High**		
University Preparation (UP)	Low Moderate **High**		

- **Learning outcomes alignment**: the level of alignment between both DP mathematics subjects, at both SL and HL, and the OSSD courses' learning outcomes is significant, as all DP themes are present in the OSSD curricula.

- **Content alignment**: the level of content alignment between DP mathematics subjects and individual OSSD courses varies. For grade 11, alignment is generally low, as courses focus primarily on functions rather than a breadth of topics – it is not until grade 12, when further mathematical areas are offered for study, that more substantial alignment can be observed. Generally, there is considerable overlap with SL mathematics subjects, but limited overlap with HL subjects, except for where 'Calculus and Vectors' is studied or where a student takes all UP \mathematics courses combined. Typically, and especially at HL, the DP contains more content than the OSSD comparison points.

- **Demand alignment**: DP mathematics courses, both at SL and HL, usually surpass the OSSD courses in demand level, especially those from grade 11. However, both 'Calculus and Vectors' and 'Data Management' score very similarly to DP SL for cognitive skills, depth of knowledge, and outstanding demand areas. When all UP courses are combined, they score similarly to DP SL for cognitive skills and depth of knowledge but score higher for outstanding areas of demand. For all OSSD courses, the main differences are with regard to volume of work, with the DP scoring significantly higher in this category.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: All eight themes extracted from the DP are identifiable in the OSSD curricula, demonstrating considerable alignment. Not only are many of the same themes present, but they are also emphasised and described in very similar ways to the DP. The learning outcomes at grade 9, in particular, show higher similarity with the DP's due to additional expectations which describe in more detail transferable learning skills and wider contexts of mathematics – these additions have been introduced in a recent curriculum update and will likely be incorporated into other grades as they go through similar reviews.

- **Similarities in content**: each mathematics course available in the UP-strand correlates to one or multiple main topic areas studied in DP mathematics subjects. Generally, OSSD content is most similar to DP SL, though some AHL sub-topics are also covered to the same depth. Specifically for AA, there is strong alignment with the topics of 'Functions' and 'Geometry and trigonometry', as the UP courses, in combination, cover vectors in comparable detail and include a broad range of functions. As for AI, there is strong alignment in the topic of 'Functions' due to a similar focus on modelling within grade 11 Functions and grade 12 Advanced Functions. Content from the other DP topics of 'Number and algebra', 'Statistics and probability', and 'Calculus' can also be identified within UP courses, though the content covered is usually aligned with SL rather than HL. The similarity in content with DP curricula is strongest when all UP courses are taken in combination, as only in this instance will each of the five main DP topics be studied. After this, the pathway with the strongest similarity is that of taking 'Calculus and Vectors', and its pre-requisites, as this enables study of four out of the five main DP content areas, as well as the study of the AHL topic of vectors.

- **Similarities in demand**: DP and OSSD both have the highest score for the presence of higher-order cognitive skills and therefore have similar emphasis on analysis, evaluation, and creation in their learning outcomes. For the rest of the scores, OSSD has higher similarity with the demand level of the DP SL than the DP HL, with 'Calculus and Vectors' being the most closely aligned with SL, closely followed by other grade 12 courses.

The **key differences** identified were the following:

- **Differences in learning outcomes:** though there is strong overlap in learning outcomes, the OSSD features slightly more emphasis on making connections within mathematics and creating a variety of representations. More notable differences occur in the new grade 9 curriculum with the introduction of 'Social-Emotional Learning Skills'. Although these overlap with some of the DP learning outcomes, they go further to encourage meta-cognition and self-regulation skills in mathematical learning. In contrast, the DP emphasises these skills primarily in a non-subject specific context – for example, through the IB learner profile.

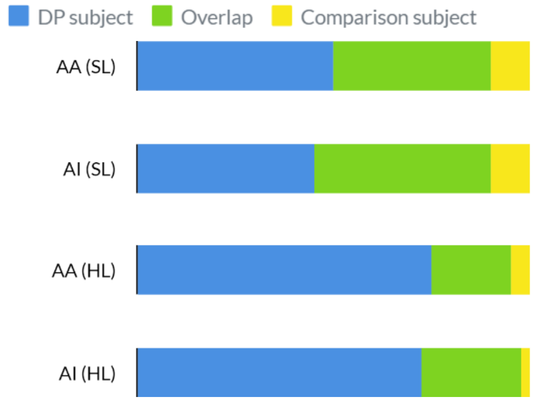
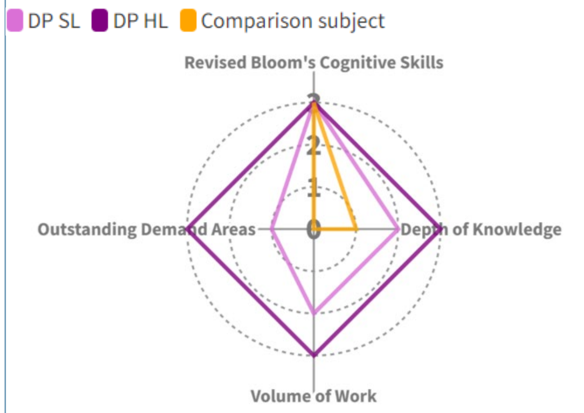
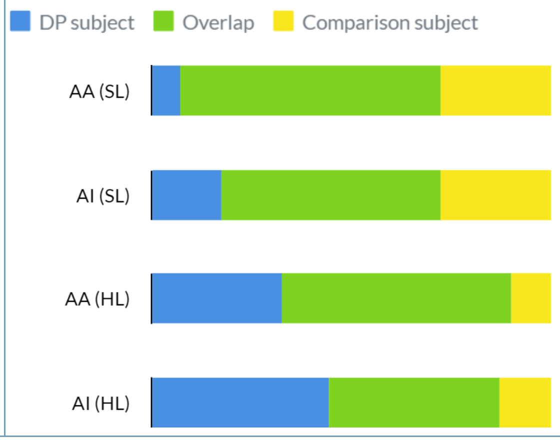
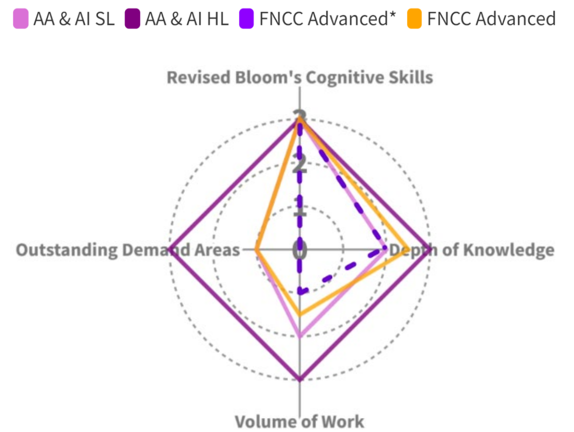
- **Differences in content:** there are various differences in mathematics content structure and alignment between the DP and OSSD mathematics. Notably, individual UP courses focus on a narrow, rather than broad, range of main topics, while each DP course includes a wider range of different mathematical areas. Therefore, the breadth of topics covered in the OSSD largely depends on the number of courses taken, whereas all DP subjects offer breadth via five main topics to be studied in each. To cover a similar breadth of main topic areas as the DP, all grade 12 OSSD courses would need to be taken, though it is unclear from the documentation how common this is. Furthermore, in some topic areas, the OSSD does not include more typically advanced concepts. Instead, the OSSD content appears to focus on the 'why' and on making connections and comparisons to develop deeper conceptual understanding. For these reasons, OSSD content is largely different to DP HL mathematics, especially with regard to 'Number and algebra', 'Calculus', and 'Statistics and probability'. Overall, the OSSD curriculum has less instances of depth in topics than DP HL, and breadth of content is more variable than it is in DP subjects.

- **Differences in demand:** the most significant difference in demand between the OSSD and DP relates to volume of work. DP mathematics covers a greater number of different topics and complex concepts within the allocated teaching hours, whereas the OSSD allocates a generous amount of teaching hours to spend on one or two main topic areas. The OSSD courses typically score lower for outstanding areas of demand, even where all UP courses are combined. Overall, OSSD UP as a pathway scores lower than DP HL subjects for demand.

5.3 Finland

The subject level alignment between the DP and FNCC mathematics subjects is represented below:

Figure 33: Visual representations of subject-level alignment (mathematics subjects)

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Basic syllabus	Low Moderate **High**		
Advanced syllabus	Low Moderate **High**		

NB: For demand, the FNCC Advanced* refers to compulsory modules only, while FNCC Advanced represents the full FNCC advanced syllabus, including both compulsory modules and the additional optional modules combined.

- **Learning outcomes alignment**: the level of alignment between both DP mathematics subjects, at both SL and HL, and the FNCC syllabi learning outcomes is significant, as all DP themes are present in the FNCC.

- **Content alignment**: though the FNCC's basic syllabus has some overlapping content with DP SL, the latter surpasses it in breadth and depth. The FNCC's advanced syllabus is slightly more aligned with AA than AI, for both SL and HL. There is a reasonable amount of alignment with DP AA HL; however, based on the documented content (which may be further developed by each provider), the advanced syllabus has less breadth and depth than DP. Notably, students in the FNCC are not required to take optional modules and in practice may study a somewhat smaller volume of content. Without the optional modules, the breadth and depth of the advanced syllabus is more comparable to DP SL.

- **Demand alignment**: the FNCC basic syllabus is the least aligned with DP subjects in terms of demand, scoring less than DP SL for all categories except Bloom's cognitive skills. The advanced syllabus is more similar in demand to DP SL than DP HL, becoming more strongly aligned when all modules are studied, both compulsory and optional. All FNCC courses scored lower than the DP subjects for volume of work – though the lack of detail in FNCC documentation, with regard to depth of content, is a factor here.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: both the DP and FNCC set out general learning outcomes that are applicable to all mathematics courses within their respective programmes. All eight themes extracted from the DP are strongly present in the FNCC, hence the mathematics learning outcome themes of the DP and FNCC are highly aligned. Both detail a wide range of skills to be developed, which altogether demonstrate a holistic approach. Indeed, it is important to both the DP and FNCC that students not only learn mathematical skills, but also cultivate a genuine interest in exploring mathematics and learn transferable skills such as collaboration, independence, confidence, and perseverance. In addition to this, understanding and application, critical and creative thinking skills, making connections, communication, and technology are all present in the DP and FNCC. Furthermore, it is important in both curricula that students will explore the wider contexts of mathematics through appreciating historical and cultural perspectives, the universality of mathematics, and reflecting on how to solve local and global challenges that face the environment and humankind. Furthermore, the FNCC has a similarly strong emphasis on inquiry-based approaches, as they aim to develop students' confidence in taking investigative and experimental actions.

- **Similarities in content**: from the FNCC, the basic syllabus content has partial alignment with DP SL, covering some similar topics, though (based on the documented content covered – which may be further developed by schools) the FNCC basic syllabus content generally lacks the breadth and depth to be comparable with the IB subjects. The advanced syllabus has more shared content with the DP curriculum. The compulsory modules alone have high alignment with DP SL content, covering a similar breadth and depth of topics. When the optional modules are included, the content of the advanced syllabus slightly surpasses DP SL in breadth and depth, though not enough to be comparable to DP HL. It can also be noted that alignment is slightly stronger with AA than AI.

- **Similarities in demand**: DP and FNCC both have the highest score for the presence of higher-order cognitive skills (taken from Bloom's Revised Taxonomy) and therefore have similar emphasis on analysis, evaluation, and creation in their learning outcomes. For the rest of the scores, the FNCC has higher similarity with the demand level of the DP SL than the DP HL, with the advanced syllabus scoring the same, or similar, for depth, volume, and outstanding areas as DP SL. Similarity with the DP SL is stronger when all compulsory and optional modules are studied from the advanced syllabus.

The **key differences** identified were the following:

- **Differences in learning outcomes:** the learning outcome themes of the DP and FNCC for mathematics are generally very similar. However, it can be noted that making connections within mathematics and modelling skills have stronger emphasis in the FNCC.

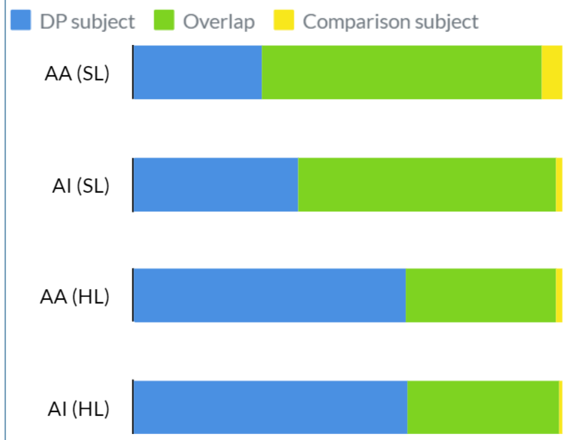
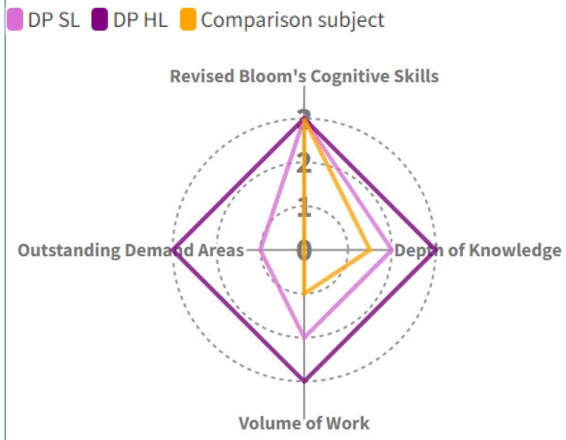
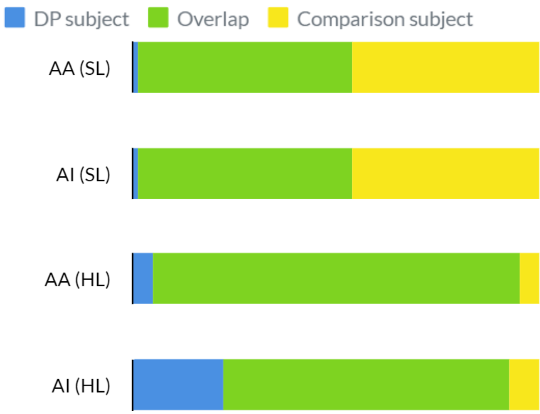
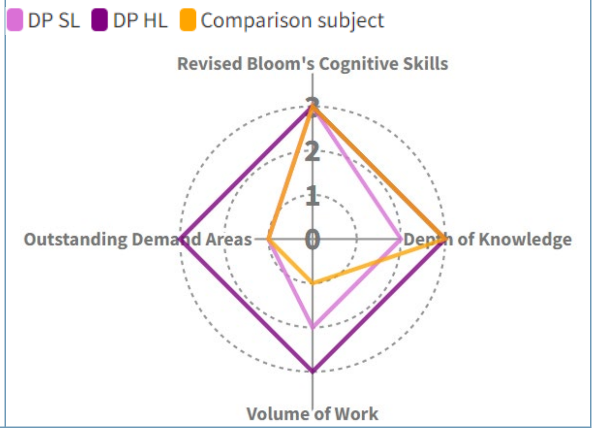
- **Differences in content:** there are key differences in the structure of the content between the DP and FNCC. Despite both offering two levels, the FNCC basic syllabus is not studied as part of advanced syllabus in the way that SL is studied as part of HL in the DP. Furthermore, the FNCC syllabi are structured into modules, some of which are optional and are not required to be studied, unlike the DP, which comprises compulsory content only. For content alignment, the basic syllabus has no alignment with AHL content and lacks the breadth and depth to be comparable to either SL or HL. For the advanced syllabus, there is only some alignment with AHL content, even when optional modules are included. Generally, the advanced syllabus has less breadth and depth than DP HL subjects according to the information available in the curriculum documentation.

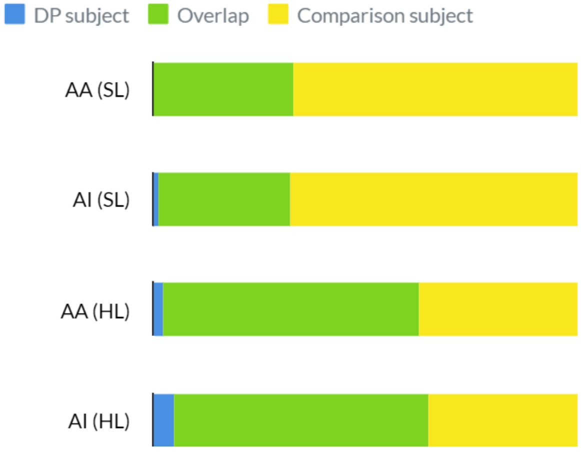
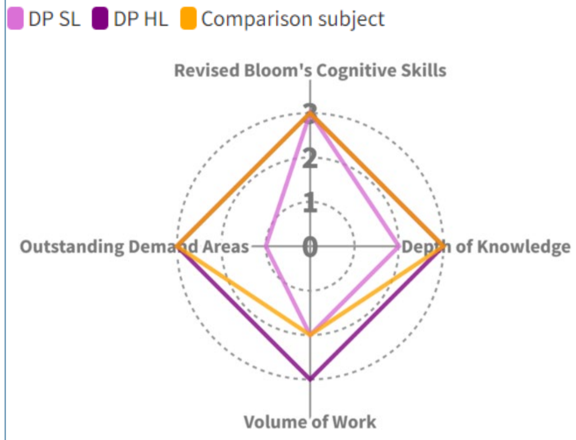
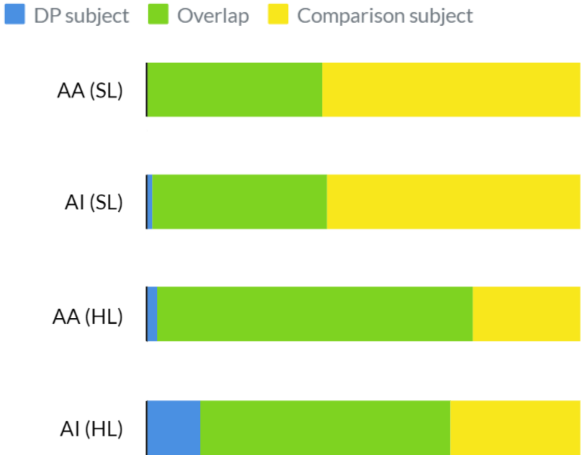
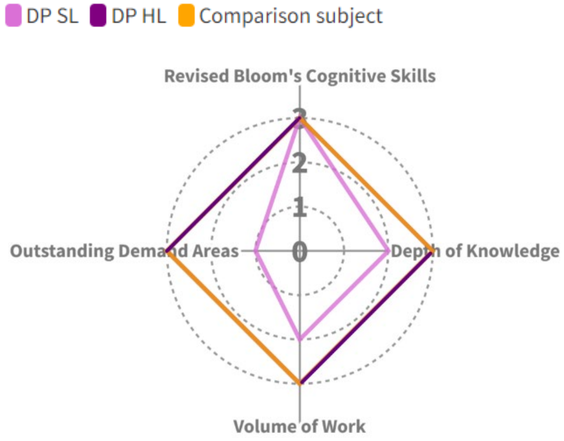
- **Differences in demand:** aside from the score for Bloom's cognitive skills, the basic syllabus scores less than the DP SL in all categories. Furthermore, the advanced syllabus scores less than the DP HL in all other categories, especially for outstanding demand areas and volume of work, thus no course in the FNCC is highly aligned with the demand scores of the DP HL. Though, the capacity for schools to further develop content beyond the documented curriculum is a relevant contextual factor with respect to these judgements.

5.4 Singapore

The subject level alignment between the DP and SGA mathematics subjects is represented below:

Figure 34: Visual representations of subject-level alignment (mathematics subjects)

<table>
<tr><th>Subject name</th><th>Learning outcomes alignment</th><th>Content alignment</th><th>Demand alignment</th></tr>
<tr><td>H1 mathematics</td><td>Low
Moderate
High</td><td>
</td><td>
</td></tr>
<tr><td>H2 mathematics</td><td>Low
Moderate
High</td><td>
</td><td>
</td></tr>
</table>

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
H2F mathematics	Low Moderate **High**		
H3 mathematics	Low Moderate **High**		

- **Learning outcomes alignment**: the level of alignment between both DP mathematics subjects, at both SL and HL, and the SGA courses' learning outcomes is significant, as nearly all DP themes are present in the SGA curricula.

- **Content alignment**: the level of content alignment between DP mathematics subjects and SGA courses varies. For H1, there is reasonable alignment with DP SL, though the former has less content than the latter, and significantly less than DP HL. There is strong alignment between H2 mathematics and AA HL, as the majority of content is overlapping. There is slightly less, but still considerable, alignment between H2 and AI HL. When considering H2F in addition to H2, these have a greater breadth and depth of content studied compared to DP HL subjects. Similarly, when H3 is offered with H2, the content of DP HL is exceeded – though through depth and practice of rigour, rather than breadth. Overall, the content within H1 and H2 is very similar to the DP curriculum, with H2 being of comparable size to DP HL, whereas the content of H2F and H3 goes beyond the scope of what is offered in the DP.

- **Demand alignment**: SGA H1 is surpassed in most demand categories by both DP SL and HL. SGA H2 scores similarly to DP HL for depth of knowledge, but its lower scores for outstanding areas and volume mean that it is generally more similar to the demand of DP SL. H2F and H3 demand scores are very comparable to DP HL subjects, scoring highly for cognitive skills, depth, volume, and outstanding areas of demand.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: seven out of the eight themes extracted from the DP are strongly present in the SGA curricula, hence there is high alignment in the mathematics learning outcome themes between the DP and SGA. Both detail a wide range of skills to be developed, which altogether demonstrate a holistic approach. Indeed, it is important to both the DP and SGA that students not only learn mathematical skills, but also cultivate a genuine interest and appreciation of the beauty of mathematics, whilst also learning transferable skills, such as reflective working, collaboration, independence, confidence, and perseverance. In addition to this, themes of understanding and application, problem-solving, reasoning, making links, communication, and technology are all present in the DP and SGA. Furthermore, it is an aim in both curricula that students will be able to use the skills and understanding learnt in their mathematics courses to engage with local and global events and issues, thus preparing them for society.

- **Similarities in content**: the key similarity in the structure of content between the DP and SGA is the distinction of different levels which students can choose from – SL/HL for DP and H1, H2, H2F, H3 for SGA. Generally, H1 content is similar to DP SL subjects, covering similar concepts in the areas of functions, statistics and probability, and calculus. However, H1 has somewhat less breadth and depth than the DP SL, and therefore they can be considered moderately, rather than highly, aligned. Like AI, H1 has a higher focus on applied mathematics, with the largest content area being 'Statistics and Probability'. H2 is very similar to DP HL subjects, with slightly more alignment with AA than AI content, especially for AHL topics. A high number of SL and

AHL topics is present in H2, which has comparable breadth and depth to DP HL subjects.

- **Similarities in demand**: DP and SGA both have the highest score for the presence of higher-order cognitive skills (taken from Bloom's Revised Taxonomy) and therefore place similar emphasis on analysis, evaluation, and creation in their learning outcomes. For the rest of the categories, H2F and H3 score the same, or similarly, to DP HL subjects, hence these subjects are highly aligned in terms of demand. H2 scores the same as DP HL subjects for depth but is closer to SL for volume and outstanding demand areas.

The **key differences** identified were the following:

- **Differences in learning outcomes:** though the SGA details outcomes that are applicable to all courses through its 21CC and MCF, unlike the DP it also outlines course-specific outcomes via 'syllabus aims'. Thus, though all courses present themes similar to the DP, the syllabus aims of H2F and H3 describe some of the more advanced skills within these themes, which are not explicit in the DP. Most significantly, H3 describes 'advanced problem-solving, mathematical rigour, and a focus on writing and evaluating proofs'. Furthermore, a few different themes and skills to those in the DP emerge from the MCF. For instance, the SGA goes further to encourage the monitoring of thinking processes and self-regulating learning, as it includes 'metacognition' as one of its five components. Moreover, the MCF is generally more prescriptive and lists a higher number of specific skills, including heuristics and spatial visualization. The prescriptive nature of the MCF means that some skills, such as modelling, have a higher emphasis in the outcomes of the SGA than in the DP. Finally, despite there being a strong presence of DP themes in the SGA, the theme of using inquiry-based approaches is not explicitly present in the outcomes of the SGA, though some of the same skills necessary for this approach are mentioned in the MCF and inquiry-based learning is present in the 'Teaching Processes'.

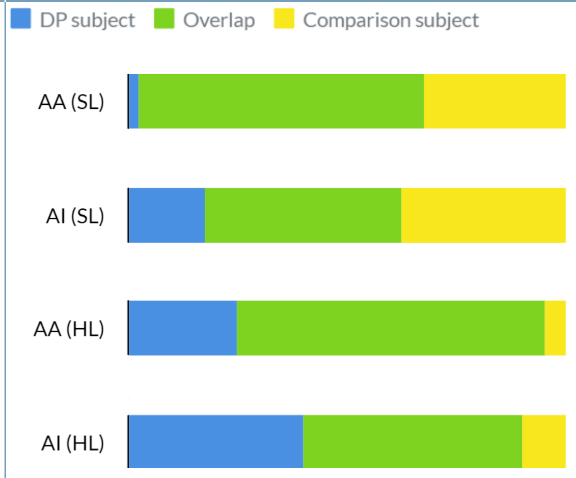
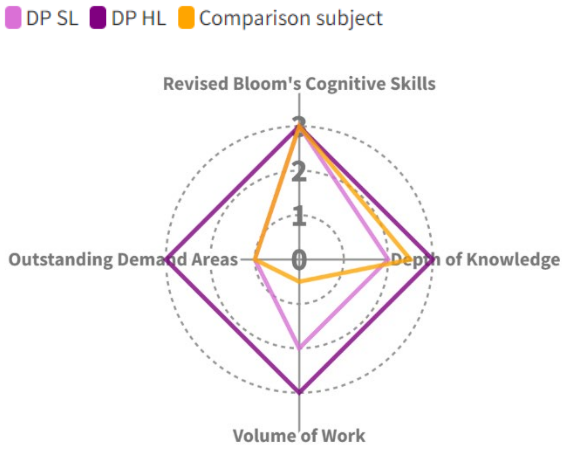
- **Differences in content:** there are several differences in the structure of mathematics content, as the SGA offers more opportunities for advanced mathematical study through H2F and H3. Also, whereas HL content builds on SL, H2 builds on O Level Additional Mathematics, not H1. With regard to main areas covered, a significant difference is that topics related to geometry and trigonometry, excluding vectors, are covered in learning prior to A Level, the depth of which depends on whether Additional Mathematics is taken. For content alignment, though the first two levels, H1 and especially H2, have similarities with the DP courses, there are larger differences with regard to H2F and H3. Despite some H2F topics being identifiable in the DP HL syllabi, the former goes into more depth in these topics and includes more complex concepts. In addition to this, H2F includes many topics which are not in the DP, and generally covers a wider range of advanced mathematical topics than any single DP HL subject; thus, SGA students taking this course will experience greater depth and breadth than DP students studying HL. Another key difference is the content of H3, which is tailored for students who specifically intend to study mathematics at university and focuses on mathematical results, rigour, and proof to a far greater extent than DP subjects.

- **Differences in demand:** for demand, the least aligned subject with the DP is H1, which scores less than DP SL in all categories except Bloom's cognitive skills. Furthermore, for both H1 and H2, the most significantly different demand category is volume of work, with this being judged as a standard (rather than heavy) amount for these courses. Moreover, though this judgement is based on a two-year duration, it can be noted that students taking these subjects in the millennia institute will have an extra year for study.

5.5 South Korea

The subject level alignment between the DP mathematics subjects and the KHSCG mathematics pathway is represented below:

Figure 35: Visual representations of subject-level alignment (mathematics subjects)

Pathway name	Learning outcomes alignment	Content alignment	Demand alignment
KHSCG mathematics pathway	Low Moderate **High**		

- **Learning outcomes alignment**: the level of alignment between both DP mathematics subjects, at both SL and HL, and the KHSCG mathematics pathway's learning outcomes is significant, as all DP themes are present in the KHSCG curricula.

- **Content alignment**: for the KHSCG mathematics pathway, alignment with AA is generally stronger than with AI, for both SL and HL. Nearly all AA SL content, and a good proportion of AI SL content, can be found in KHSCG, though the pathway exceeds the breadth and depth of DP SL by featuring a significant amount of additional content. Most of this content can, however, be identified in DP HL, resulting in considerable alignment with AA HL and reasonable alignment with AI HL. Generally, the DP HL subjects have a slightly higher volume of content than this pathway. However, it can be noted that the amount of overlapping content is dependent on the KHSCG electives chosen – in practice, students have a great degree of flexibility regarding the breadth and depth of their mathematics study.

- **Demand alignment**: the KHSCG mathematics pathway is more similar in demand to DP SL than DP HL. Though it scores similarly to DP HL for Bloom's cognitive skills and depth of knowledge, the pathway scores the same as SL for outstanding areas of demand and significantly less than both DP subjects for volume of work.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: All eight themes extracted from the DP learning outcomes are identifiable in the KHSCG curricula, hence there is considerable alignment between the mathematics learning outcome themes of the DP and KHSCG. Five out of eight DP learning outcomes are strongly present in the KHSCG, especially with regard to critical and creative thinking skills. Both also consider the attitudes and learning skills of students and aim to develop independence, confidence, and interest. Other skills of communication, making connections, and understanding and application have similar emphasis in the KHSCG. Other DP themes such as wider contexts, technology, and inquiry-approaches are present, though with less emphasis.

- **Similarities in content**: when considering the pathway content, a large amount of the DP SL topics can be identified, as well as a considerable number of AHL topics. Therefore, this pathway exceeds the DP SL in breadth and depth of content and is more comparable to that of DP HL subjects. Furthermore, outside of this pathway, the KHSCG curriculum offers the Mathematical Inquiry Task elective, which has similarities to the mathematical exploration in the DP syllabus.

- **Similarities in demand**: DP and KHSCG both have the highest score for the presence of higher-order cognitive skills (taken from Bloom's Revised Taxonomy) and therefore have similar emphasis on analysis, evaluation, and creation in their learning outcomes. For the rest of the scores, the KHSCG mathematics pathway has higher similarity with the demand level of the DP SL than the DP HL, scoring the same for outstanding areas of demand and slightly more for depth of knowledge.

The **key differences** identified were the following:

- **Differences in learning outcomes:** though there are considerable similarities in learning outcome themes, the KHSCG has significantly less emphasis on the wider contexts of mathematics than the DP, only referring to enabling students to participate in society. Furthermore, inquiry-based approaches, including using technology to explore, has less emphasis in the KHSCG, with no explicit reference to investigative or experimental actions except when in the context of information-processing (e.g. collecting data). Conversely, the KHSCG has some specific concepts in their learning outcomes which, though similar to the DP, are not explicitly mentioned, such as convergent thinking and information-processing.

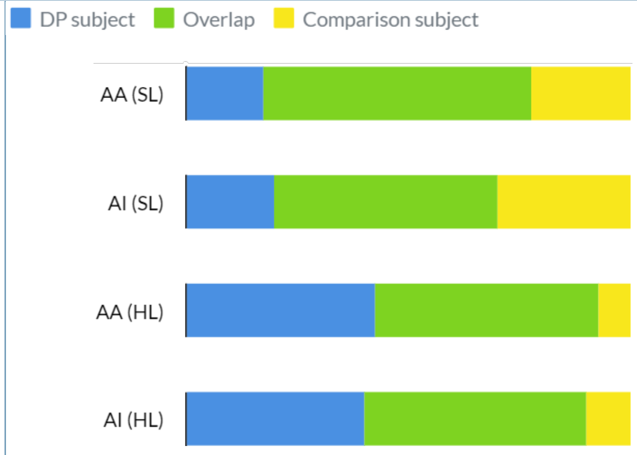
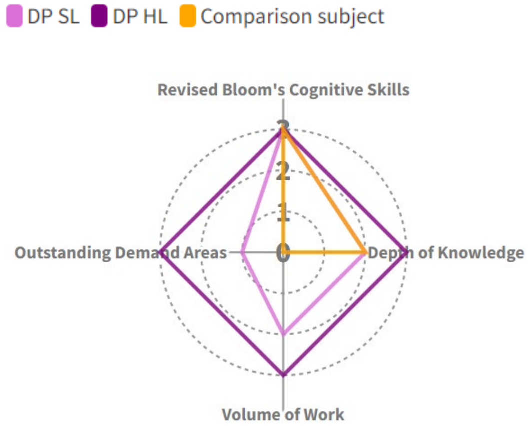
- **Differences in content:** the structure of mathematical content varies significantly between the DP and KHSCG. Whereas the DP integrates five main mathematical areas into all its courses, the KHSCG separates large content areas into stand-alone electives. Therefore, the breadth of study could vary significantly between KHSCG students, depending on the electives selected. Generally, there is a higher degree of flexibility in the KHSCG curriculum, which offers a wider range of options and allows students to study few or several electives. Hence, there is only significant alignment with DP content when students take a pathway that will lead to a similar breadth and depth of content to be covered. Furthermore, the pathway more strongly aligns with AA content than AI content.

- **Differences in demand:** the most significant difference in demand between the KHSCG and DP relates to volume of work. DP mathematics covers a greater number of different topics and complex concepts within the allocated teaching hours, whereas the KHSCG allocates a generous amount of teaching hours to cover a similar breadth and depth of content in the selected pathway. Furthermore, the pathway scores significantly lower than DP HL subjects for outstanding demand areas, thus the pathway has more similarity with SL than HL for demand.

5.6 United States (CCSS)

The subject level alignment between the DP mathematics subjects and the CCSSM is represented below:

Figure 36: Visual representations of subject-level alignment (mathematics subjects)

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
CCSSM	Low **Moderate** High		

- **Learning outcomes alignment**: the level of alignment between both DP mathematics subjects, at both SL and HL, and the CCSSM learning outcomes is moderate. The DP and CCSSM share similar learning outcome themes involving critical thinking skills, use of technology, understanding and application, forming links and generalisations, communication skills, and learning skills. However, DP themes involving wider contexts and inquiry-based approaches are not evident in the CCSSM.

- **Content alignment**: the level of content alignment between DP mathematics subjects and the CCSSM is varied. The AA SL and CCSSM have the most content overlap, though the CCSSM do not share any calculus content, contain different geometry material and feature only a few AHL sub-topics. Moreover, the CCSSM do not contain enough AHL content to have significant overlap with HL overall. In summary, the CCSSM have slightly more depth in some topics than the DP SL subjects, but less breadth; and have both less breadth and depth than HL subjects.

- **Demand alignment**: all DP mathematics courses, both at SL and HL, considerably surpass the CCSSM in demand in terms of volume of work and stretch areas.

The key similarities identified were the following:

- **Similarities in learning outcomes**: like the DP, the CCSSM lay out general expectations which are applicable to all mathematics study. Six out of the eight themes extracted from the DP mathematics subject group are present in the CCSSM, hence there is considerable overlap between the DP and the CCSS in terms of learning outcomes. Overlapping themes – including problem-solving, reasoning, reflecting on and critiquing work, use of technology, and accuracy in the communication of mathematics – are generally emphasised and described in similar ways.

- **Similarities in content**: the content in the CCSSM is structured in a similar way to the DP, with the standards being organised into main topics involving functions, geometry, algebra, number, and statistics and probability. Furthermore, for all topics except calculus, much of the content in DP SL mathematics subjects is present in the CCSSM, especially for AA. With regard to HL content, there are strong similarities to the DP topic ‘Number and algebra’, due to the presence of complex numbers and matrices in the CCSSM. There is some limited similarity between the CCSSM and the HL content of ‘Functions’ (for both AA and AI) and ‘Geometry and trigonometry’ (AI only).

- **Similarities in demand**: both the DP and CCSSM score highly for the presence of higher order cognitive skills described in Bloom’s Revised Taxonomy, by having a strong presence of evaluation and creation in their learning outcomes. Furthermore, the CCSSM score the same as the DP SL subjects for depth of knowledge, as content in many topics appears to be covered in considerable detail and depth.

The **key differences** identified were the following:

- **Differences in learning outcomes**: though the CCSSM learning outcomes have similarities with the DP mathematics subject group, the themes of inquiry-based approaches and wider contexts are not present. Furthermore, some of the detail in the DP's learning outcomes is missing – for example, learning skills such as working collaboratively and having curiosity are not features of the expectations, nor are making links to other disciplines. In addition, the CCSSM list two unique expectations involving searching for patterns in structure and looking for regularity in repeated reasoning. Both of these have similarities to the DP's theme of making links and generalisations, but both are more specific regarding the types of thinking students should engage in. The CCSSM also place more emphasis on modelling within their learning outcomes.

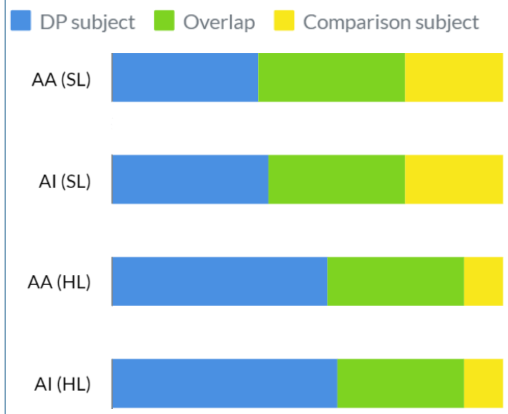
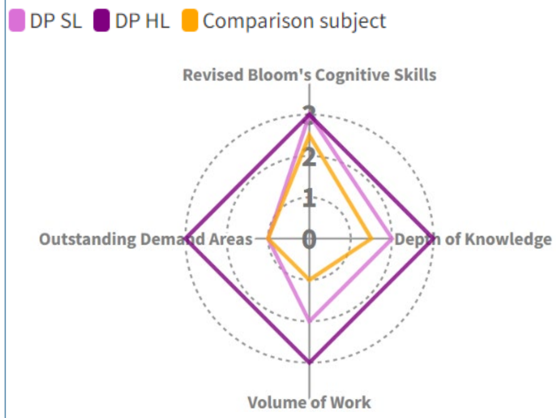
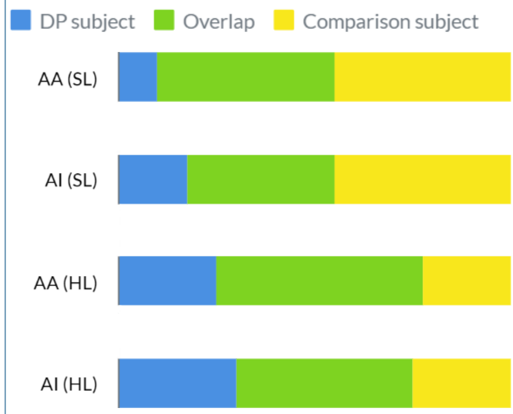
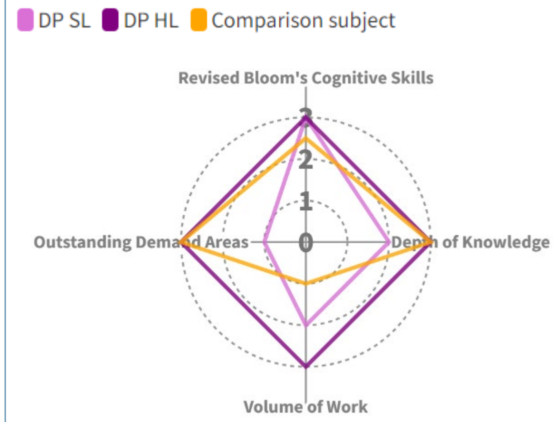
- **Differences in content:** a key difference is that the CCSSM content applies to four years of study, as opposed to two years in the DP. Moreover, unlike the DP, the CCSSM do not constitute a comprehensive curriculum and therefore do not describe all content in the same level of detail, especially for fourth course options. This leads to some strong differences in content. Most notably, the CCSSM do not detail content for the topic of calculus, which is one of the five main topics in DP mathematics subjects. Calculus (and pre-calculus) is usually the focus of fourth course study in the US. Furthermore, since the CCSSM content does not focus on more advanced mathematical study typically covered in fourth courses, DP HL content is largely not present in the CCSSM. Therefore, the CCSSM lack the breadth and depth of DP HL mathematics.

- **Differences in demand**: though scoring similarly to DP mathematics subjects for cognitive skills and depth of knowledge, there are significant differences in other demand categories. The most considerable difference between the DP and CCSSM is volume of work. Where DP subjects have moderate to heavy volume, the CCSSM appear to have a generous amount of time to cover content. However, as the CCSSM do not detail recommended teaching hours, the volume of work score is likely to vary from state-to-state implementation. The CCSSM also score lower for outstanding areas of demand, again because the standards do not comprehensively detail content studied in fourth courses. Overall, the demand level of the CCSSM is considerably lower than that of the DP HL subjects and somewhat lower than SL subjects.

5.7 France

The subject level alignment between the DP and FB mathematics subjects is represented below:

Figure 37: Visual representations of subject-level alignment (mathematics subjects)

<table>
<tr><th>Subject name</th><th>Learning outcomes alignment</th><th>Content alignment</th><th>Demand alignment</th></tr>
<tr><td>FB mathematicsP</td><td>Low
Moderate
High</td><td>
</td><td>
</td></tr>
<tr><td>FB mathematicsT</td><td>Low
Moderate
High</td><td>
</td><td>
</td></tr>
</table>

- **Learning outcomes alignment:** the level of alignment between the learning outcomes of both DP mathematics subjects, both at SL and HL, and those of the FB subjects is moderate, as most, though not all, DP themes are present in the FB curricula.

- **Content alignment:** the level of content alignment between DP mathematics subjects and FB subjects is generally moderate. Indeed, although FB subjects may have similar breadth and depth to the DP subjects, the topics and subtopics covered in each curricula vary, resulting in a moderate amount of shared content. The breadth and depth of FB mathematicsP is aligned with DP SL subjects and FB mathematicsT is aligned with DP HL subjects. FB mathematics overall has slightly more alignment with DP AA content than DP AI.

- **Demand alignment:** FB mathematics is well aligned with the DP SL and HL subjects with regards to demand, scoring similarly in most categories. FB mathematicsP is most aligned with DP SL subjects and FB mathematicsT is most aligned with DP HL subjects. However, a key difference is that FB mathematics scores lower for volume of work than DP subjects, at both SL and HL.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: both the DP and FB set out general learning outcomes that are applicable to all mathematics courses within their programmes. Six of the eight themes extracted from the DP are evidenced in the FB mathematics curricula, hence there is reasonable overlap in mathematics learning outcome themes between the DP and FB. Like the DP, FB promotes critical thinking skills, communication in different contexts, the use of technology in investigation and problem-solving, using generalisation and abstraction, understanding concepts and application, and the use of inquiry approaches.

- **Similarities in content**: similar to SL and HL in the DP, FB mathematics offers two levels of study, mathematicsP and mathematicsT, with the latter building on the content of the former. The level of overall content alignment between DP and FB mathematics subjects is moderate. Indeed, FB mathematicsP has similar breadth and depth to DP SL subjects and includes some SL content from each of the DP's five main topics. As such, FB mathematicsP has partial alignment with SL content across all topics, except the DP AI's Statistics and Probability topic. FB mathematicsP also covers some AHL vectors content that is present in the DP Geometry and Trigonometry topic. FB mathematicsT (including mathematicsP) has similar breadth and depth as DP HL subjects and includes most SL content across all topics as well as some AHL content, particularly from the DP's Geometry and Trigonometry, and Calculus topics.

- **Similarities in demand**: FB mathematicsP scores the same as DP SL subjects for outstanding demand areas and scores similarly, though slightly lower, for Bloom's cognitive skills and depth of knowledge. Therefore, FB mathematicsP aligns reasonably well with the demand of DP SL. Moreover, FB mathematicsT scores the same as DP HL for depth of knowledge and outstanding areas of demand, and scores similarly, though slightly lower, for Bloom's cognitive skills. Thus, FB mathematicsT aligns reasonably well with the demand of DP HL.

The **key differences** identified were the following:

- **Differences in learning outcomes:** though there is overlap in learning outcomes, the FB does not evidence a similar emphasis on DP themes of engaging with mathematics in wider contexts and developing transferable learning skills. There is also no explicit reference to students making links to other subjects. Conversely, the FB curricula places a higher emphasis than the DP on oral communication of mathematics, as well as the use of representations.

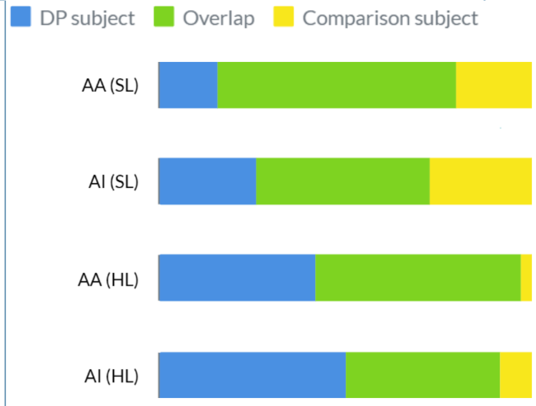
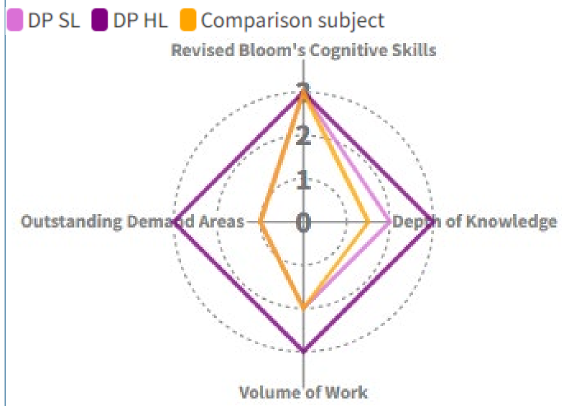
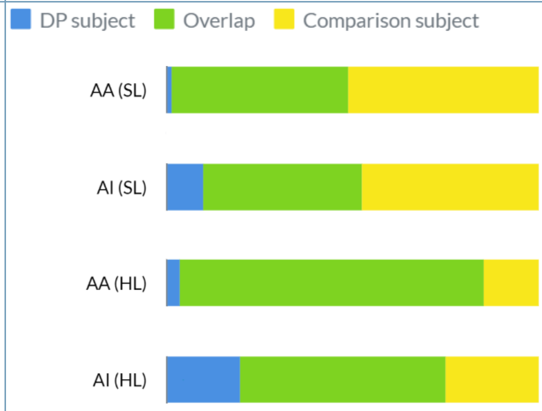
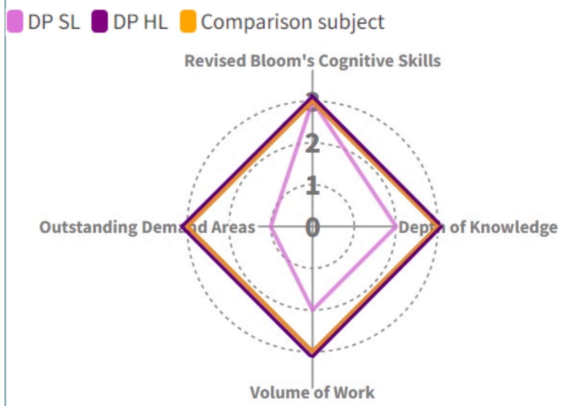
- **Differences in content:** FB mathematicsP only partially aligns with DP SL content, due to often not including some key subtopics. Instead, FB mathematicsP covers a topic named Algorithms and Programming, which consists of content that is not present in the DP, such as the use of Python. FB mathematicsT has moderate alignment with DP HL, due to not having any alignment with AHL content in some topics, and only partial in others. Indeed, for both AA and AI, FB mathematicsP has no alignment with AHL Functions or Statistics and Probability content. In addition, FB mathematicsP has no alignment with DP AI AHL content in Number and Algebra. Again, FB mathematicsT instead covers different content to the DP, such as the Algorithms and Programming topic, limits of sequences, and other vectors and statistics subtopics. Finally, it can be noted that FB subjects are slightly more aligned with DP AA than DP AI.

- **Differences in demand:** the most significant difference in demand between FB and DP mathematics relates to volume of work. DP mathematics covers a similar breadth and depth of content in a short amount of allocated teaching hours, whereas the FB allocates a generous amount of teaching hours to content coverage, increasing the time per week in the second year to cover the more complex content. The difference is most notable when comparing the FB courses to DP HL, which has a heavy volume of work, as opposed to a standard volume observed in the FB courses.

5.8 Spain

The subject level alignment between the DP and SB mathematics subjects is represented below:

Figure 38: Visual representations of subject-level alignment (mathematics subjects)

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Mathematics I	Low Moderate **High**		
Mathematics II	Low Moderate **High**		

- **Learning outcomes alignment**: the level of alignment between the learning outcomes of both DP mathematics subjects, at both SL and HL, and those of the SB mathematics subjects is high, as all DP learning outcome themes are present in the SB curricula.

- **Content alignment**: SB mathematics I is well aligned with DP AA SL content, as they share a considerable number of subtopics and are of similar breadth and depth. SB mathematics I has moderate alignment with DP AI SL, as it shares some, but slightly fewer, subtopics with this subject. SB mathematics II (which requires prior study of mathematics I) is strongly aligned with DP AA HL, as it shares a considerable amount of content and has similar breadth and depth. SB mathematics II has moderate, rather than strong, alignment with DP AI HL, as each subject features several areas which are not covered by the other.

- **Demand alignment**: the demand level of SB mathematics I and SB mathematics II strongly aligns with the demand of DP SL and HL mathematics subjects, respectively. Indeed, SB mathematics I scores the same as, or very similarly to, the DP's SL mathematics subjects for Bloom's cognitive skills, depth of knowledge, volume of work, and outstanding demand areas. Likewise, SB mathematics II scores the same as DP HL for all demand categories.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: similarly to the DP, the SB sets out general learning outcomes that are applicable to both years of mathematics study in the Science and Technology modality. Overall, the level of alignment between the SB courses mathematics' learning outcomes and the DP's is high. All eight themes extracted from the DP are well evidenced in the SB's Specific Competencies and corresponding Evaluation Criteria. Like the DP, the SB promotes critical thinking skills, consideration of global issues and contexts, transferable learning skills, clear communication, use of technology, making connections, and use of inquiry approaches. Thus, the SB takes a similarly holistic approach to the DP with regards to mathematics learning outcomes.

- **Similarities in content**: both the DP and SB's Science and Technology modality offer two levels of study, with the second course following directly on from the first. With regards to content alignment, SB mathematics I covers a significant number of subtopics from the DP's Number and Algebra and Geometry and Trigonometry topic areas, covering most SL and some AHL content. Overall, SB mathematics I has similar breadth and depth as DP SL, though has more depth in some topics and less in others. SB mathematics I has reasonable, though slightly less alignment, with DP AI SL, as the former's subtopics are more similar to those in DP AA SL. SB mathematics II has similarity to DP HL mathematics subjects in terms of the breadth and depth of content covered. With regards to content alignment, SB mathematics II strongly aligns with DP AA HL, as it covers most of the SL and AHL subtopics in every topic area. Furthermore, SB mathematics II also has moderate content alignment with DP AI HL, as it covers most subtopics from Number and Algebra, and a considerable number from Geometry and Trigonometry and Calculus. Matrices is a key subtopic shared by DP AI HL and SSB mathematics II.

- **Similarities in demand**: SB mathematics I aligns very closely with DP SL mathematics subjects in all demand categories, with both evidencing higher Bloom's cognitive skills, similar depth of knowledge, a moderate-heavy volume of work, and a small number of outstanding demand areas. SB mathematics II aligns very closely with DP HL mathematics subjects for demand, scoring highly across all demand categories and similarly demonstrating higher order cognitive skills, considerable depth across topics, a heavy volume of work and a high number of areas that are considered challenging for upper-secondary mathematics.

The **key differences** identified were the following:

- **Differences in learning outcomes:** although there is strong alignment between DP and SP mathematics learning outcomes, a few small differences can be noted. Indeed, the SB more explicitly emphasises computational thinking, use of representations, making connections within mathematics, and personal and social skills.

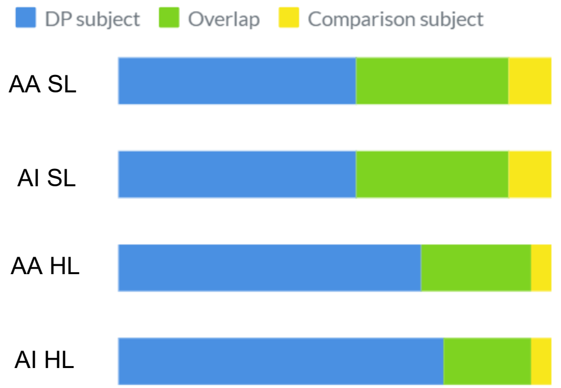
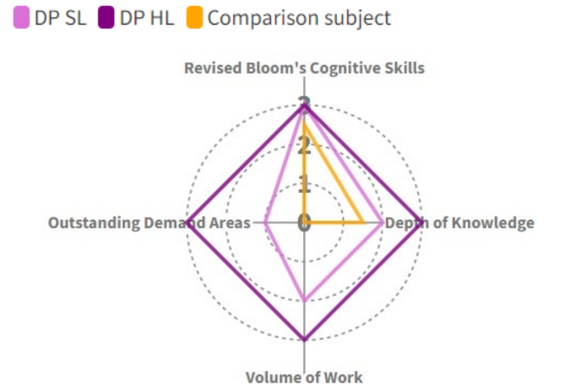
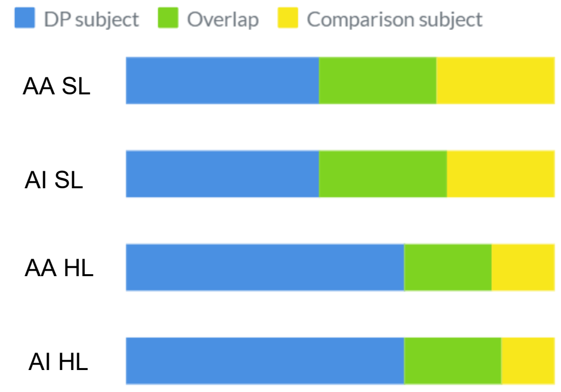
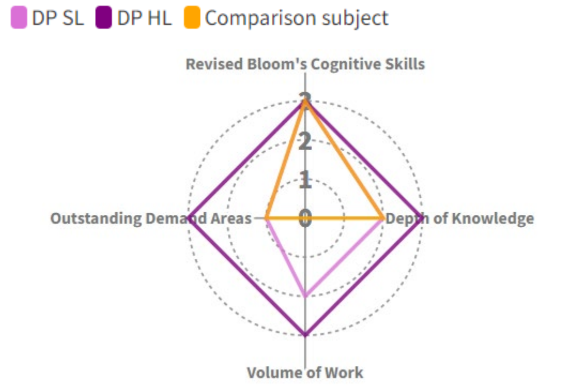
- **Differences in content:** whilst similar to DP SL subjects, SB mathematics I has less depth in statistics and calculus content and more depth in number and algebra and geometry and trigonometry content. With regards to SB mathematics II, there is only moderate content alignment with DP AI HL, due to content being more similar to DP AA HL. Indeed, SB mathematics II does not include the same emphasis on modelling or the subtopics relating to graph theory, decision mathematics, or any AHL Statistics and Probability content. Although matrices is a key topic in SB mathematics II, the latter extends these to different areas to DP AI HL. Furthermore, SB mathematics II contains a considerable amount of content which is not on the DP curriculum, particularly with regards to conics, vectors, and matrices.

- **Differences in demand:** there are no significant differences with regards to demand, as SB mathematics I aligns closely with DP SL subjects and SB mathematics II aligns closely with DP HL subjects

5.9 Brazil

The subject level alignment between DP (AA and AI, SL and HL) and BHSC Mathematics and Technology (MAT) is represented below:

Figure 39: Visual representations of subject-level alignment (mathematics)

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
BHSC Mathematics and Technology (Basic General Education) *BHSC MAT (BGE)*	Low Moderate **High**		
BHSC Mathematics and Technology (Formative Itinerary) *BHSC MAT (FI)**	Low Moderate **High**		

* BHSC MAT (FI) represents the pathway of studying MAT in basic general education and then specialising in a MAT formative itinerary.

- **Learning outcomes alignment**: there is a high level of alignment between the learning outcomes of DP mathematics and BHSC MAT. Indeed, all the DP mathematics' learning outcome themes are present in BHSC MAT, including a strong emphasis on considering and using mathematics with respect to wider contexts (such as local and global issues) and developing critical thinking, technological, communication, and transferable learning skills within the subject.

- **Content alignment**: the documentation indicates that there is a low-moderate level of content alignment between BHSC MAT and DP mathematics (AA and AI), with the level of content alignment with DP mathematics being very similar for both BHSC MAT (BGE) and BHSC MAT (FI). Indeed, BHSC MAT (BGE) does not covers some, but not all, DP SL content and overall has less breadth and depth than DP SL and HL mathematics subjects. BHSC MAT (FI) increases the breadth of mathematical application; however, the formative itinerary component does not focus on extending the complexity of the mathematics covered; thus, it does not present a stronger alignment with DP mathematics than BHSC MAT (BGE). The content for MAT formative itineraries is described in limited detail, thus the breadth and depth of BHSC MAT (FI) is challenging to ascertain. Generally, it is indicated that the breadth and depth of BHSC MAT (FI) may be similar to DP SL subjects, but with differences in the content covered.

- **Demand alignment**: it can be noted that the demand scores for BHSC subjects are based on documentation which has somewhat limited detail regarding subject content. Generally, the documentation reviewed indicates a low-moderate alignment between the demand scores of DP mathematics (AA and AI) and BHSC MAT. BHSC MAT (BGE) scores lower than DP SL subjects in all categories, whereas BHSC MAT (FI) has stronger alignment with DP SL subjects, scoring the same for most demand categories. BHSC MAT subjects score lower overall compared to DP HL subjects, and lower than all DP mathematics subjects for volume of work.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: The mathematics learning outcomes for BHSC MAT have strong similarities with DP mathematics with regards to considering and using mathematics in wider contexts and developing critical thinking skills. Furthermore, both encourage inquiry-based approaches by expecting students to investigate conjectures, analyse information, and draw conclusions. Problem-solving skills are also a key emphasis in both mathematics curricula, as well as the use of technology, effective and accurate communication, making connections and generalisations, and understanding and applying mathematical concepts and procedures. Overall, there is a strong alignment in the mathematics learning outcomes of DP mathematics and BHSC MAT.

- **Similarities in content**: Whilst not strongly similar, BHSC MAT and DP SL mathematics subjects have partial content alignment, sharing some content in most topic areas. Moreover, it can be noted that the specific skills for BHSC MAT often include similar real-world contexts, interdisciplinary links, and uses of technology to those suggested in the 'Connections' sections of the DP mathematics subject guides, which reinforces the learning outcome findings that BHSC MAT has a similar emphasis on these aspects. Moreover, while BHSC MAT (FI) does not particularly increase alignment with DP mathematics content beyond that observed for BHSC MAT (BGE), the applied focus of the formative itinerary component has similarities with the applied thematic focus of DP AI.

- **Similarities in demand**: There is moderate alignment between the demand scores of DP SL mathematics subjects and BHSC MAT (FI). Indeed, these profiles receive the same score in all categories except volume of work. These scores reflect a similarity in the emphasis on critical thinking in their learning outcomes and the presence of tasks that promote a deeper engagement with mathematics content (though it can be noted that BHSC MAT content is overall less complex than DP SL and HL mathematics). Moreover, while less aligned with DP HL, BHSC MAT scores similarly with regards to Bloom's cognitive skills.

The **key differences** identified were the following:

- **Differences in learning outcomes:** Though DP mathematics and BHSC MAT share very similar learning outcomes, it can be noted that the formative itinerary skills of BHSC MAT have a greater focus than the DP on some skills involving entrepreneurship and proposing mediation and intervention strategies for global and local issues.

- **Differences in content:** BHSC MAT (BGE) and BHSC MAT (FI) do not cover a considerable amount of content which is in DP mathematics. Indeed, no DP AHL content is covered and, in contrast to DP mathematics subjects, BHSC MAT does not cover any calculus content. Moreover, BHSC MAT content overlaps some DP SL content, but does not cover topics such as functions and statistics in as much detail and depth. Moreover, the formative itinerary component is application-focused and covers areas that are not featured in DP mathematics, such as logic, robotics, and gamification. Overall, the level of content alignment is relatively low, and BHSC MAT features less breadth and depth than DP HL subjects and less depth in the topics it shares with DP SL mathematics.

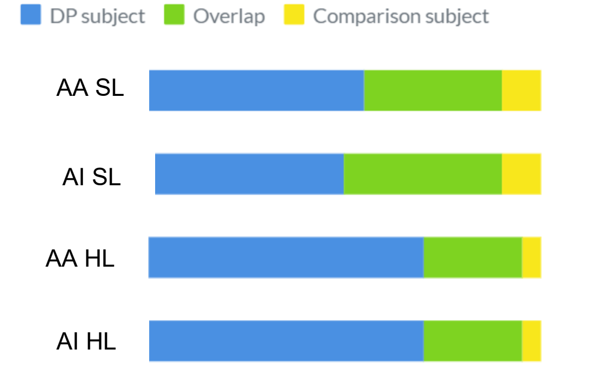
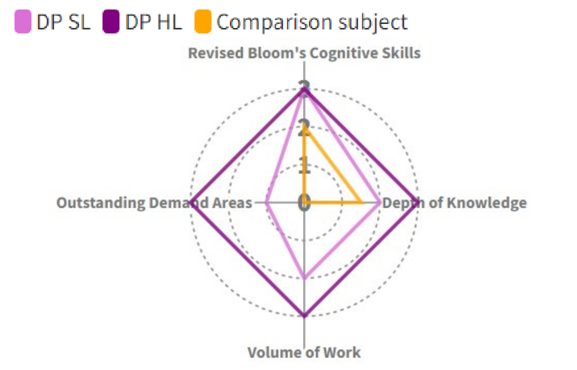
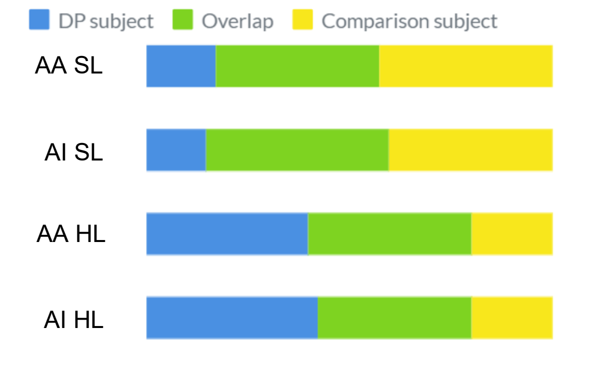
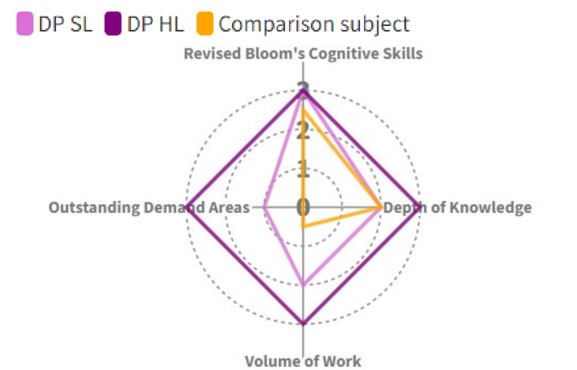
- **Differences in demand:** the demand scores of DP HL subjects and BHSC MAT subjects are not strongly aligned. Indeed, DP HL subjects score higher with regards to depth of knowledge, volume of work, and outstanding demand areas. Moreover, a generous amount of time is allocated for BHSC MAT subjects, contributing to a lower volume of work score compared to all DP mathematics subjects.[133]

[133] Time allocations were drawn from the RJRC.

5.10 Mexico

The subject level alignment between the DP mathematics subjects (AA and AI, SL and HL) and MBG Mathematical Thinking units is represented below:

Figure 40: Visual representations of subject-level alignment (mathematics)

Subject name	Learning outcomes alignment	Content alignment	Demand alignment
Compulsory Mathematical Thinking Units	Low Moderate **High**		
Compulsory and Optional Mathematical Thinking Units	Low Moderate **High**		

MBG Compulsory units: Mathematical Thinking I, II, and III.
MBG Optional units: Probability and Statistics I and II, Financial Mathematics I and II, Drawing I and II, Differential Calculus, and Integral Calculus.

- **Learning outcomes alignment**: there is an overall high level of alignment between the mathematics learning outcomes of the DP and MBG. Indeed, all themes extracted from the DP mathematics learning outcomes are present in the Mathematical Thinking units of the MBG.

- **Content alignment**: there is a low-moderate level of content alignment between the MBG compulsory Mathematical Thinking units and DP mathematics. Indeed, the compulsory units combined have lesser breadth and depth than both DP SL and HL and do not cover a significant amount of AA and AI content. Additionally, there is a moderate level of content alignment between the MBG's compulsory and optional Mathematical Thinking units and the DP mathematics subjects, with the optional Mathematical Thinking units covering further content from some DP topics, as well as some technical drawing content that is not in the DP. Overall, the Mathematical Thinking units combined have somewhat greater breadth than DP SL and more depth in certain topics, though an overall lesser breadth and depth than DP HL mathematics. Notably, no combination of units in the MBG presents a high level of content alignment with DP HL mathematics (either AA or AI). Also, it should be noted that MBG students choosing to specialise in mathematics need not take all the optional units, therefore, the breadth and depth of actual mathematics study is variable and depends on student choices.

- **Demand alignment**: there is a low-moderate level of alignment between the demand scores of the MBG's Mathematical Thinking units and the DP SL mathematics subjects. Indeed, the compulsory Mathematical Thinking units score less than DP SL in all demand categories, and the combination of compulsory and optional units score similarly in some, but not all, categories. There is a low level of alignment between the demand scores of Mathematical Thinking units and DP HL subjects, with the latter scoring higher in all demand categories.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: DP and MBG mathematics learning outcomes place a high emphasis on problem-solving and reasoning, communicating mathematics clearly in various forms, applying concepts to a range of contexts, and making connections within mathematics and to other subjects.

- **Similarities in content**: being of lesser breadth and depth, the compulsory Mathematical Thinking units alone do not have strong similarities with DP mathematics content. That said, the compulsory units feature some SL mathematics content from most DP topics, particularly Functions, Statistics and probability, and Calculus. The addition of optional Mathematical Thinking units strengthens the alignment with DP mathematics, as these add to the content of compulsory units. The units all combined cover some SL content in all DP topics, most particularly Calculus and Statistics and probability. Furthermore, the optional units of Differential Calculus and Integral Calculus have strong alignment with SL and AHL Calculus content in the DP, thus having similar depth to DP HL in this topic. The units align slightly more strongly with AA (SL and HL) for Calculus content and AI (SL) for Number and algebra. For the most part, however, the DP content that they do cover is usually that which is present in both DP mathematics subjects.

- **Similarities in demand**: The MBG Mathematical Thinking units score the same or similarly to DP SL mathematics subjects in the depth of knowledge demand category, due to featuring content requiring considerable pre-requisite knowledge and the consistent requirement for a deeper engagement with content. Furthermore, the compulsory and optional Mathematical Thinking units combined score similarly to DP mathematics subjects (SL and HL) for the Bloom's cognitive skills demand category, due to a similar emphasis on analysis, synthesis, and evaluation.

The **key differences** identified were the following:

- **Differences in learning outcomes:** there are no significant differences between the learning outcomes of DP mathematics and Mathematical Thinking in the MBG. However, it can be noted that the DP places a higher emphasis on the use of technology and inquiry-based approaches, as well as considering and using mathematics in respect of wider contexts (such as local and global issues). Conversely, the MBG units identify some specific reasoning skills to be developed, such as heuristic and intuitive thinking, that are not explicitly highlighted within DP mathematics learning outcomes.

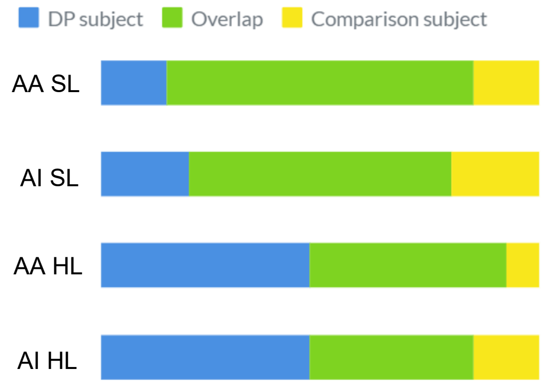
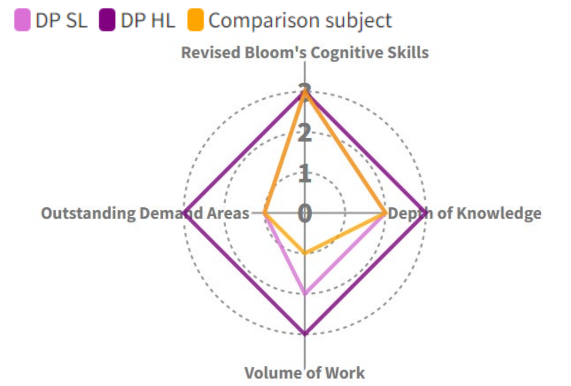
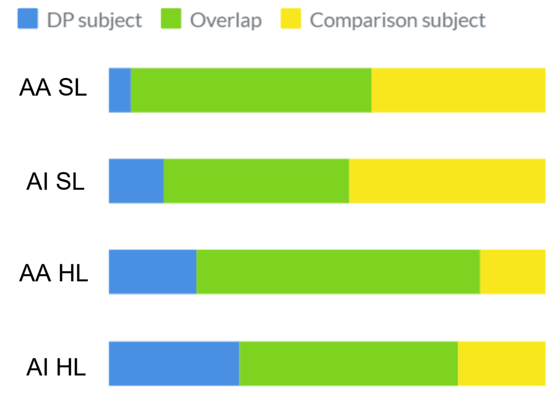
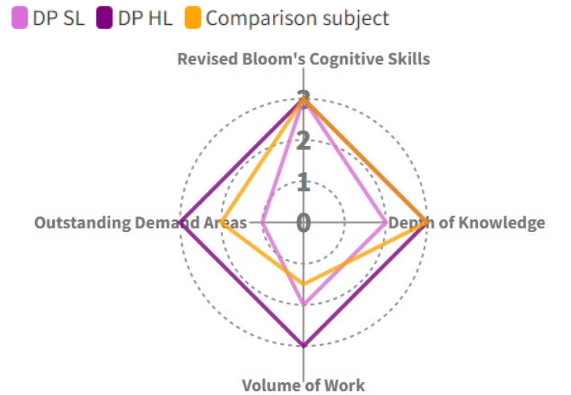
- **Differences in content:** the compulsory Mathematical Thinking units in the MBG do not cover a significant amount of content from DP mathematics. Indeed, they cover very little similar content to Number and algebra and only some of the more basic concepts in Geometry and trigonometry. Whilst there is better alignment with SL content in other DP topics, certain key sub-topics, such as integration and binomial distribution, are not included in the compulsory units. Moreover, the compulsory units generally do not include similar content to AHL sub-topics in DP mathematics subjects. The combination of compulsory and optional Mathematical Thinking units in the MBG present a stronger alignment with DP content; however, except for Calculus, the units do not cover similar content to AHL in the DP. Therefore, there is no subject (or combination of units) in the MBG that has a high-level of content alignment with DP HL mathematics subjects. It can be noted that the MBG offers Drawing I and II units, which involve the application of mathematics in technical drawing, which is not covered in DP mathematics. Overall, however, the breadth and depth of MBG Mathematical Thinking is lesser than DP HL mathematics subjects.

- **Differences in demand:** the MBG Mathematical Thinking units score significantly lower than DP SL and HL mathematics subjects for volume of work. Indeed, the time allocated to each unit is generous for the complexity of its content and the number of topics covered, resulting in a lighter volume of work than that of DP mathematics. Moreover, no significant outstanding areas of demand are present in the Mathematical Thinking units; thus, they score somewhat lower than DP SL subjects and significantly lower than DP HL subjects in this regard.

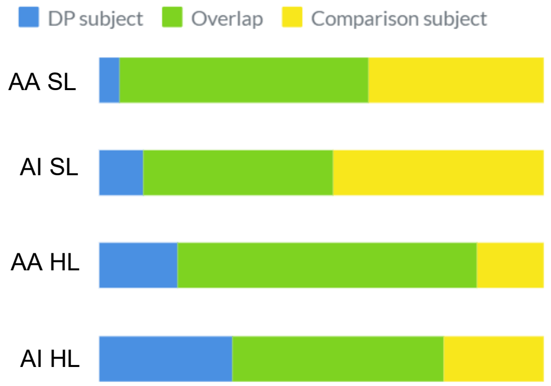
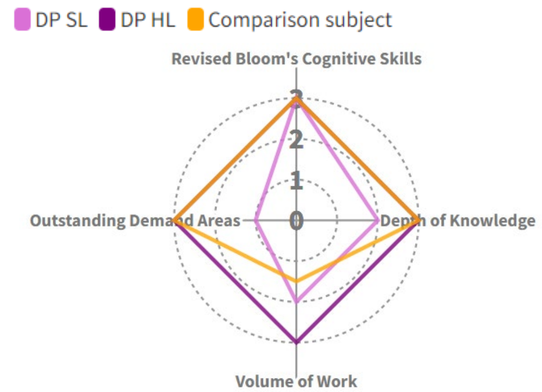
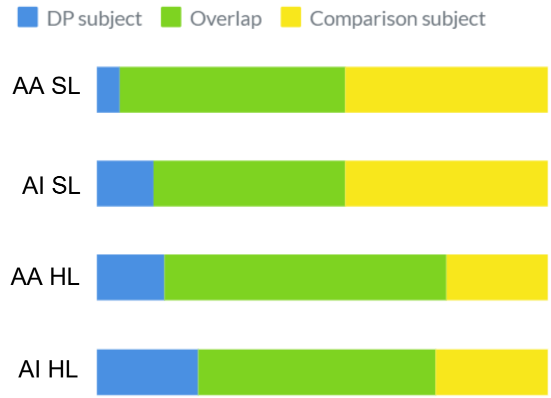
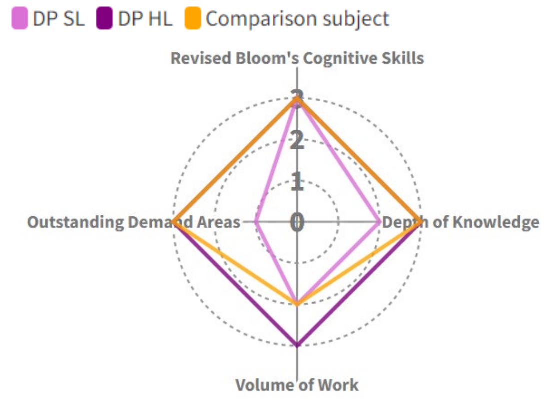
5.11 Japan

The subject level alignment between DP mathematics (AA and AI, SL and HL) and the JHSC mathematics pathways is represented below:

Figure 41: Visual representations of subject-level alignment (mathematics).

Pathway name	Learning outcomes alignment	Content alignment	Demand alignment
JHSC General Subjects Pathway I	Low Moderate **High**		
JHSC General Subjects Pathway II	Low Moderate **High**		

- **General Subjects Pathway I**: Mathematics I and II, Mathematics A and B, and Basic Inquiry.
- **General Subjects Pathway II**: Mathematics I, II, and III, Mathematics A, B and C, and Basic Inquiry and Inquiry.

Pathway name	Learning outcomes alignment	Content alignment	Demand alignment*
JHSC Specialised Subjects Pathway I	Low Moderate **High**		
JHSC Specialised Subjects Pathway II	Low Moderate **High**		

- **Specialised Subjects Pathway I**: Mathematics I and II (SMC) and Inquiry subjects.
- **Specialised Subjects Pathway II**: Mathematics I and II (SMC), Advanced Mathematics (SMC), and Inquiry subjects.

- **Learning outcomes alignment**: There is a high level of alignment between the learning outcomes of DP and JHSC mathematics. Most DP learning outcome themes are strongly present in the JHSC objectives, with two partially present, and only one absent.

- **Content alignment**: General Subjects Pathway I has strong alignment with SL content and has comparable breadth and depth. In addition to strong alignment with SL content, the other pathways also have good alignment with the AHL content in certain topics, with Specialised Subjects Pathway II having the highest presence of AHL content overall. These pathways also cover different content to DP mathematics and overall have similar breadth and depth to DP HL subjects, if not slightly more in the case of Specialised Subjects Pathway II. It can be noted that all pathways align more with AA content than AI content.

- **Demand alignment**: General Subjects Pathway I strongly aligns with the demand scores given to DP mathematics subjects at SL. The other pathways align well with the demand scores given to DP mathematics subjects at HL, with Specialised Subjects Pathway II having the strongest alignment.

The **key similarities** identified were the following:

- **Similarities in learning outcomes**: Most DP themes are well-evidenced in the JHSC objectives, including understanding and application, communicating mathematically, developing learning skills, using critical thinking, and making links and generalisations.

- **Similarities in content**: General Subjects Pathway I aligns strongly with the SL content of most topics. There is strong alignment with AA SL content for all topics except *Calculus*, which it partially aligns with. Additionally, there is strong alignment with the AI SL content of most topics, with partial alignment for *Number and algebra* and *Functions*. With regards to AA content, the other JHSC pathways have strong alignment with the SL content in every topic, as well as the AHL content in *Number and algebra*. General Subjects Pathway II also has partial alignment with the AHL content of *Geometry and trigonometry* and *Calculus*. Both Specialised Subject pathways have strong alignment with the AHL content of *Calculus* and Specialised Subjects Pathway II also has partial alignment with the AHL content in *Geometry and trigonometry*. Regarding AI content, the other pathways' alignment with SL content remains the same as General Subjects Pathway I. However, the pathways have good alignment with the AHL content *in Number and algebra* and partial alignment with *Calculus*. The second general and specialised pathways also have partial alignment with the AHL content in *Geometry and trigonometry*. Lastly, the inclusion of inquiry-based subjects in the pathways leads to alignment with the DP mathematics exploration.

- **Similarities in demand**: General Subjects Pathway I scores the same as DP subjects at SL for all demand categories, except volume of work where it scores slightly less. The other pathways score the same as DP subjects at HL for both Bloom's cognitive skills and depth of knowledge, with the Specialised Subjects pathways also scoring the same for outstanding demand areas.

The **key differences** identified were the following:

- **Differences in learning outcomes**: The DP themes of engaging with mathematics in a wider context and inquiry approaches are only partially present in the JHSC objectives. Most JHSC subjects do not emphasise broader contexts such as global issues and inquiry approaches are limited to non-compulsory inquiry-based subjects. Furthermore, the JHSC objectives place significantly less emphasis on the use of technology compared to the DP learning outcomes. While no significantly different themes emerge from the JHSC objectives, the inquiry-based and specialised subjects place greater emphasis on making connections between science and mathematics.

- **Differences in content**: As noted, the JHSC pathways align more closely with AA content than AI content. All pathways only partially alignment with the AI SL content in *Number and algebra* and *Functions*, as there is less presence of financial applications and modelling. In terms of AHL content, the JHSC pathways do not align with the *Functions* contain of either AA or AI. In addition, there is at most only partial alignment with *Geometry and trigonometry* AHL content, as key subtopics such as reciprocal trigonometric ratios (AA), graph theory (AI), and decision maths (AI) are not covered. Furthermore, none of the JHSC pathways align with AI AHL content from *Statistics and probability*. Instead, the pathways cover different content to one or both DP subjects. This includes different geometry content (e.g. conics), limits of sequences, and topics which focus on linking mathematics to the everyday world.

- **Differences in demand**: JHSC pathways generally score less for volume of work compared to DP subjects. For this category, only Specialised Subjects Pathway II scores the same as DP subjects at SL, and no pathways score similarly to DP subjects at HL. Moreover, it can be noted that General Subjects Pathway II, while aligning generally well to DP subjects at HL, scores less for outstanding demand areas.

6. Cross-cutting Findings

This section summarises the main cross-cutting trends, similarities and differences found when comparing the DP subjects against the comparison subjects in the eleven comparison programmes. In particular, the section focuses on the cross-cutting findings uncovered from the learning outcomes, content and demand analyses.

6.1 Learning Outcomes

Table 71: Presence of the DP's mathematics learning outcomes in the comparison curricula.

Learning outcome themes extracted from the DP mathematics subject group	Presence of DP themes in the learning outcomes of the comparison curricula										
	VCE (Victoria)	OSSD (Ontario)	FNCC (Finland)	SGA (Singapore)	KHSCG (South Korea)	CCSS (US)	FB (France)	SB (Spain)	BHSC (Brazil)	MBG (Mexico)	JHSC (Japan)
1. Wider contexts											
2. Transferable learning skills											
3. Inquiry-based approaches											
4. Understanding and application											
5. Links and generalisations											
6. Critical/creative thinking skills											
7. Communicating mathematics											
8: Technology											

Key:

	This theme is well-evidenced in the learning outcomes of the comparison curriculum.		*This theme is partially evidenced in the learning outcomes of the comparison curriculum.*		*This theme is not evidenced in the learning outcomes of the comparison curriculum.*

The table above displays the learning outcome themes extracted from the DP mathematics subjects and summarises their presence in the mathematics curricula of the eleven comparison programmes/standards. Additionally, to complement the above, the table below lists the main themes found in the curricula of each of the eleven comparison programmes/standards which were not present in the DP or were emphasised more strongly.

Table 72: Summary of different themes/emphases featured in the learning outcomes of the comparison curricula.

Comparison	Different themes/emphases to those of DP mathematics
Australia (Victoria)	Modelling; analysis.
Canada (Ontario)	Social-emotional learning skills (including metacognition) *(grade 9)*; connections within mathematics.
Finland (FNCC)	Modelling; connections within mathematics.
Singapore	Metacognition; modelling; heuristics; mathematical rigor *(H2F and H3).*
South Korea	Convergent thinking; information-processing.
US (CCSS)	Modelling.
France	Oral communication.
Spain	Connections within mathematics; computational thinking; personal and social skills.
Brazil	Proposing mediation and intervention strategies to sociocultural and economic problems and an emphasis on entrepreneurship *(formative itinerary).*
Mexico	Certain mathematical reasoning skills such as heuristic and intuitive thinking.
Japan	No significantly different themes.

Regarding structure and presentation of learning outcomes, the DP, CCSS, FNCC, and FB similarly describe one set of outcomes which are applicable to all mathematics courses that they offer – i.e. they do not outline specific outcomes for each mathematics subject/course. In contrast, the other curricular complement their overarching outcomes for their mathematics subject area with course-specific outcomes. That said, the KHSCG and JHSC subject-specific outcomes only slightly differ where they are contextualised for the content of the subject. Similarly, the OSSD course-specific expectations are primarily content-focused. However, for the VCE, SGA, and SB the course-specific outcomes describe somewhat different skills for each course, as well as content. For example, VCE Foundation Mathematics outcomes reflect the more basic skills of routine problem-solving, whereas VCE Specialist Mathematics reflect more advanced skills of problem-solving in non-routine and abstract contexts. For the SB, there are slight differences in skills between courses, which shows a progression in some skills from first to second year. For the BHSC and MBG, there are skills which apply to the common mathematics learning, then additional skills are included for the optional courses that students can take – i.e. the formative itinerary in the BHSC and the optional units in the MBG.

Generally, the comparison curricula often feature similar learning outcomes to those in DP mathematics. In particular, the themes of *Transferable learning skills*, *Understanding and application*, *Making links and generalisations*, *Critical and creative thinking skills*, *Communication*, and *Using technology* are usually well-evidenced in the comparison curricula. However, the themes of *Linking mathematics to wider contexts* and *Using inquiry-based approaches* are the least consistently evidenced in the learning outcomes of the curricula, though it can be noted that they are sometimes present in other sections of their curricula.

The comparison curricula with the strongest alignment to the DP, regarding mathematics learning outcomes, are the OSSD, FNCC, SB and BHSC, closely followed by the SGA. Indeed, all the above reflect a holistic approach in their learning outcomes and feature key DP outcomes such as thinking about mathematics and global issues, the universality of mathematics, understanding historical and cultural perspectives, using inquiry approaches, and developing curiosity, independence, and perseverance.

The comparison curricula with the least alignment to DP mathematics learning outcomes are the VCE, CCSS, and FB, as they comprise a narrower range of learning outcomes, mostly focusing on understanding and application, problem-solving, and technology. In the VCE, the least demanding subjects in particular, show low alignment with DP mathematics. Mathematical Methods and Specialist Mathematics, on the other hand, are more closely aligned with DP mathematics in terms of learning outcomes.

In terms of themes or emphases featured in the comparison curricula but not in the DP, a few patterns emerge. Most commonly, the skill of modelling has greater emphasis in the learning outcomes of the comparison curricula, with VCE, FNCC, SGA, and the CCSS explicitly detailing this skill. However, it can be noted that, whilst not as strongly emphasised in the learning outcomes, the DP does focus on modelling in other sections of its documentation and is a key skill, particularly in mathematics: applications and interpretation. The OSSD and SGA have a stronger emphasis on encouraging students to engage in certain types of deeper thinking during their mathematical learning, which for the SGA involves developing metacognition skills and for the OSSD involves developing social-emotional learning skills. While these are not a feature of DP mathematics' learning outcomes, they are somewhat present in the wider philosophical underpinnings of the DP (e.g. in the learner profile). Finally, a few curricula emphasise the development of computational and heuristic thinking.

Overall, most comparison curricula have a high degree of alignment with the DP's mathematics learning outcomes.

6.2 Content

6.2.1 Structure

The organisation of the mathematics subject area into mathematics: analysis and approaches (AA) and mathematics: applications and interpretation (AI) is unique to the DP. Indeed, none of the comparison curricula offer two subjects that cover the same broad range of topics while differing in their thematic focus. Instead, other curricula may have courses that integrate content that appears in AA and AI or will offer applied mathematics courses that cover different areas to the 'pure' mathematics courses.

Another unique feature of DP mathematics is that of having five main topics which are studied by all students, regardless of subject and course choice. In other mathematics curricula, the main topics greatly vary depending on the subject chosen, or the number of courses taken.

The DP feature of including content from a broad range of mathematical areas in one course is seen in some of the comparison curricula. The VCE, SGA, FNCC, SB, FB, MBG, and JHSC offer stand-alone courses which span a similarly broad range of topics to the DP, i.e. one subject/course will include content from number and algebra, functions, geometry and trigonometry, statistics and probability, and calculus. The common learning in the CCSS and BHSC also span a broad range, though do not include calculus. In contrast, the OSSD, KHSCG, and JHSC (general subjects) organise the mathematics content of their upper-secondary years into courses or electives that each focus on a narrower range of mathematical areas. For example, topics like calculus, functions, and statistics are taught separately in different courses. While the common mathematics learning in the BHSC and MBG spans a range of areas, the specialisation options in these programmes are narrower in their focus. Indeed, the MBG optional units each focus on one area of mathematics (e.g. financial mathematics) and the BHSC's formative itinerary component focuses on applied mathematics (e.g. robotics).

Like SL and HL, other curricula offer different 'levels' of study to students, albeit in different forms. Most recognisably similar to the DP is the offering of H1, H2, H2F, and H3 in the SGA, the option of the basic or advanced syllabus in the FNCC, one or two years of study in the FB and SB, and Mathematical Methods and Specialist Mathematics in the VCE. Most of the comparison curricula cater to a wider range of mathematical ability, as they offer options pitched at a more foundational level than DP mathematics SL, one or two also offer more specialised courses than DP mathematics.

Finally, another feature that the DP shares with most of the comparison mathematics curricula is that, once chosen, all content within a specific course is compulsory. This is true for all curricula except the FNCC, VCE, and JHSC, which contain modules/subjects featuring optional content

6.2.2 Content Alignment

The following tables summarise the presence of DP mathematics SL and AHL content in the mathematics curricula of the eleven comparison programmes/standards. Here, all the mathematics content within each comparison curriculum is considered. To view the mathematics content within specific subjects, and how it aligns with DP mathematics subjects, refer to the relevant country in Section 4. Subject-Level Alignment.

Table 73: Presence of DP mathematics: analysis and approaches content in the mathematics curricula of the comparison programmes/standards.

Mathematics: analysis and approaches topics		Presence in the comparison curricula										
		VCE	OSSD	FNCC	SGA	KHSCG	CCSS	FB	SB	BHSC	MBG	JHSC
SL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
AHL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
SL/HL	Mathematics exploration											

Table 74: Presence of DP mathematics: applications and interpretation content in the mathematics curricula of the comparison programmes/standards.

Mathematics: applications and interpretation topics		Presence in the comparison curricula										
		VCE	OSSD	FNCC	SGA	KHSCG	CCSS	FB	SB	BHSC	MBG	JHSC
SL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
AHL	1. Number and algebra											
	2. Functions											
	3. Geometry and trigonometry											
	4. Statistics and probability											
	5. Calculus											
SL/HL	Mathematics exploration											

Key:

	Strong presence of this topic in the comparison curriculum.		*Partial presence of this topic in the comparison curriculum.*		*Little or no presence of this topic in the comparison curriculum.*

The table below lists any significant content found in the comparison mathematics curricula which was not present in DP mathematics or covered differently.

Table 75: Topics not found in the DP's content, or that follow a significantly different approach

Comparison	Topics
VCE	*Specialist Mathematics*: 'Mechanics'; 'Logic and Algebra'; conics; polar coordinates *Further Mathematics*: time series; financial applications (some different coverage)
OSSD	No large topics that are significantly different
FNCC	*All:* some economics modules *Advanced:* 'Algorithm and number theory' (some different coverage)
SGA	*Higher 2 Further Mathematics*: polar coordinates; recurrence relations; conics; linear spaces; non-parametric tests *Higher 3 Mathematics:* proof (beyond scope of AA)
KHSCG	*Economic Mathematics* (elective) *Artificial Intelligence Mathematics* (elective) *Calculus*: limit of a sequence *Geometry:* conics
CCSS	Some different geometry coverage (circle theorems and conics)
FB	Algorithms and programming; limits of sequences; different statistics content
SB	Conics and different matrices, vectors, and limits content
BHSC	Algorithms and programming and conics *Formative itinerary*: Logic, gamification, robotics, fiscal and financial mathematics (different coverage), and financial citizenship
MBG	Linear programming Technical drawing and various applications
JHSC	*From various electives:* Linear programming Some different geometry content – loci, equation of a circle, Euclidean algorithm, properties, and conics Sequences – recurrence sequences and limits of sequences Mathematics and human activities Mathematics and social life

Content from each of the DP five main topics of *Number and algebra*, *Functions*, *Geometry and trigonometry*, *Statistics and probability*, and *Calculus* is found in all comparison curricula except the CCSS and BHSC which do not include any calculus content.

The comparison curricula tend to align more strongly with AA SL content than AI SL content. With a few exceptions, AA SL content is strongly present across the comparison curricula. AI SL content is strongly present in some curricula, however, it has more of a partial presence in several others. Notably, the AI SL sub-topic of Voronoi diagrams is not present in any of the comparison curricula and hypothesis testing is also often absent.

To varying degrees, all the comparison curricula (except the BHSC) contain some AHL content, however, it's not strongly present across the comparison curricula. The specific AHL topics and sub-topics which are covered vary across curricula – i.e. there are very few AHL sub-topics that are consistently covered in all, or most, of the curricula. That said, the comparison curricula (excluding the BHSC and MBG) typically include some content involving vectors, albeit to varying degrees of depth. This contributes to some alignment being present with the AA and/or AI AHL content in *Geometry and trigonometry* across most curricula. However, certain areas of DP AI AHL content are particularly rare the comparison curricula. Indeed, across the curricula, there is often limited or no alignment with AI *Statistics and probability* AHL content, which includes sub-topics such as further hypothesis testing, nonlinear regression, transition matrices, and Markov chains. Other AI sub-topics which are often not present in the curricula include eigenvalues and eigenvectors, graph theory, adjacency matrices, slope fields, and phase portraits.

Generally, most of the DP sub-topics which are identified as 'Outstanding areas of demand' in the demand judgements are absent or weakly present across the comparison curricula. For example, Maclaurin series is only present in the SGA. Furthermore, vectors are often not covered to the same depth as AA.

Only five comparison curricula have content which aligns with the DP's mathematical exploration, namely the VCE, OSSD, KHSCG, BHSC, and JHSC. The strongest similarities to this DP component are observable in the 'Mathematical Inquiry Task' elective from the KHSCG and the 'Inquiry-Based Study of Science and Mathematics' elective in the JHSC.

As can be seen in the tables, the curricula that include the most DP mathematics content and have the strongest content alignment with the latter are the VCE, SGA, and SB. However, it can be noted that the SB in particular is more strongly aligned with AA than AI content. Whereas the VCE and SGA include a significant amount of content from both DP subjects.

Regarding content not covered by the DP, there is no large mathematical area which is commonly covered by the comparison curricula and not by the DP. However, it can be noted that it is common for other curricula to include some different geometry content, such as conics, as well as polar coordinates and limits of sequences. A notable difference, which is not common, is the coverage of algorithms and programming in the FB and BHSC.

6.3 Demand

The following tables provide a visual representation of the demand scores the expert panel awarded to the DP mathematics subjects and respective comparison subjects in each of the eleven comparison programmes/standards.

Key:

VCE: FM: Foundation Mathematics GM: General Mathematics FRM: Further Mathematics MM: Mathematics Methods SM: Special Mathematics	OSSD: F&A: Functions & Applications F: Functions AF: Advanced Functions CV: Calculus & Vectors DM: Data Management UP: University Pathway	FNCC: B; Basic A (C): Advanced (compulsory modules) A (C+O): Advanced (compulsory and optional modules)	SGA: H1: Higher 1 H2: Higher 2 H2F: Higher 2 Further H3: Higher 3	KHSCG: Pathway: Mathematics, Mathematics I, Mathematics II, Calculus, Probability and Statistics, and Geometry	CCSS: CCSSM: Common Core State Standards for Mathematics
FB: Math P: Mathematics *Première* Math T: Mathematics *Terminale*	SB: Mathematics I Mathematics II	BHSC BGE: Basic General Education FI: Formative Itinerary	MBG C: Compulsory units C+O: Compulsory and optional units	JHSC GSPI: General Subjects Pathway I GSPII: General Subjects Pathway II SSPI: Specialised Subjects Pathway I SSPII: Specialised Subjects Pathway II	

Table 76: The demand scores of mathematics subjects from the DP and comparison programmes (2022 studies; Australia, Canada and Finland)

Demand Category	Scores																	
	DP (IB)				VCE (Australia)					OSSD (Canada)						FNCC/GUSE (Finland)		
	AA SL	AA HL	AI SL	AI HL	FM	GM	FR M	MM	SM	F&A 11	F 11	AF 12	CV 12	DM 12	UP	B	A (C)	A (C+O)
Revised Bloom's Cognitive Skills	3	3	3	3	1	2	2	2	2.5	3	3	3	3	3	3	3	3	3
Depth of Knowledge	2	3	2	3	0	1	1.5	2	3	1	1	2	2	1.5	2	1	2	2.5
Volume of Work	2	3	2	3	0	1	1	1	2	0	0	0.5	0.5	0.5	0.5	0	1	1.5
Outstanding Demand Areas	1	3	1	3	0	0	1	1	3	0	0	0	1	1	2	0	0	1

Table 77: The demand scores of mathematics subjects in the DP and comparison programmes (2022 studies; Singapore, South Korea, and US)

Demand Category	Scores									
	DP (IB)				SGA (Singapore)				KHSCG (S. Korea)	CCSS (US)
	AA SL	AA HL	AI SL	AI HL	H1	H2	H2F	H3	Pathway	CCSSM
Revised Bloom's Cognitive Skills	3	3	3	3	3	3	3	3	3	3
Depth of Knowledge	2	3	2	3	1.5	3	3	3	2.5	2
Volume of Work	2	3	2	3	1	1	2	3	0.5	0
Outstanding Demand Areas	1	3	1	3	0	1	3	3	1	0

Table 78: The demand scores of mathematics subjects in the DP and comparison programmes (2023, 2024, and 2025 studies; France, Spain, Brazil, Mexico, and Japan).

Demand Category	Scores															
	DP (IB)				FB (France)		SB (Spain)		BHSC (Brazil)		MBG (Mexico)		JHSC (Japan)			
	AA SL	AA HL	AI SL	AI HL	Math P	Math T	Math I	Math II	BGE	FI	C	C+O	GSPI	GSPII	SSPI	SSPII
Revised Bloom's Cognitive Skills	3	3	3	3	2.5	2.5	3	3	2.5	3	2	2.5	3	3	3	3
Depth of Knowledge	2	3	2	3	1.5	3	1.5	3	1.5	2	1.5	2	2	3	3	3
Volume of Work	2	3	2	3	1	1	2	3	0	0	0	0.5	1	1.5	1.5	2
Outstanding Demand Areas	1	3	1	3	1	3	1	3	0	1	0	0	1	2	3	3

Except for the VCE, FB, and MBG, there is at least one subject/pathway in each comparison curriculum which scores the same as DP mathematics subjects for Bloom's cognitive skills, with all receiving the highest score for their evidence of analysis, critical thinking, and creativity in their learning outcomes. VCE Specialist Mathematics, FB subjects and MBG combined units receive similar, but slightly lower scores in this category compared to the DP and others.

As can be seen in the tables above, all the comparison curricula have at least one subject/course which scores the same, or similar, as DP mathematics SL for depth of knowledge. In contrast, only the VCE, SGA, FB, SB, and JHSC have courses/pathways which score the same as DP mathematics HL for this category. However, the FNCC and KHSCG receive very similar scores where specific courses are taken. The VCE, OSSD, and FNCC all include at least one subject or course which scores one point less than DP mathematics SL in this category. This would also be true for the KHSCG and JHSC, depending on the electives chosen.

Apart from some subjects/pathways in the VCE, SGA, SB, and JHSC the subjects offered in the comparison curricula score lower for volume of work than DP mathematics subjects, often significantly so. Many comparison subjects/courses were considered to have a generous or standard amount of time allocated to teach the syllabus, rather than requiring a quicker pace of learning. Notably, however, teaching hours are not described in the SGA or CCSS, meaning that some caution is necessary when interpreting the judgements for these.

With the exception of the CCSS and MBG, all the comparison curricula offer at least one subject or course that features at least one area of outstanding demand – the same as DP mathematics SL. In contrast, only the VCE, SGA, FB, SB, and JHSC offer subjects/pathways that have a comparable number of outstanding areas to DP mathematics HL. Several comparison curricula have at least one subject/course that features no areas of outstanding demand (for KHSCG and JHSC this would be certain stand-alone electives).

Overall, most comparisons offer a subject, course or pathway which is similar in demand to the DP mathematics SL subjects. However, only the VCE, SGA, SB and JHSC offer subjects which closely align with DP mathematics HL subjects for demand, with FB also having alignment, though less strongly. Notably, volume of work is the primary category where the comparison subjects often score significantly lower than the DP subjects, followed by outstanding areas of demand.

Finally, it is worth considering the minimum mathematics requirements and demand levels that students will encounter in their respective programmes. While the SGA, VCE, and SB offer courses that align with DP mathematics HL, it is not a requirement for students to study any mathematics at upper-secondary in these programmes. Most comparison curricula, including those with mathematical requirements, offer some courses which score significantly lower than DP mathematics SL and HL for demand.

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Appendix A

This Appendix provides further detail on the criteria utilised by Ecctis' experts and external panel members with subject expertise to measure demand for each of the subjects analysed in this study.

Demand Profile – Subject-level Judgement

- **Revised Bloom's cognitive** skills score (0-3): this is an overall score of course demand, based entirely on a review of learning outcomes. Levels have been defined based on increasing emphasis on Bloom's Higher Order Thinking Skills.

    - Level 0 – remembering and understanding: learning outcomes (as well as assessment and content) are primarily focused on recall and understanding, with limited or no evidence of higher order thinking skills.
    - Level 1 – applying: learning outcomes (as well as assessment and content) comprise a mix of recall-, understanding- and application-focused objectives, with only limited presence of higher order thinking skills.
    - Level 2 – analysing: learning outcomes (as well as assessment and content) comprise a mix of recall-, understanding and application-focused goals but also feature a substantial focus on analysis. Learning outcomes can also potentially feature some (though limited) evidence of evaluation and creation-focused goals.
    - Level 3 – evaluating and creating (or synthesising): learning outcomes (as well as assessment and content) feature a predominant focus on analysis-, evaluation- and creation/synthesis.

- **Depth of knowledge** (adapted from Webb's) score (0-3): this is an overall score evaluating the depth of knowledge or complexity of knowledge required by curriculum standards and expectations. The score is focused on subject content and learning outcomes, complemented by assessment where relevant/possible. Levels have been defined based on the level of detail studied per topic, as well as the levels of thinking described in Webb's Depth of Knowledge framework.

    - Level 0 – All or most topics are studied in limited detail (pre-upper secondary level). Only basic pre-requisite knowledge is required in order to grasp ideas. The level of cognitive complexity of the information students are expected to know is low (e.g. many tasks may require recall and reproduction of information such as facts, definitions, terms, or simpler procedures – acquired knowledge).
    - Level 1 – Some topics are studied in considerable detail. Moderate levels of pre-requisite knowledge are required in order to grasp ideas in some topics. The level of cognitive complexity of the information students are expected to know is low to moderate (e.g. many tasks may require engagement of some mental processing beyond habitual responses, including comparison and basic reasoning – knowledge application).

  - Level 2 – Most topics are studied in considerable detail. Considerable pre-requisite knowledge is required in order to grasp ideas in some topics. The level of cognitive complexity of the information students are expected to know is average to high (e.g. some tasks require complex reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. The cognitive demands are often complex and abstract – analysis).
  - Level 3 – All or most topics are studied in very high detail. Considerable pre-requisite knowledge is required in order to grasp ideas in most topics. The level of cognitive complexity of information students are expected to know is mostly high (e.g. many tasks may require complex reasoning, planning, developing, information synthesis, interpretation of data for problem solving, and thinking most likely over an extended period – extended thinking).

- **Volume of work** score (0-3): this is a trifactor score, considering breadth of content and depth of content, evaluated against the programme's specified timeframe. The three factors – breadth, depth, and time – were all considered in defining the levels.

  - Level 0 – light: small number of themes and sub-themes covered; a significant majority of time is spent on straightforward or basic themes; generous time allocation per theme.
  - Level 1 – moderate: typical number of themes and sub-themes covered; more time spent on conceptually complex themes compared to Level 1 (though majority of time still spent on themes of basic depth); standard time allocation per theme.
  - Level 2 – moderate heavy: typical to high number of themes and sub-themes covered; a significant proportion of time spent on issues beyond basic conceptual depth; standard to short time allocation per theme.
  - Level 3 – heavy: high number of themes and sub-themes covered; a large proportion of time spent on issues beyond basic conceptual depth; short time allocation per theme.

- **Outstanding areas of subject demand** score (0-3): this score reflects the number of content areas typically viewed as more challenging and/or conducive to intellectual stretching of learners. Levels have been defined on a scale of increasing presence of 'stretch areas'.

  - Level 0 – no stretch areas (0)
  - Level 1 – few stretch areas (1-2)
  - Level 2 – a significant number of stretch areas (3-4)
  - Level 3 – a high number of stretch areas (>4)

Appendix B

Learner profile	Approaches to learning	Approaches to teaching	International-mindedness
Inquirers: We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life. **Knowledgeable**: We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance. **Thinkers**: We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions. **Communicators**: We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups. **Principled**: We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences. **Open Minded**: We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.	In all IB programmes, there are five categories of skills including: **Thinking skills:** including areas such as critical thinking, creative thinking, and ethical thinking **Research skills:** including skills such as comparing, contrasting, validating, and prioritizing information **Communication skills:** including skills such as written and oral communication, effective listening, and formulating arguments **Social skills:** including areas such as forming and maintaining positive relationships, listening	In all IB programmes, teaching is: **Based on inquiry:** A strong emphasis is placed on students finding their own information and constructing their own understandings. **Focused on conceptual understanding:** Concepts are explored in order to both deepen disciplinary understanding and to help students make connections and transfer learning to new contexts. **Developed in local and global contexts:** Teaching uses real-life contexts and examples, and students are encouraged to process new information by connecting it to their own experiences and to the world around them. **Focused on effective teamwork and collaboration:** This includes promoting teamwork and collaboration between students, but also refers to the collaborative relationship between teachers and students.	The aim of all IB programmes is to develop internationally minded people who recognize their common humanity and shared guardianship of the planet. Central to this aim is international-mindedness. International-mindedness is a multifaceted concept that captures a way of thinking, being and acting characterised by an openness to the world and a recognition of our deep interconnectedness to others. To be open to the world, we need to understand it. IB programmes therefore provide students with opportunities for sustained inquiry into a range of local and global issues and ideas. This willingness to see beyond immediate situations and boundaries is essential as globalization and emerging technologies continue to blur traditional distinctions between the local, national and international. An IB education fosters international-mindedness by helping students reflect on their own perspective, culture and identities, as well as those of others. By engaging with diverse beliefs, values and experiences, and by learning to think and collaborate across cultures and disciplines, IB learners gain the understanding necessary to make progress towards a more peaceful world.

Caring: We show empathy, compassion, and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us. **Risk-Takers**: We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change. **Balanced**: We understand the importance of balancing different aspects of our lives – intellectual, physical, and emotional – to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live. **Reflective**: We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development**.**	skills, and conflict resolution **Self-management skills:** including both organizational skills, such as managing time and tasks, and affective skills, such as managing state of mind and motivation.	**Designed to remove barriers to learning:** Teaching is inclusive and values diversity. It affirms students' identities, and aims to create learning opportunities that enable every student to develop and pursue appropriate personal goals. **Informed by assessment:** Assessment plays a crucial role in supporting, as well as measuring, learning. This approach also recognizes the crucial role of providing students with effective feedback.	An IB education further enhances the development of international-mindedness through multilingualism. All IB programmes require students to study, or study in, more than one language. This is because we believe that communicating in more than one language helps students to appreciate that his or her own language, culture and world view are just one of many. In this way, it provides excellent opportunities to develop intercultural understanding and respect. International-mindedness is also encouraged through a focus on global engagement and meaningful service with the community. These elements challenge students to critically consider power and privilege, and to recognize that they hold this planet and its resources in trust for future generations. They also highlight the focus on action in all IB programmes: a focus on moving beyond awareness and understanding to engagement, action and bringing about meaningful change to make a more peaceful and sustainable world for everyone.

Appendix C

CONFIDENTIAL

Task brief – Expert Demand Panel – [Subject]

For each subject, highlight in yellow the descriptor(s) deemed to best fit each demand category, using the following criteria (please refer to the demand tables for descriptors of the levels):

- **Revised Bloom's cognitive skills** score (0-3): this is an overall score of course demand, based entirely on a review of learning outcomes. Levels have been defined based on increasing emphasis on Bloom's Higher Order Thinking Skills.

- **Depth of knowledge** (adapted from Webb's) score (0-3): this is an overall score evaluating the depth of knowledge or complexity of knowledge required by curriculum standards and expectations. The score is focused on subject content and learning outcomes, complemented by assessment where relevant/possible. Levels have been defined based on the level of detail studied per topic, as well as the levels of thinking described in Webb's Depth of Knowledge framework.

- **Volume of work** score (0-3): this is a trifactor score, considering breadth of content and depth of content, evaluated against the programme's specified timeframe. The three factors – breadth, depth and time – were all taken into account in defining the levels.

- **Outstanding areas of subject demand** score (0-3): this score reflects the number of content areas typically viewed as more challenging and/or conducive to intellectual stretching of learners. Levels have been defined on a scale of increasing presence of 'stretch areas'.

Demand Judgements – [Subject]

Table 79: [Subject]

Demand Judgement	Score Descriptors (highlight the best-fit descriptor)	Judgement and Key Evidence
Revised Bloom's Cognitive Skills[134]	Level 0 – remembering and understanding: learning outcomes are primarily focused on recall and understanding, with limited or no evidence of higher order thinking skills.	
	Level 1 – applying: learning outcomes (as well as assessment and content) comprise a mix of recall-, understanding- and application-focused objectives, with only limited presence of higher order thinking skills.	
	Level 2 – analysing: learning outcomes (as well as assessment and content) comprise a mix of recall-, understanding and application-focused goals but also feature a substantial focus on analysis. Learning outcomes can also potentially feature some (though limited) evidence of evaluation and creation-focused goals.	
	Level 3 – evaluating and creating (or synthesising): learning outcomes feature a predominant focus on analysis-, evaluation- and creation/synthesis.	
Depth of Knowledge[135]	Level 0 – All or most topics are studied in limited detail (pre-upper secondary level). Only basic pre-requisite knowledge is required in order to grasp ideas. The level of cognitive complexity of the information students are expected to know is low (e.g. many tasks may require recall and reproduction of information such as facts, definitions, terms, or simpler procedures – acquired knowledge).	
	Level 1 – Some topics are studied in considerable detail. Moderate levels of pre-requisite knowledge are required in order to grasp ideas in some topics. The level of cognitive complexity of the information students are expected to know is low to moderate (e.g. many tasks may require engagement of some mental processing beyond habitual responses, including comparison and basic reasoning – knowledge application).	
	Level 2 – Most topics are studied in considerable detail. Considerable pre-requisite knowledge is required in order to grasp ideas in some topics. The level of cognitive complexity of the information students are expected to know is average to high (e.g. some tasks require complex reasoning, planning, using evidence, and a higher level of thinking than the previous two	

[134] Evidence pool: Learning outcomes
[135] Evidence pool: Learning outcomes, subject content, assessment types

Demand Judgement	Score Descriptors (highlight the best-fit descriptor)	Judgement and Key Evidence
	levels. The cognitive demands are often complex and abstract – analysis).	
	Level 3 – All or most topics are studied in very high detail. Considerable pre-requisite knowledge is required in order to grasp ideas in most topics. The level of cognitive complexity of information students are expected to know is mostly high (e.g. many tasks may require complex reasoning, planning, developing, information synthesis, interpretation of data for problem solving, and thinking most likely over an extended period of time – extended thinking).	
Volume of work[136]	Level 0 – light: small number of themes and sub-themes covered; a significant majority of time is spent on straightforward or basic themes; generous time allocation per theme.	
	Level 1 – moderate: typical number of themes and sub-themes covered; more time spent on conceptually complex themes compared to Level 1 (though majority of time still spent on themes of basic depth); standard time allocation per theme.	
	Level 2 – moderate heavy: typical to high number of themes and sub-themes covered; a significant proportion of time spent on issues beyond basic conceptual depth; standard to short time allocation per theme.	
	Level 3 – heavy: high number of themes and sub-themes covered; a large proportion of time spent on issues beyond basic conceptual depth; short time allocation per theme.	
Outstanding areas of subject demand[137]	Level 0 – no stretch areas (0)	
	Level 1 – few stretch areas (1-2)	
	Level 2 – a significant number of stretch areas (3-4)	
	Level 3 – a high number of stretch areas (>4)	

[136] Evidence pool: Subject content; assessment types and number; course duration; time allocated per topic/sub-topic (where available).
[137] Evidence pool: Subject content.

Appendix D

This appendix displays the BNCC's specific competencies and specific skills for Mathematics and Technology in high school.

Table 80: Specific competencies and specific skills for Mathematics and Technology.

1. Use mathematical strategies, concepts and procedures to interpret situations in different contexts, whether daily activities, facts from Natural and Human Sciences, socioeconomic or technological issues, disseminated through different media, in order to contribute to general training.
(EM13MAT101) Critically interpret economic and social situations and facts related to Natural Sciences that involve the variation of quantities, by analyzing the graphs of the represented functions and the rates of variation, with or without the support of digital technologies.
(EM13MAT102) Analyze tables, graphs and samples of statistical research presented in reports published by different media, identifying, when applicable, inadequacies that could lead to interpretation errors, such as inappropriate scales and samples.
(EM13MAT103) Interpret and understand scientific texts or those published by the media, which use units of measurement of different magnitudes and the possible conversions between them, whether adopted or not by the International System (SI), such as those of storage and data transfer speed, linked to technological advances .
(EM13MAT104) Interpret rates and indices of a socioeconomic nature (human development index, inflation rates, among others), investigating the calculation processes of these numbers, to critically analyze reality and produce arguments.
(EM13MAT105) Use the notions of isometric transformations (translation, reflection, rotation and compositions thereof) and homothetic transformations to build figures and analyze elements of nature and different human productions (fractals, civil constructions, works of art, among others).
(EM13MAT106) Identify situations in everyday life in which it is necessary to make choices taking into account probabilistic risks (using this or that contraceptive method, opting for one medical treatment over another, etc.).
2. Propose or participate in actions to investigate challenges in the contemporary world and make ethical and socially responsible decisions, based on the analysis of social problems, such as those related to health situations, sustainability, the implications of technology in the world of work, among others, mobilizing and articulating concepts, procedures and languages specific to Mathematics.
(EM13MAT201) Propose or participate in actions suited to the demands of the region, preferably for your community, involving measurements and calculations of perimeter, area, volume, capacity or mass.
(EM13MAT202) Plan and carry out sample research on relevant issues, using data collected directly or from different sources, and communicate the results through a report containing graphs and interpretation of measures of central tendency and measures of dispersion (amplitude and standard deviation), using or not technological resources.
(EM13MAT203) Apply mathematical concepts in the planning, execution and analysis of actions involving the use of applications and the creation of spreadsheets (for family budget control, simple and compound interest calculation simulators, among others), to make decisions.

3. Use strategies, concepts, definitions and mathematical procedures to interpret, build models and solve problems in different contexts, analyzing the plausibility of results and the adequacy of proposed solutions, in order to build consistent arguments.
(EM13MAT301) Solve and elaborate everyday problems, in Mathematics and other areas of knowledge, which involve simultaneous linear equations, using algebraic and graphical techniques, with or without the support of digital technologies.
(EM13MAT302) Build models using 1st or 2nd degree polynomial functions, to solve problems in different contexts, with or without the support of digital technologies.
(EM13MAT303) Interpret and compare situations involving simple interest with those involving compound interest, through graphical representations or spreadsheet analysis, highlighting the linear or exponential growth in each case.
(EM13MAT304) Solve and elaborate problems with exponential functions in which it is necessary to understand and interpret the variation of the quantities involved, in contexts such as Financial Mathematics, among others.
(EM13MAT305) Solve and elaborate problems with logarithmic functions in which it is necessary to understand and interpret the variation of the quantities involved, in contexts such as earthquakes, pH, radioactivity, Financial Mathematics, among others.
(EM13MAT306) Solve and elaborate problems in contexts involving real periodic phenomena (sound waves, phases of the moon, cyclical movements, among others) and compare their representations with the sine and cosine functions, in the Cartesian plane, with or without the support of algebra and geometry applications.
(EM13MAT307) Use different methods to obtain the measurement of the area of a surface (reconfigurations, approximation by cuts, etc.) and deduce calculation expressions to apply them in real situations (such as the relocation and distribution of plantations, among others), with or without the support of digital technologies.
(EM13MAT308) Apply metric relationships, including the laws of sine and cosine or the notions of congruence and similarity, to solve and elaborate problems involving triangles, in various contexts.
(EM13MAT309) Solve and develop problems involving the calculation of total areas and volumes of prisms, pyramids and round bodies in real situations (such as calculating the cost of material for coating or painting objects whose shapes are compositions of the solids studied), with or without support of digital technologies.
(EM13MAT310) Solve and elaborate counting problems involving sortable or non-orderable groupings of elements, using multiplicative and additive principles, using different strategies, such as tree diagrams.
(EM13MAT311) Identify and describe the sample space of random events, counting possibilities, to solve and elaborate problems involving the calculation of probability.
(EM13MAT312) Solve and elaborate problems that involve calculating the probability of events in successive random experiments.
(EM13MAT313) Use, when necessary, scientific notation to express a measurement, understanding the notions of significant figures and doubtful figures, and recognizing that every measurement is inevitably accompanied by error.
(EM13MAT314) Solve and elaborate problems that involve quantities determined by the ratio or product of others (speed, population density, electrical energy, etc.).
(EM13MAT315) Investigate and record, through a flowchart, when possible, an algorithm that solves a problem.

(EM13MAT316) Solve and elaborate problems, in different contexts, that involve calculation and interpretation of central tendency measures (mean, mode, median) and dispersion measures (amplitude, variance and standard deviation).
4. Understand and use, with flexibility and precision, different mathematical representation registers (algebraic, geometric, statistical, computational, etc.), in the search for solutions and communication of problem results.
(EM13MAT401) Convert algebraic representations of 1st degree polynomial functions into geometric representations in the Cartesian plane, distinguishing the cases in which the behavior is proportional, whether or not using algebra and dynamic geometry software or applications.
(EM13MAT402) Convert algebraic representations of 2nd degree polynomial functions into geometric representations in the Cartesian plane, distinguishing the cases in which one variable is directly proportional to the square of the other, whether or not using algebra and dynamic geometry software or applications, among other materials.
(EM13MAT403) Analyze and establish relationships, with or without the support of digital technologies, between the representations of exponential and logarithmic functions expressed in tables and in a Cartesian plane, to identify the fundamental characteristics (domain, image, growth) of each function.
(EM13MAT404) Analyze functions defined by one or more sentences (Income Tax table, electricity bills, water, gas, etc.), in their algebraic and graphical representations, identifying domains of validity, image, growth and decrease, and converting these representations into a to another, with or without the support of digital technologies.
(EM13MAT405) Use initial concepts of a programming language to implement algorithms written in common language and/or mathematics.
(EM13MAT406) Build and interpret frequency tables and graphs based on data obtained in research by statistical samples, including or not the use of software that interrelates statistics, geometry and algebra.
(EM13MAT407) Interpret and compare statistical data sets through different diagrams and graphs (histogram, box-plot, branches, and leaves, among others) others), recognizing the most efficient ones for their analysis.
5. Investigate and establish conjectures regarding different concepts and mathematical properties, employing strategies and resources, such as observation of patterns, experiments and different technologies, identifying the need, or not, for an increasingly formal demonstration in validating said conjectures.
(EM13MAT501) Investigate relationships between numbers expressed in tables to represent them in the Cartesian plane, identifying patterns and creating conjectures to generalize and algebraically express this generalization, recognizing when this representation is a 1st degree polynomial function.
(EM13MAT502) Investigate relationships between numbers expressed in tables to represent them in the Cartesian plane, identifying patterns and creating conjectures to generalize and algebraically express this generalization, recognizing when this representation is a 2nd degree polynomial function of the type y = ax2.
(EM13MAT503) Investigate maximum or minimum points of quadratic functions in contexts involving surfaces, Financial Mathematics or Kinematics, among others, with the support of digital technologies.
(EM13MAT504) Investigate processes for obtaining the measurement of the volume of prisms, pyramids, cylinders and cones, including Cavalieri's principle, to obtain formulas for calculating the measurement of the volume of these figures.
(EM13MAT505) Solve problems about tiling the plane, with or without the support of dynamic geometry applications, to conjecture about the types or composition of polygons that can be used in tiling, generalizing observed patterns.
(EM13MAT506) Graphically represent the variation in the area and perimeter of a regular polygon when the lengths of its sides vary, analyzing and classifying the functions involved.

(EM13MAT507) Identify and associate arithmetic progressions (AP) with related functions in discrete domains, for analyzing properties, deducing some formulas and solving problems.
(EM13MAT508) Identify and associate geometric progressions (PG) with exponential functions of discrete domains, for analyzing properties, deducing some formulas and solving problems.
(EM13MAT509) Investigate the deformation of angles and areas caused by different projections used in cartography (such as cylindrical and conical), with or without the support of digital technology.
(EM13MAT510) Investigate sets of data relating to the behavior of two numerical variables, using or not using information technologies, and, when appropriate, take variation into account and use a straight line to describe the observed relationship.
(EM13MAT511) Recognize the existence of different types of sample spaces, discrete or not, and events, equiprobable or not, and investigate implications in the calculation of probabilities.