M06/5/MATME/SP2/ENG/TZ2/XX

22067304

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 4 May 2006 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 16]*

Let S_n be the sum of the first n terms of the arithmetic series $2+4+6+\dots$.

(a) Find

(i) S_4;

(ii) S_{100}. *[4 marks]*

Let $\boldsymbol{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(b) (i) Find $\boldsymbol{M}^2$.

(ii) Show that $\boldsymbol{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$. *[5 marks]*

It may now be assumed that $\boldsymbol{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$, for $n \ge 4$. The sum $\boldsymbol{T}_n$ is defined by

$$\boldsymbol{T}_n = \boldsymbol{M}^1 + \boldsymbol{M}^2 + \boldsymbol{M}^3 + \dots + \boldsymbol{M}^n.$$

(c) (i) Write down $\boldsymbol{M}^4$.

(ii) Find $\boldsymbol{T}_4$. *[4 marks]*

(d) Using your results from part (a) (ii), find $\boldsymbol{T}_{100}$. *[3 marks]*

2. *[Maximum mark: 18]*

Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$.

(a) Find the inverse function, f^{-1}. *[3 marks]*

(b) Given that $g^{-1}(x) = x + 2$, find $(g^{-1} \circ f)(x)$. *[2 marks]*

(c) Given also that $(f^{-1} \circ g)(x) = \dfrac{x+3}{3}$, solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$. *[2 marks]*

Let $h(x) = \dfrac{f(x)}{g(x)}$, $x \neq 2$.

(d) (i) **Sketch** the graph of h for $-3 \leq x \leq 7$ and $-2 \leq y \leq 8$, including any asymptotes.

(ii) Write down the **equations** of the asymptotes. *[5 marks]*

(e) The expression $\dfrac{3x-5}{x-2}$ may also be written as $3 + \dfrac{1}{x-2}$. Use this to answer the following.

(i) Find $\int h(x)\,\mathrm{d}x$.

(ii) **Hence**, calculate the **exact** value of $\int_3^5 h(x)\,\mathrm{d}x$. *[5 marks]*

(f) On your sketch, shade the region whose area is represented by $\int_3^5 h(x)\,\mathrm{d}x$. *[1 mark]*

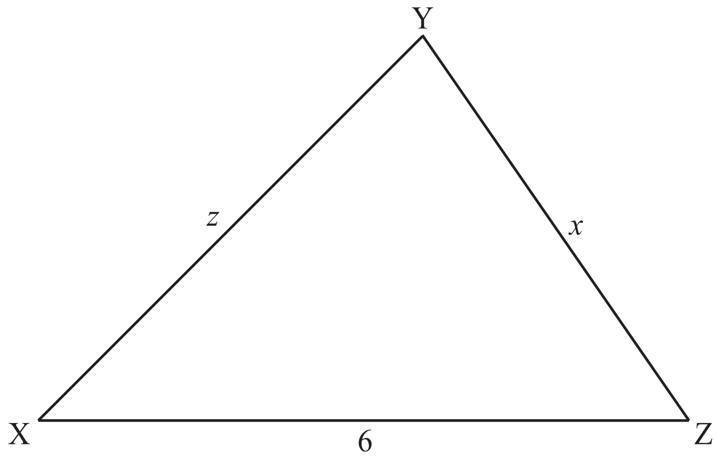
3. *[Maximum mark: 20]*

(a) Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

(i) the value of x giving the maximum value of y;

(ii) this maximum value of y. *[4 marks]*

The triangle XYZ has $\text{XZ} = 6$, $\text{YZ} = x$, $\text{XY} = z$ as shown below. The perimeter of triangle XYZ is 16.



(b) (i) Express z in terms of x.

(ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$.

(iii) Hence, show that $\cos Z = \dfrac{5x - 16}{3x}$. *[7 marks]*

Let the area of triangle XYZ be A.

(c) Show that $A^2 = 9x^2 \sin^2 Z$. *[2 marks]*

(d) Hence, show that $A^2 = -16x^2 + 160x - 256$. *[4 marks]*

(e) (i) Hence, write down the maximum area for triangle XYZ.

(ii) What type of triangle is the triangle with maximum area? *[3 marks]*

4. *[Maximum mark: 17]*

In a large school, the heights of all fourteen-year-old students are measured.

The heights of the girls are normally distributed with mean 155 cm and standard deviation 10 cm.

The heights of the boys are normally distributed with mean 160 cm and standard deviation 12 cm.

(a) Find the probability that a girl is taller than 170 cm. *[3 marks]*

(b) Given that 10% of the girls are shorter than $x\text{ cm}$, find x. *[3 marks]*

(c) Given that 90% of the boys have heights between $q\text{ cm}$ and $r\text{ cm}$ where q and r are symmetrical about 160 cm, and $q < r$, find the value of q and of r. *[4 marks]*

In the group of fourteen-year-old students, 60% are girls and 40% are boys.
The probability that a girl is taller than 170 cm was found in part (a).
The probability that a boy is taller than 170 cm is 0.202.

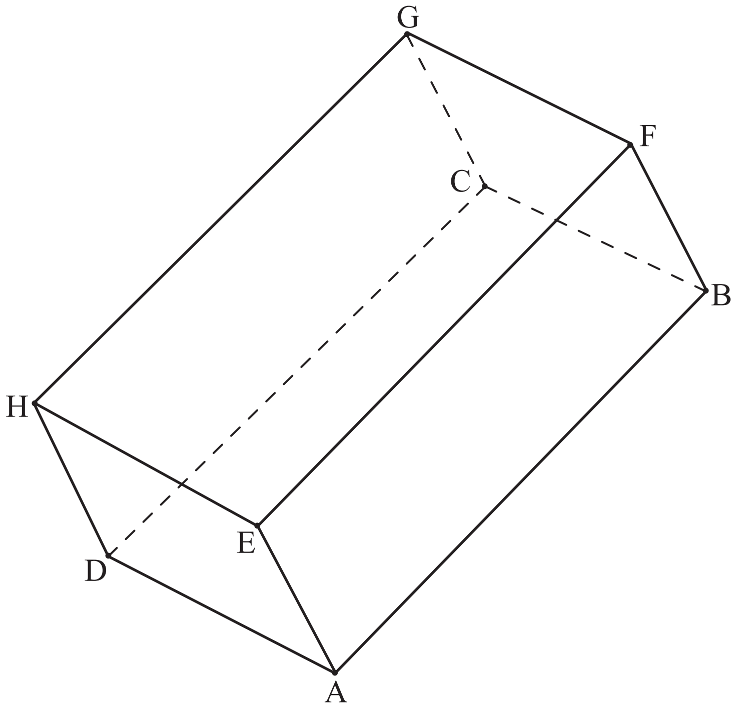
A fourteen-year-old student is selected at random.

(d) Calculate the probability that the student is taller than 170 cm. *[4 marks]*

(e) Given that the student is taller than 170 cm, what is the probability the student is a girl? *[3 marks]*

5. *[Maximum mark: 19]*

The following diagram shows a solid figure ABCDEFGH. Each of the six faces is a parallelogram.



The coordinates of A and B are $A(7, -3, -5)$, $B(17, 2, 5)$.

(a) Find

(i) $\overrightarrow{AB}$;

(ii) $\left|\overrightarrow{AB}\right|$. *[4 marks]*

(This question continues on the following page)

(Question 5 continued)

The following information is given.

$$\vec{AD}=\begin{pmatrix}-6\\6\\3\end{pmatrix},\ \left|\vec{AD}\right|=9,\ \vec{AE}=\begin{pmatrix}-2\\-4\\4\end{pmatrix},\ \left|\vec{AE}\right|=6$$

(b) (i) Calculate $\vec{AD}\cdot\vec{AE}$.

(ii) Calculate $\vec{AB}\cdot\vec{AD}$.

(iii) Calculate $\vec{AB}\cdot\vec{AE}$.

(iv) Hence, write down the size of the angle between any two intersecting edges. *[5 marks]*

(c) Calculate the volume of the solid ABCDEFGH. *[2 marks]*

(d) The coordinates of G are $(9, 4, 12)$. Find the coordinates of H. *[3 marks]*

(e) The lines (AG) and (HB) intersect at the point P.

Given that $\vec{AG}=\begin{pmatrix}2\\7\\17\end{pmatrix}$, find the acute angle at P. *[5 marks]*